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Strain-hardening effects in reinforced concrete*

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In this contribution, I would like to comment on the following aspects of the paper under discussion: (1) the value of ε_u adopted; (2) the assumed stress-strain relationship for concrete in compression; (3) the way rotation was calculated over the 356 mm gauge length of beams; (4) Figure 8; and (5) the conclusion regarding the energy absorption capacities of structures reinforced with steels having different lengths of the yield range. I would also like to present some results of an independent investigation into the effects of strain-hardening of steel upon the behaviour of reinforced concrete sections.

It is usually recognized that a proper definition of the parameter ε_u, ultimate strain, is important in the development of a rational theory for reinforced concrete sections subject to flexure. The value adopted for this parameter has a decisive influence upon the deformational characteristics of reinforced concrete sections and members, although its effects upon strength are rather insignificant. The authors implicitly adopted the conventional definition of ε_u (strain at which concrete on the compression side of a beam begins to crush) but assumed the highly unconventional value of $\varepsilon_u = 0{\cdot}7\%$. The fact that spalling of the 75 × 150 mm cylinders accompanying their test beams occurred at an average strain of 0·7% does not necessarily mean that the compression concrete in the test beams would crush at the same strain value. Besides, the ε_m values reported are very high in comparison with the usual value of around 0·2%[1]. Although the values of ε_u cannot be directly compared because of the arbitrary nature of the definition of this parameter, they are very likely to be high when the values of ε_m are high. The authors did verify, by strain measurements on the surface of the compression concrete in one test series, that the value of 0·7% for crushing strain was reasonable. Figure 10 also shows that crushing of the compression concrete in the test beams could be predicted reasonably well on the basis of $\varepsilon_u = 0{\cdot}7\%$. There is, however, a subjective element in these verifications. The points indicating the crushing of concrete in Figure 10 were defined on the basis of visual observation. The verification by strain measurements would be somewhat more reliable if decreases in the concrete strains were observed at around a strain value of 0·7%. Such decreases should result from a release of strains on the gauges during crushing[2]. But, even in these cases, the measured values of the crushing strain would depend upon the position of the gauge in relation to the section at which crushing starts.

The most satisfactory definition of ε_u so far has been the one suggested by Rüsch[2]. He defines ε_u as the extreme compression fibre strain at which a reinforced concrete section reaches its maximum load- or moment-carrying capacity. This definition is very logical if it is accepted that the primary function of a member or a structure is to carry loads. Unlike the conventional definition of ε_u, Rüsch's definition has a mathematical significance. For given axial load, ε_u corresponds to the peak of the curve relating moment and extreme compression fibre strain. The effects of such factors as lateral reinforcement, strain gradient and so on can also be taken into account when ε_u is defined according to Rüsch. This is an improvement over the conventional practice of assigning a constant value to ε_u.

The stress-strain curve for concrete in compression adopted in the paper is not quite realistic. The authors noted that the assumed concrete behaviour had relatively little influence upon sectional moment-curvature relationships. But this is true only as long as sections are relatively lightly reinforced. The authors were mainly concerned with lightly reinforced sections, since strain-hardening effects upon highly reinforced sections are not very pronounced. The assumed shape of the concrete stress-strain relationship was, therefore, immaterial within the limited range of the investigation. The assumption of an unrealistic σ—ε curve for concrete, however, involved a loss of generality. To be more explicit, the effects of such factors as tie spacing, duration of loading and so on, which have more pronounced influences upon over-reinforced than upon under-reinforced sections, cannot be

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assessed realistically by using the σ—ε curve for concrete adopted in the paper.

In calculating rotation over the 356 mm gauge length of a beam, the authors multiplied the plastic portion of the peak curvature by a yield length equal to the effective depth of the beam. It would be of interest to know what led to the decision to use this particular value of the yield length. When a beam has undergone large inelastic deformations and, therefore, extensive cracking, the integration of curvatures over a section of the beam would yield a value of rotation that is less than the actual rotation over that beam section. The authors' approximate method leads to rotation values that are less even than those obtained by integration of curvatures. This would seem to indicate that the approximate method is unrealistically conservative at high levels of deformation.

It would normally be expected that the difference between the 'ACI ultimate moment' and the maximum moment predicted by the method used in the paper would diminish as the reinforcement content of sections increases and the strain-hardening effects become less pronounced. Figure 8 indicates, however, that this difference increases as the steel ratio, p, goes up from 0·77 to 1·19%. Is this due to the fact that the strain-hardening modulus of the steel used in beam series 2 was larger than that of the steel in series 1? It may be for the same reason that the authors' method predicted almost equal reserve strength over the ACI ultimate moment for critical sections with $p = 1{\cdot}69$ and 2·30%. Figure 8 also indicates that the predicted maximum moments are larger than the observed maximum moments for three out of six critical sections in test series 1 ($p = 0{\cdot}77\%$) and 4 ($p = 2{\cdot}30\%$). For all the sections in test series 2 ($p = 1{\cdot}19\%$), predicted strength values are higher than the observed ones. The predicted and observed values nearly converge for the sections in series 3 ($p = 1{\cdot}69\%$). Whether the authors' method would overestimate or underestimate sectional strength does not, therefore, depend upon the steel ratio, p. It is also interesting to note that the experimental maximum moments of two sections in series 4 were less even than the calculated plastic moments, whereas three other sections reached or exceeded the predicted maximum moments.

Figure 10 indicates that the authors' theory leads to an underestimation of the deformational capacity of a ductile structure. This capacity is better estimated as the steel ratio increases and the structure becomes less ductile. The energy absorption capacity of ductile structures would, therefore, be underestimated and that of brittle and semi-brittle structures would be more or less accurately estimated, when this theory is used. Considering this, one would hesitate to accept without qualifications the conclusion that a decrease in the length of the yield range of reinforcing steel does not necessarily lead to a reduction in the energy absorption capacity of a structure.

A comprehensive investigation into the inelastic behaviour of reinforced concrete structures is now in progress at the University of Waterloo under the direction of Professor M. Z. Cohn. The effects of strain-hardening upon structural response are being investigated, together with the effects of numerous other factors. The presentation of a few results of this investigation may be of interest here. Figure I shows the theoretical moment-curvature relationships for three singly reinforced concrete sections under pure flexure, identical in all respects except that they are reinforced with steels having different strain-hardening moduli. All three steels have the same modulus of elasticity, yield strength and strain at the onset of hardening. The assumed concrete and steel stress-strain relationships and the method by which the M-φ curves are derived have been described elsewhere[3]. Figures II to IV show the effects of different strain-

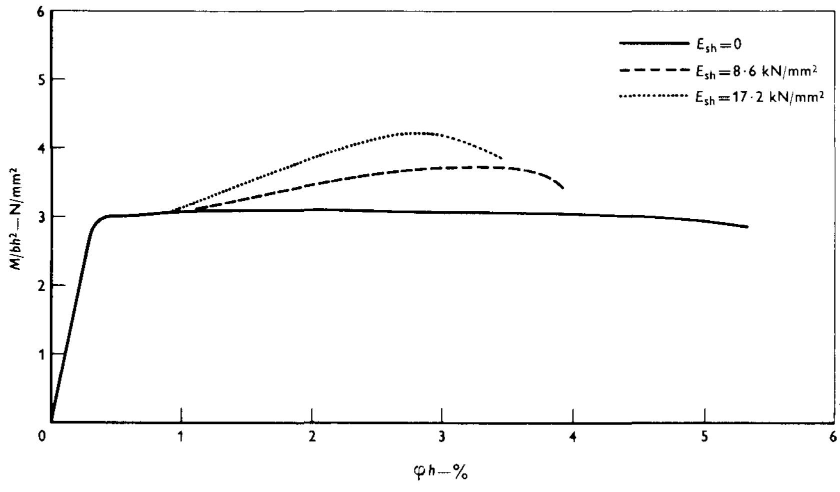

Figure I: Moment-curvature diagrams for different strain-hardening moduli.

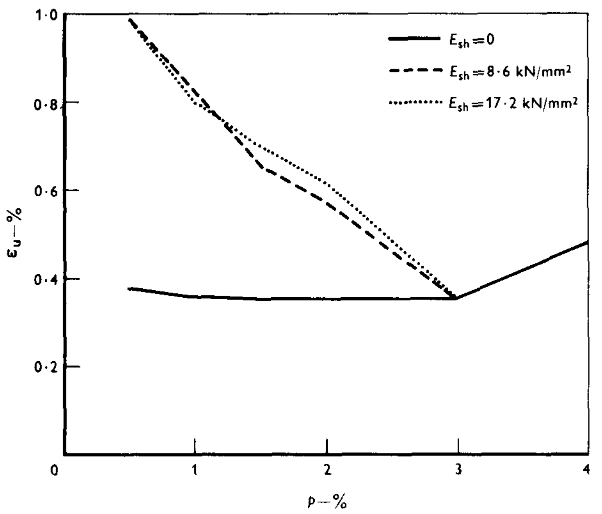

Figure II: Variation of ultimate strain with steel ratio for different strain-hardening moduli.

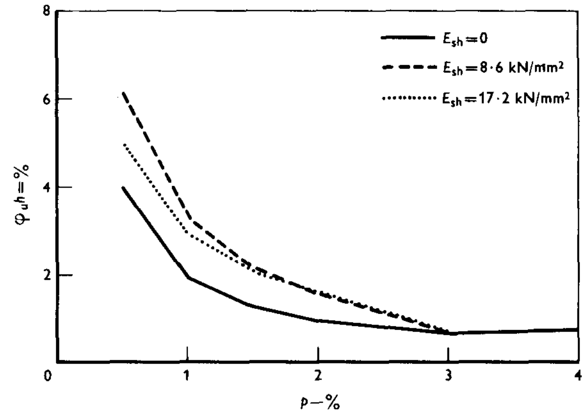

Figure IV: Variation of ultimate curvature with steel ratio for different strain-hardening moduli.

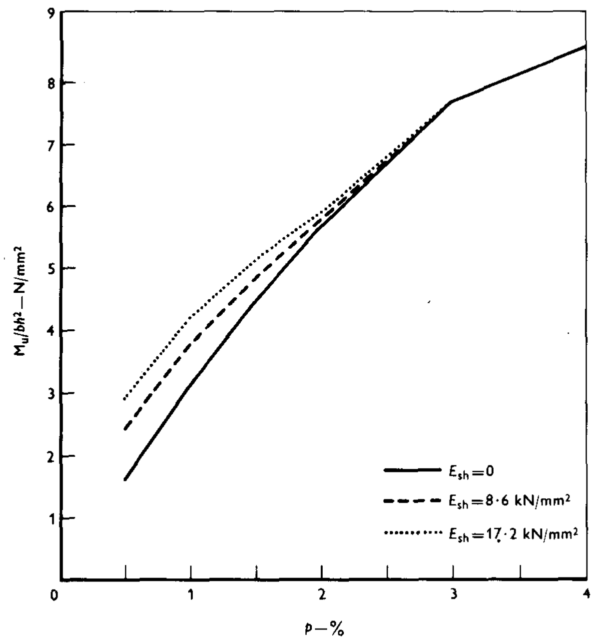

Figure III: Variation of ultimate moment with steel ratio for different strain-hardening moduli.

hardening moduli upon the ultimate strain, the ultimate moment and the corresponding curvature, respectively, for different percentages of tensile reinforcement. It should be mentioned that the ultimate stage is defined, according to Rüsch[1], as the stage at which a section reaches its maximum moment-carrying capacity. M_u is this maximum moment; φ_u and ε_u are the curvature and the extreme compression fibre strain, respectively, corresponding to this moment. In Figures III and IV, h is the over-all depth of the section. Figure I indicates that the sectional strength increases and the ductility decreases somewhat with increasing strain-hardening moduli of the reinforcing steel. Figures II to IV illustrate clearly that the effects of strain-hardening upon highly reinforced sections are negligible. Figures II and IV tend to indicate that after E_{sh} has reached a certain value, further increases in the value of this parameter do not have any significant influence on ε_u or φ_u. The authors' comments on the results presented in Figures I to IV would be of interest.

Reply by the authors

We are grateful to Mr Ghosh for his discussion of some of the assumptions and results of our paper, which attempted to explain some observed results in as simple and straightforward manner as possible. The points which he has raised will be answered one at a time.

The value $\varepsilon_u = 0{\cdot}007$ in/in is unlikely to be obtained in a conventional cylinder test under uniform strain. However, owing to the strain gradient which is present in the compression zone of a beam under high moment, the most highly strained extreme fibres get sufficient support from the inner, less deformed fibres so that such strain values can well be reached; this was borne out by tests of Clark, Gerstle and Tulin[4]. The longitudinal moment gradient also contributes to the ductility of the compression concrete, as, for instance, predicted by the well-known formulas of Corley and Mattock (given in references 7 and 4 of the paper). If Mattock's recommendation is used, the predicted value of ε_u would be 0·01226, whilst Corley's evaluation would be 0·0075.

The argument concerning the magnitude of ε_m is long-standing. Desayi and Krishman[5] suggested its value as 0·2%, but we at that time in a discussion[6] offered experimental evidence indicating observations of ε_m in excess of 0·3% for concrete containing local Colorado aggregates. The larger deformations may be the result of coarseness of the local sands, but the reasons not withstanding, $\varepsilon_m = 0{\cdot}3\%$ is not an unreasonable value. These observations were later corroborated by Clark et al.[4] in 1967.

A reasonably successful attempt was made to verify the assumed value of ε_u. Mr Ghosh himself points out the difficulty of exact measurement due to the concentrated nature of this strain which depends, among other factors, upon the way in which the load is applied. It is also well known that the rotation capacity is strongly affected by lateral reinforcing, but it was not within the scope of this work to study this effect; no ties were provided within a distance of $d/2$ of the critical section.

The use of Rüsch's definition of ε_u as the fibre strain corresponding to the maximum moment has two disadvantages for the present study:

(1) it does not represent a pure material property which would seem desirable from a structural mechanics viewpoint;

(2) as borne out by the moment-rotation curves of Mr Ghosh's Figure I, the maximum moment can occur long before crushing takes place, depending upon the assumed shape of the concrete stress-strain curve, so, in the curve for $E_{sh} = 0$, the maximum moment appears under a rotation of only about two-fifths of the ultimate value—it is obvious that, under this definition, ε_u would have to have a lower value than 0·007 to yield realistic values for the ductility of the beam.

Associated with the question of ultimate strain is the corresponding length over which plastification is assumed, since the real quantity of physical interest is the total hinge rotation. As a matter of fact, it could well be argued that, because of the discontinuous nature of the strain field in the heavily cracked hinging region, any assumed average strain value is a fiction, and will only lead to useful, if approximate, results when used in conjunction with a suitable plastic length. In our case, the 356 mm gauge length was chosen because all of the inelastic behaviour appeared to occur within this region, with the major cracks occurring within a length equal to the beam depth. Thus, the choice of the plastic hinge length seemed to us, as it had to others (references 4 and 9 of the paper), to be in accordance with the physical facts, though it is certainly true that a lesser ultimate strain used in conjunction with a longer hinge length could lead to similar results.

Because of the discontinuities associated with crack formation in the hinging region, one could question the existence of continuous curvature and therefore of the method of integration of curvatures suggested by Mr Ghosh. Any judgement as to the validity of the moment-rotation relationships given should be based upon comparison with experiment, not upon a questionable theory. In this sense, we considered the linear strain-hardening moment-rotation relationships, such as shown in Figure 7, a reasonable compromise between reality and simplicity. The same comment holds for the assumed bi-linear concrete stress-strain curve, the use of which was documented earlier[6].

Mr Ghosh complains that the paper described a *limited* investigation. Any experimental study must of necessity be limited. This cannot be interpreted as criticism, but is the very nature of the experimental process, since choices concerning dimensions and proportions must be made specifically, with a resulting loss of generality. However, if experimental investigators withheld reports of their studies until all possible combinations had been examined, communications between researchers would come to a standstill.

Mr Ghosh's argument, however, that effects of tie spacing, duration of loading, etc., should be investigated, simply points out the fact that there is no shortage of topics for experimental study, but is not pertinent to a report on the effects of strain-hardening in steel because strain-hardening rarely, if ever, occurs in over-reinforced sections.

As already surmised by Mr Ghosh, the trends resulting from varying the steel ratios in Figures 8 and 11 are somewhat obscured by the different strain-hardening properties of the different sizes of reinforcing bar. The purpose of the tests was to verify the validity of the approach, and we consider that the results shown in these Figures do provide this verification. In general, the correlation appears well within the spread expected of concrete structures, so that Figures 8 and 11 may be considered evidence that the proposed approach furnishes adequate prediction about the ultimate capacity of concrete beams with strain-hardening steel.

The load-deflexion curves of Figure 10 indicate that the analytical piecewise-linear curves are, at least for all cases studied, well able to predict the onset of crushing. In discussing the ductility of actual structures, this appears to be a better criterion than the spalling associated with loss of cross-section, especially if, as in the case of earthquakes, reversal of loading may be expected. It was this reasoning which led to the conclusions regarding constancy of the energy absorption capacity.

The investigation at the University of Waterloo is of considerable interest to us, and we have no doubt that it will verify our conclusions regarding strain-hardening effects. It might be noted that almost half of the range of the steel ratios studied is beyond the values permitted by the American Concrete Institute. Also, the physical meaning of φ_u remains obscure to us. The value $\varphi_u h = 1{\cdot}9$ for $E_{sh} = 0$ certainly has no relation to the actual beam ductility. Similarly, both Mr Ghosh's Figure I and our own results show decreasing ductility with increasing strain-hardening modulus, but Mr Ghosh's Figure IV shows the opposite trend for φ_u.

In summary, we would like to point out that the purpose of this paper was to explain some observed phenomena of continuous concrete beams, and we believe that this purpose has been achieved. We shall be interested to see whether additional insight will be gained by refinement of the assumptions.

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