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Editorial comment

A CONFERENCE ON THE FUTURE of one major aspect of concrete research was held in September this year by West Virginia University in the USA. This was the second in a series of conferences, aimed at identifying future research needs, which it is intended should be held approximately every two years. The first was held in 1969 and dealt with the behaviour of structural concrete subjected to combined loadings. The subject of the conference this last September was the behaviour of concrete structural systems.

These conferences, conducted by the Department of Civil Engineering, West Virginia University, are sponsored jointly by the American Concrete Institute, American Society of Civil Engineers, Reinforced Concrete Research Council, Portland Cement Association, Prestressed Concrete Institute and National Science Foundation, and are held in a delightful lodge in excellent surroundings in a state park in West Virginia. This means to say that the participants are completely isolated for the period of the conference, and therefore can really get down to discussing the subjects in hand. The participants are research workers, practising engineers, and contractors. They are chosen by invitation, to ensure an appropriate balance between the diverse interests. A member of the Editorial Advisory Board took part in the conference this year, and it is this participation which has given rise to this Editorial comment.

The proceedings of each day are organized in the following way. During the morning, there is a presentation of what is intended to be a controversial and stimulating paper on some particular aspect of research. Following the initial presentation of this paper, there is a discussion on the whole subject in which all can take part. The discussion is not recorded but a reporter is appointed for each session. The afternoon is left free so that the participants can enjoy the surroundings and continue the discussions privately. Then in the evening a further session, following exactly the procedure of the morning session, is held on a new subject.

This year some sixty people attended, and apparently the conference stimulated a lot of heated discussion and debate, which is likely to have a considerable effect upon the future thoughts and aims of those attending. This approach is one that seems to be of considerable value in ensuring that the research needs of all the various parties in the construction industry are satisfactorily debated, and thereafter dealt with by all concerned. The findings from the West Virginia conference are being reported and made available to the various sponsoring bodies. These are bodies which have some responsibility for concrete research and therefore the proceedings from the conference can be used as guide-lines in future research planning and policy.

A seminar of this type seems to have a particular advantage in isolating those taking part from the outside world; this is something which is very difficult to achieve in this present age. Perhaps it is time that comparable conferences took place in this country, instead of the separate short meetings, lasting a matter of an hour or two, reached with difficulty through traffic jams, and sometimes prematurely quitted to attend the next meeting, which have tended to be the rule in this period so often said to be of great debate.

Concrete research news

from four Belgian Universities

Influence of long-term loading upon the behaviour of reinforced concrete beams

BACKGROUND AND SPONSORSHIP

This is part of a comprehensive research programme, which is being carried out under the auspices of the Nationaal Opzoekingscentrum der Burgerlijke Bouwkunde (National Civil Engineering Research Centre) and subsidized by the Fund for Collective Fundamental Research. The programme is being undertaken jointly by the following laboratories at four universities in Belgium:

Liège State University, Laboratoire des Constructions du Génie Civil, Director: Professor R. Baus

Public University of Brussels, Laboratoire des Constructions Civiles, Director: Professor P. Moenaert

Roman Catholic University of Louvain, Laboratorium voor Proeven op Materialen, Director: Professor C. G. Reyntjens

Ghent State University, Laboratorium Magnel voor Gewapend Beton, Director: Professor F. G. Riessauw

The subject of the whole programme (Research No. 547 of the Fund for Collective Fundamental Research) is the influence of long-term loading upon the behaviour and resistance of concrete constructions, reinforced or prestressed concrete structures, and of composite (steel-concrete) structures as well as of metal structures.

PRESENT STAGE

The first stage of the research now in process includes the study of the influence of long-term loading upon the behaviour of reinforced concrete beams. The number of parameters is limited to two: the percentage of reinforcement and the percentage of the service load. For this stage of the research, both concrete and steel quality are being kept practically constant (cube strength after 28 days = 350 kg/cm^2; hard-grade deformed steel bars of 14 mm diameter with warranted yield point 40 kg/cm^2).

The beams being tested, which all have the dimensions $l = 3{\cdot}2$ m, $b = 150$ mm and $h_t = 280$ mm, vary in reinforcement as follows.

Type I: 0·82% tensile reinforcement (Brussels)
Type II: 2·17% tensile reinforcement (Ghent)
Type III: 3·52% tensile reinforcement (Liège)
Type IV: 2·17% tensile reinforcement and 1·23% compression reinforcement (Louvain)

On each type of beam six different tests are made, and each test is carried out twice. The six tests are as follows.

(1) A static test up to rupture (equal concentrated loads at each third-point of the 2·80 m span).
(2) A shrinkage test on an unloaded beam.
(3) Four creep tests. The applied load is either a fraction of the failure load P_r determined by the static test or the service load P_{serv} of the beam, determined by ultimate load analysis (CEB method). Thus the four loads will be $0{\cdot}9P_r$, $0{\cdot}8P_r$, $0{\cdot}7P_r$ and P_{serv}.

During the various tests, the following measurements are made:

(1) deflection (at mid-span and under the loads).
(2) concrete deformation (central cross-section, top and bottom fibre).
(3) the width of a certain number of cracks.

NEXT STAGE

The tests described above have recently been in progress. A second stage of the research programme is contemplated, namely a study of the influence of long-term loading upon the behaviour of prestressed concrete beams. The programme for this is now being studied.

CONTACTS

Communications regarding this programme at the four Belgian Universities may be addressed to:

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The bearing strength of reinforced concrete blocks under concentrated loading

PREVIOUS WORK ON UNREINFORCED BLOCKS

In many structures concentrated loading occurs. In 1969 an investigation of the strength of unreinforced concrete blocks under concentrated loading was carried out, 135 blocks being tested. The variables in this investigation were the dimensions of the blocks and the area over which the concentrated loading was applied.

PRESENT INVESTIGATION

Reinforced concrete is used to increase the bearing capacity under concentrated loading. An investigation of the most suitable reinforcement is being carried out at the moment. In the first stage of this investigation, the variables are:

(1) area over which the concentrated loading is applied;
(2) type of reinforcement: stirrups, spiral or mat;
(3) position of the reinforcement;
(4) amount of reinforcement;
(5) distribution of the reinforcement.

In the first stage, the concrete blocks have constant dimensions of 200 × 200 × 500 mm. The quality of the concrete is K350 (cube strength in kg/cm^2 at 28 days). The quality of the steel is Qr 24 (minimum yield strength = 2400 kg/cm^2). Both cracking load and ultimate load are being measured.

In the second stage, the variables will be:

(1) area over which the loading is applied;
(2) position of loading (symmetrical or asymmetrical);
(3) external and internal (cone) loading;
(4) type of reinforcement;
(5) amount of reinforcement.

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Behaviour of reinforced concrete slender columns under sustained load

REASON FOR RESEARCH

The creep of concrete is very important for the stability of structures. The deformation of eccentrically loaded columns is increased by creep. Because of this, the ultimate load will be reduced. To complement a method of calculation developed by CUR-TNO, this phenomenon is being investigated in a programme which began in May 1971.

PRESENT PROGRAMME

The following factors are varied:

(1) slenderness (length/side) = 25·7, 15·7, 8 and 4;
(2) reinforcement (Qr 40) on each face: 0·2%, 0·4%, 0·8% and 1·6%;
(3) eccentricity
    (a) symmetrical single curvature at eccentricities of 10, 30 50 and 100 mm;
    (b) symmetrical double curvature at eccentricities of 10, 30, 50 and 100 mm;
(4) load P = 55%, 70% and 85% of the short-term strength.

The cross-section of the specimens is 150 × 150 mm and the stirrup reinforcement is of 4 mm diameter steel (Qr 24) at 100 mm spacing. The quality of the concrete is K 300. The investigation will be done in a room with a relative humidity of 60% and a temperature of 20°C.

The programme of measurements is as follows:

(1) horizontal displacements;
(2) compression and tension strains;
(3) position and width of cracks;
(4) the duration of the test.

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Behaviour of lightweight concrete under uniaxial loading

REASON FOR RESEARCH

In the past four years an investigation has been carried out into the behaviour of normal concrete under biaxial loading. To investigate whether there is any difference in behaviour in deformation and rupture between normal and lightweight concrete and also to investigate the consequences of this for our structures, a similar programme on various types of lightweight concrete has been planned.

PRESENT PROGRAMME

A series of uniaxial tests is being carried out to study the influence of the following factors:

(1) quality of concrete: K 200 and K 325 (cube strength in kg/cm^2 at 28 days);
(2) type of aggregate: Argex, Argex S, Hollith, Korlin;
(3) rate of stressing $\partial\sigma/\partial t$ = 200, 20, 2, 0·2 and 0·02 kg/cm^2 s;
(4) rate of deformation:
    (a) $\partial\varepsilon_1/\partial t$ = 1000, 100, 10, 1 and 0·1 × 10^{-6}/s;
    (b) $\partial\varepsilon_2/\partial t$ = 200, 20, 2 and 0·2 × 10^{-6}/s;
(5) uniaxial compression and uniaxial tension.

The dimensions of the specimens are 100 × 100 × 300 mm.

From the measurements taken, the following diagrams will be prepared:

$$\sigma/\varepsilon_1,\ \sigma/\varepsilon_2,\ \sigma_1/\nu \text{ and } \sigma_1/(\varepsilon_1 + 2\varepsilon_2)$$

In the above,

σ = stress in kg/cm^2
ε_1 = longitudinal strain
ε_2 = transverse strain
$\nu = \left|\dfrac{\varepsilon_2}{\varepsilon_1}\right|$
t = time in seconds

FUTURE WORK

After the results of the uniaxial tests have been studied, a programme for a series of biaxial tests will be planned.

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Thermodynamics of sorption in porous building materials: measurement of sorption isosteres

BACKGROUND

Surface thermodynamic data are indispensable in a scientific approach to the technology of porous building materials. Almost all the important technical and physical properties of these porous—sometimes microporous—substances are influenced significantly by the nature of their porous texture. This porous texture is characterized in terms of pore statistical parameters (e.g. pore size distribution functions), as well as the nature of the internal surface. Both of these structure parameters can be obtained from sorption data.

PRESENT PHASE OF RESEARCH PROGRAMME

The material system that is being investigated at the present time, in this continuing research programme at the Building Materials Laboratory of the Technical University of Denmark, is the system hardened-cement-paste-water. Sorption isosteres (vapour pressure as a function of temperature, at constant moisture content) are measured directly; sorption isotherms are generated from the isosteres; thermodynamic data (such as entropy and enthalpy of sorption) are computed.

APPARATUS AND MEASUREMENTS

The experimental techniques for a vacuum apparatus, in which sorption isosteres will be measured automatically, were developed during a pilot programme. The isosteres obtained with the help of this apparatus were measured by a dynamic procedure: the temperature was changed 10°C every hour and the pressure was read just before the temperature change. The extent of hysteresis between cooling and heating curves is a measure of departure from equilibrium.

That we can obtain isosteres dynamically is of great practical importance. It means that this mode of the operation can be automated, and equilibrium can be approached as closely as desired by adjusting the heating and cooling rates. A new apparatus operating on these principles has been designed and is being assembled. The major features of this apparatus are as follows:

(1) pressure range $\hat{=} 10^{-4}$ to 300 Torr;
(2) temperature range $\hat{=} -30$ to $+150$°C;
(3) automatically controlled heating and cooling rates;
(4) high precision of temperature control and read-out;
(5) high precision of pressure read-out;
(6) digital recording of temperature and pressure.

The temperature and pressure ranges could probably be extended to ≏ −200°C and 10^{-6} Torr, respectively. Investigations at very low pressures are particularly interesting, since in this region adsorbed water and other adsorbates will act as true 'surface probes'. On the other hand, measurements at sub-zero temperatures will help to establish a definite thermodynamic theory for the freezing process.

ANALYSIS OF RESULTS

The sorption data will be analysed by using regression techniques. The sorption data from the pilot project are at present being analysed. The results of this analysis, together with a more detailed description of the experimental techniques will be published shortly.

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Low-frequency dynamic mechanical testing of hardened cement paste

An experimental programme is being undertaken to discover the dynamic mechanical response of hardened cement paste (hcp) beams in bending in the frequency range from 10^{-3} to 10^{2} Hz. Other experimental variables will be temperature, hcp structural characteristics and equilibrium moisture content. The structure of the hcp will be controlled by the water/cement ratio and the curing temperature, thus producing well hydrated pastes with different total porosities, pore size distributions, internal surface areas etc. Appropriate parameters characterizing these properties will be derived from water sorption isotherms.

The central goal of the research is to relate deformational behaviour to the structure of hcp. The response curves will be characterized with viscoelastic parameters, thus allowing quantitative comparison of the response of different pastes. Apparent activation energies will be computed as an aid to the identification of diffusion or flow processes responsible for the deformation.

Low-frequency tests have been chosen because previous short-term creep recovery tests* have established the presence of a deformational mechanism within this time range. The present experiments will extend the time range of the previous tests, and also provide a unique opportunity to test the applicability of linear viscoelastic theory in this system.

In the creep recovery research project, the tentative conclusion was drawn that the deformation was caused by a redistribution of 'capillary' water in response to the load. The extension of the time range to shorter times in the present project will make it possible to investigate this hypothesis further.

The experiments will be performed using an electromagnetic exciter and a nontouching displacement transducer. A special electronic circuit has been designed which is able to measure the phase difference between the applied load and the resulting displacement to an absolute accuracy of 0·1°.

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Future 'Concrete research news'

The Editor will welcome submissions for future editions of 'Concrete research news'. Entries should relate to programmes of research which are just starting, or which are just entering a substantially new phase. Each item must be submitted, or at least authorized in its final form, by the head of department or director of research concerned.

Entries for 'Concrete research news' will be considered by the Editorial Advisory Board in the usual way and so, if it is convenient to submit up to eight copies (at least two, please), this is helpful. It is hoped to publish all items accepted in one of the two issues following the date of their receipt. As a general guide only, items for the March issue should reach the Editor by the end of January at the latest, and so on quarterly.

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*SELLEVOLD, E. J. *Anelastic behaviour of hardened portland cement paste.* Thesis submitted to Stanford University, California, Department of Civil Engineering, for the degree of PhD. Stanford Report No. 113.

UDC 624.023.934.012.45

A theoretical investigation of ultimate torque as calculated by truss theory and by the Russian ultimate equilibrium method

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SYNOPSIS

An analysis is given of the distribution of forces in reinforced concrete elements under pure torsion. It is assumed that the elements are under-reinforced and that failure is initiated by yielding of reinforcement. Two methods of calculation are taken into consideration, the space truss analogy and the 'ultimate equilibrium method' as adopted in the Soviet code. It is shown that the latter method will give the same results as the truss theory, provided that proper assumptions are made with respect to the form of the failure surface and the magnitude of the stirrup stresses at the smaller and the larger sides of the beam respectively.

Introduction

The truss analogy may be considered the method of calculation normally accepted in Western Europe for the determination of the internal forces acting in elements subjected to torsion. In this method—which dates back to Mörsch(1)—an element is conceived as a space truss model in which the reinforcements form a system of steel tension members, the concrete taking the function of compression diagonals between the nodal points of the system.

In 1959, Lessig(2) introduced a new conception, which has since been included in the Soviet code. This 'ultimate equilibrium method' which, to judge from its application in numerous technical papers, has met wide acceptance, considers the equilibrium in a possible failure surface, as illustrated in Figure 1.

It should be noted, however, that this method provides no information about which failure surface will occur or what the steel stresses will be in the several parts of the reinforcement crossing this failure surface. For these questions, therefore, assumptions have to be made. As an example may be mentioned here the assumption(3-5) that, in all reinforcement passing through the failure surface, the yield point of the steel is reached.

This paper gives a comparison of the two methods (the truss analogy and the ultimate equilibrium method) as a basis for determining what assumptions must be made when applying the Russian approach to reach the same results as are obtained with the truss theory.

The behaviour of reinforced concrete beams under torsion is very different before and after cracking. Test results(6-8) show that hollow sections have a smaller cracking torque than equivalent solid sections of the same size and the same concrete quality. This difference can be ascribed to the contribution made by the concrete core to the cracking torque.

The same results also show that such a difference does not occur in the ultimate torques. This can be explained by assuming(6) that, owing to the elongation of the stirrups, after cracking a supporting shell is formed that completely resists the torsional moment. Thus, after cracking, the concrete core will no longer contribute in any way to the ultimate torque; only a narrow 'concrete skin' around the reinforcement will remain active in resisting the torsional moment.

This knowledge will be applied here to both the truss theory and the equilibrium method. In dealing with the truss theory, we may therefore base the calculation of the torsional shear forces in the cracked state upon the behaviour of hollow sections (since the contribution of the core may be neglected). Likewise, we may assume, with respect to the failure surface, that the depth of the neutral axis will remain small, and has the same dimensions as the width of the concrete compression struts in the truss model.

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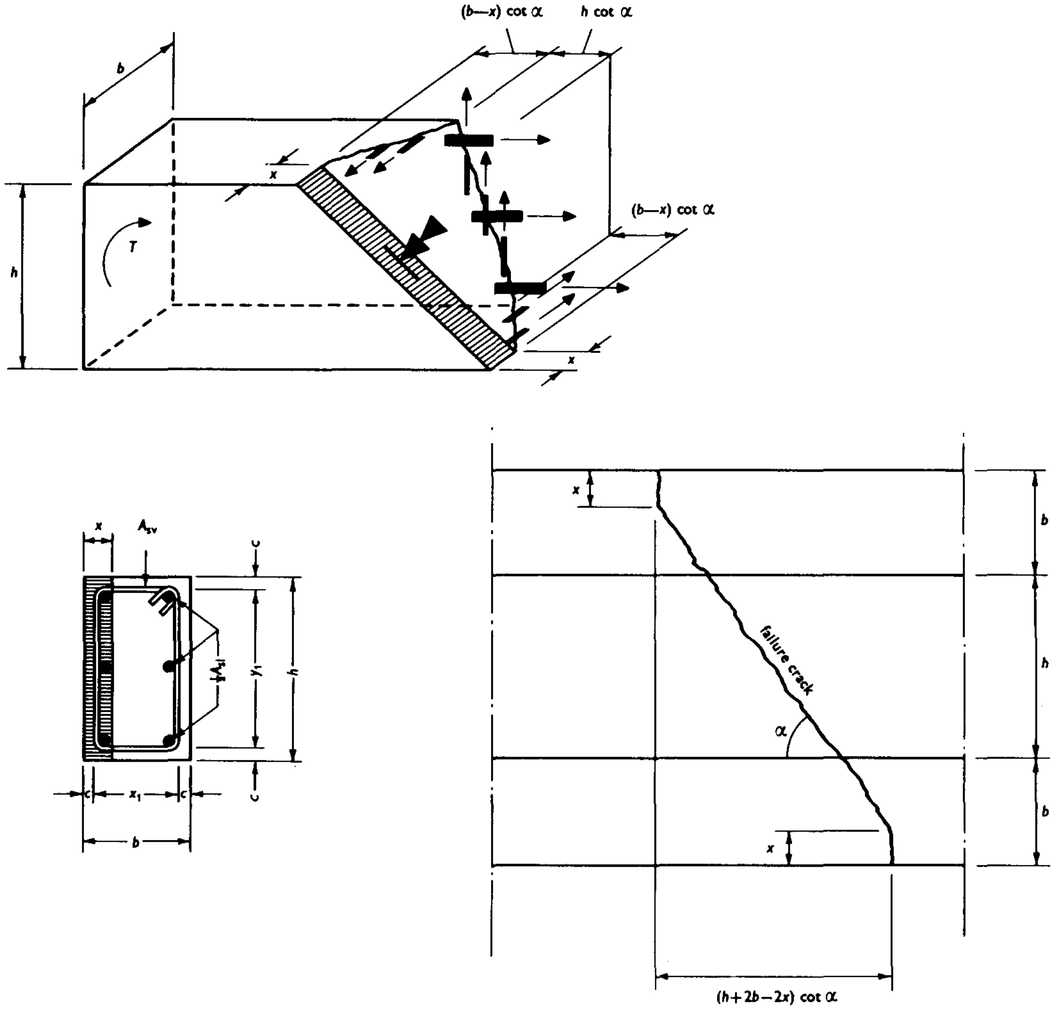


Figure 1: Assumed failure surface of reinforced concrete beams under pure torsion.

Notation

T = torsional moment

T_u = ultimate torsional moment

V_w = $T/2A_{cl}$ = shear force in walls of a hollow beam

h = larger over-all dimension of a rectangular cross-section

b = smaller over-all dimension of a rectangular cross-section

y_l = $h - 2c$ = larger centre-to-centre dimension of a stirrup

x_l = $b - 2c$ = smaller centre-to-centre dimension of a stirrup

c = concrete cover, measured to centre of stirrup

A_c = hb = total area of cross-section

A_{cl} = $y_l x_l$ = area of cross-section within stirrups

u_{cl} = $2(x_l + y_l)$ = perimeter of cross-section within stirrups

K = $2A_c A_{cl}/u_{cl}$ = shape factor of cross-section

A_{sl} = cross-sectional area of all longitudinal bars

A_{sv} = cross-sectional area of one leg of stirrup

s_v = spacing of stirrups in the longitudinal direction

σ_{sl} = steel stress in longitudinal bars

σ_{vy} = steel stress in stirrups at larger side of the beam

σ_{vx} = steel stress in stirrups at smaller side of the beam

f_{yl} = yield strength of longitudinal bars

f_{yv} = yield strength of stirrups

f_{cyl} = cylinder compressive strength of concrete

ρ_l = A_{sl}/A_c = volume percentage of longitudinal bars

ρ_v = $A_{sv}u_{cl}/s_v A_c$ = volume percentage of stirrups

ω_l = $\rho_l f_{yl}/f_{cyl}$ = reinforcement index of longitudinal reinforcement

ω_t = $\rho_v f_{yv}/f_{cyl}$ = reinforcement index of transverse reinforcement

ω_{tr} = $\sqrt{\omega_l \omega_t}$ = effective reinforcement index for torsion; geometric mean of reinforcement indices of longitudinal bars and stirrups

δ_s = $\dfrac{\rho_l f_{yl}}{\rho_v f_{yv}} = \dfrac{\omega_l}{\omega_t}$ = ratio of reinforcement indices for longitudinal bars and stirrups

n_h = number of longitudinal bars at larger side of the beam

n_b = number of longitudinal bars at smaller side of the beam

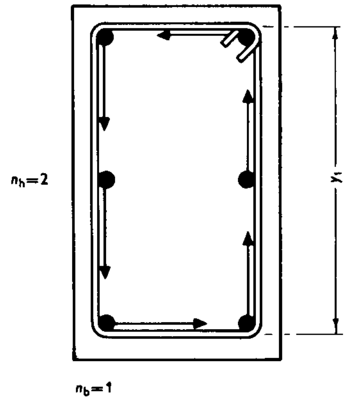


Figure 2: Meaning of the numbers n_b and n_h.

Derivation of general equations according to the space truss theory

We assume a uniform distribution of the stirrups throughout the beam: $(n_h + 1)$ longitudinal bars along the longer sides of the beam and $(n_b + 1)$ longitudinal bars along the shorter sides (see Figure 2). The equations derived for this general case will be simplified in the next two sections for specific arrangements of longitudinal bars.

As is known from elementary mechanics, in (thin-walled) sections under torsion, the external torque is balanced by internal shear forces $V_w = T/2A_{c1}$ per unit length. The larger sides of the beam therefore have to carry a shear force $(T/2A_{c1})y_1$ and the smaller sides a shear force $(T/2A_{c1})x_1$. The first shear force must be resisted by n_h bars and the latter by n_b bars.

It is assumed that the compression diagonals in the front face of the beam are inclined at an angle α to the axis of the beam, and in the top face of the beam at an angle β (see Figure 3). The equilibrium equations for the front face of the beam are:

$$A_{sv}\frac{y_1 \cot\alpha}{n_h s_v}\sigma_{vy} = \frac{T}{2A_{c1}}\frac{y_1}{n_h} \quad \text{..........(1)}$$

$$\frac{A_{sl}}{2(n_b + n_h)}\sigma_{sl} = \frac{T}{2A_{c1}}\frac{y_1}{n_h}\cot\alpha \quad \text{......(2)}$$

Eliminating $\cot\alpha$, we obtain:

$$\frac{T}{2A_{c1}} = \sqrt{\frac{A_{sv}\sigma_{vy}}{s_v}\frac{A_{sl}\sigma_{sl}}{u_{c1}}}\sqrt{\frac{u_{c1}}{4y_1}\frac{2n_h}{n_b + n_h}} \quad \text{......(3)}$$

Likewise, the equilibrium equations for the top face of the beam are:

$$A_{sv}\frac{x_1 \cot\beta}{n_b s_v}\sigma_{vx} = \frac{T}{2A_{c1}}\frac{x_1}{n_b} \quad \text{..........(4)}$$

$$\frac{A_{sl}}{2(n_b + n_h)}\sigma_{sl} = \frac{T}{2A_{c1}}\frac{x_1}{n_b}\cot\beta \quad \text{......(5)}$$

From equations 2 and 5 we find as necessary condition:

$$\cot\beta = \frac{n_b}{n_h}\frac{y_1}{x_1}\cot\alpha \quad \text{..........(6)}$$

From equations 1 and 4 it also follows that:

$$\sigma_{vx} = \sigma_{vy}\frac{x_1}{y_1}\frac{n_h}{n_b} = \sigma_{vy}\frac{\cot\alpha}{\cot\beta} \quad \text{..........(7)}$$

It can be seen from this derivation that, generally speaking, the stresses in the stirrups will be not the same for the front and the top faces of the beam.

From equations 1 and 2 we finally obtain:

$$\cot^2\alpha = \frac{A_{sl}\sigma_{sl}}{u_{c1}}\frac{s_v}{A_{sv}\sigma_{vy}}\frac{u_{c1}}{4y_1}\frac{2n_h}{n_h + n_b} \quad \text{........(8)}$$

Case 1: The longitudinal reinforcement is evenly distributed along the four sides of the beam (truss theory)

For this particular case,

$$\frac{n_h}{n_b} = \frac{y_1}{x_1} \quad \text{..................(9)}$$

Substitution into equation 3 gives:

$$\frac{T}{2A_{c1}} = \sqrt{\frac{A_{sv}\sigma_{vy}}{s_v}\frac{A_{sl}\sigma_{sl}}{u_{c1}}} \quad \text{............(10)}$$

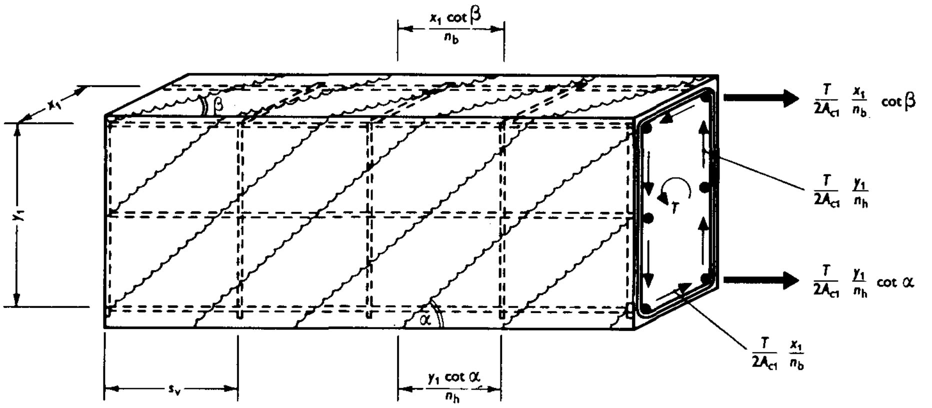
The optimum failure torque will be reached if both longitudinal bars and stirrups yield:

$$\frac{T_u}{2A_{c1}} = \sqrt{\frac{A_{sv}f_{yv}}{s_v}\frac{A_{sl}f_{yl}}{u_{c1}}} \quad \text{............(11)}$$

Substitution of equation 9 into equation 6 gives further: $\cot\beta = \cot\alpha$. The angle $\alpha = \beta$ between the inclination of the compression diagonals and the axis of the beam is then given by:

$$\cot^2\alpha = \frac{A_{sl}f_{yl}}{u_{c1}}\frac{s_v}{A_{sv}f_{yv}} = \frac{\rho_l f_{yl}}{\rho_v f_{yv}} = \delta_s \quad \text{......(12)}$$

The objection is sometimes raised to the foregoing derivation that the supposed degree of freedom does not actually exist; the angle α could only be 45° in view of the cracks already formed. However, recent publications(2,6) state—on the grounds of measurements—that, as soon as yield occurs in either the longitudinal bars or the stirrups, inelastic deformations take place that ensure redistribution of stresses between transverse and longitudinal reinforcement. Because of this process of rearrangement of forces, the concrete compression diagonals acquire a new inclination in spite of the existing cracks. We may therefore conclude that the actual distribution of forces after cracking depends mainly upon the arrangement of reinforcement (ratio of stirrups to longitudinal



Condition for internal equilibrium: $\frac{x_1}{n_b}\cot\beta = \frac{y_1}{n_h}\cot\alpha$

Figure 3: Analysis of a reinforced concrete beam under torsion as a space truss.

bars), in agreement with the behaviour of other types of statically indeterminate concrete structures.

Equation 11 can be written as:

$$\frac{1}{\sqrt{\delta_s}}\frac{A_{sl}f_{yl}}{u_{c1}} = \sqrt{\delta_s}\frac{A_{sv}f_{yv}}{s_v} = \frac{T_u}{2A_{c1}} \quad \ldots\ldots(13)$$

The total amount of reinforcement can therefore be expressed as:

$$\left(\frac{A_{sl}f_{yl}}{u_{c1}} + \frac{A_{sv}f_{yv}}{s_v}\right) = \left(\sqrt{\delta_s} + \frac{1}{\sqrt{\delta_s}}\right)\frac{T_u}{2A_{c1}} \quad \ldots.(14)$$

Denoting A_{sl} and A_{sv} in terms of the volume percentages of longitudinal bars and stirrups, respectively, we get:

$$(\rho_l f_{yl} + \rho_v f_{yv}) = \left(\sqrt{\delta_s} + \frac{1}{\sqrt{\delta_s}}\right)\frac{T_u u_{c1}}{2A_c A_{c1}}$$

$$= \left(\sqrt{\delta_s} + \frac{1}{\sqrt{\delta_s}}\right)\frac{T_u}{K} \quad \ldots\ldots(15)$$

In this equation $\rho_l = A_{sl}/A_c$ and $\rho_v = A_{sv}u_{c1}/s_v A_c$.

The minimum amount of reinforcement is found for $\sqrt{\delta_s} = 1$. In that case, the equations simplify to:

$$\frac{A_{sl}f_{yl}}{u_{c1}} = \frac{A_{sv}f_{yv}}{s_v} = \frac{T_u}{2A_{c1}} \quad \ldots\ldots\ldots\ldots(16)$$

These are the well-known equations of Rausch(9).

For $\delta_s = 1$, it follows from equation 12 that $\alpha = \beta = 45°$; the inclination of the concrete compression diagonals then coincides with the existing pattern of cracking. From this derivation, it clearly follows that the ratio $\delta_s = 1$ (as tacitly assumed by Rausch) is not a necessary condition for the internal equilibrium, but has more the meaning of a structural condition leading to the minimum amount of reinforcement.

If $\delta_s \neq 1$, after yield of the reinforcement a rearrangement of forces will occur; the inclination of the concrete compression diagonals will then no longer be 45°, but will follow from equation 12.

Depending upon the arrangement of the reinforcement, this process of redistribution can involve relatively large deformations leading to excessive cracking. In view of this fact, it seems advisable, for practical applications, to limit the ratio δ_s, for example to $\frac{1}{2}\sqrt{2} \leqslant \delta_s \leqslant \sqrt{2}$, in order to avoid inadmissible cracking(2).

Case 2: The longitudinal reinforcement consists of four corner bars (truss theory)

For this particular case,

$$n_b = n_h = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots(17)$$

Substitution into equation 3 gives:

$$\frac{T}{2A_{c1}} = \sqrt{\frac{A_{sl}\sigma_{sl}}{u_{c1}}\frac{A_{sv}\sigma_{vy}}{s_v}}\sqrt{\frac{u_{c1}}{4y_1}} \quad \ldots\ldots\ldots(18)$$

The optimum failure torque will be reached if both longitudinal bars and stirrups yield:

$$\frac{T_u}{2A_{c1}} = \sqrt{\frac{A_{sl}f_{yl}}{u_{c1}}\frac{A_{sv}f_{yv}}{s_v}}\sqrt{\frac{u_{c1}}{4y_1}} \quad \ldots\ldots\ldots(19)$$

Substitution of equation 17 into equation 8 gives:

$$\cot^2\alpha = \frac{\rho_l f_{yl}}{\rho_v f_{yv}}\frac{u_{c1}}{4y_1} = \delta_s\frac{u_{c1}}{4y_1} \quad \ldots\ldots\ldots(20)$$

To make $\alpha = 45°$, there must be a surplus of longitudinal reinforcement, which follows from:

$$\delta_s = 4y_1/u_{c1} = 2y_1/(x_1 + y_1) \quad \ldots\ldots\ldots\ldots(21)$$

For $y_1 = 2x_1$ we would thus find $\delta_s = 1{\cdot}33$, whilst for $y_1 = 4x_1$ we would find $\delta_s = 1{\cdot}60$.

A comparison of equations 11 and 19 shows that they have the term

$$\sqrt{\frac{A_{sl}f_{yl}}{u_{cl}}\frac{A_{sv}f_{yv}}{s_v}}$$

in common. For the case of four corner bars, a reduction factor $\sqrt{u_{cl}/4y_1}$ is added, because of the non-uniform distribution of the longitudinal reinforcement around the perimeter of the beam.

To avoid excessive cracking, it will be necessary again to limit the ratio of longitudinal bars to stirrups, for instance to:

$$\tfrac{1}{2}\sqrt{2} \leqslant \delta_s u_{cl}/4y_1 \leqslant \sqrt{2}$$

Derivation of equations for ultimate torque from the equilibrium conditions in a failure surface

In the following derivation it is assumed—as is done in truss theory—that all the reinforcement is loaded only in tension, possible shearing forces (resisted by dowel action of the bars) not being taken into consideration. As already mentioned in the introduction, we also assume that the depth of the neutral axis is small. Taking this depth, x, to be approximately equal to twice the concrete cover c (measured to the centre of the stirrups) the internal lever arm becomes equal to x_1 (see Figure 1).

If these assumptions are made, the equilibrium conditions in the failure surface become very simple. For the case of uniformly distributed longitudinal reinforcement, the torsional moment is then given by:

$$T = A_{sv}\sigma_{vy}\frac{y_1 \cot\alpha}{s_v}x_1 + A_{sv}\sigma_{vy}\frac{x_1\cot\alpha}{s_v}y_1$$

$$T = A_{sv}\sigma_{vy}\frac{2A_{cl}\cot\alpha}{s_v} \quad\ldots\ldots\ldots\ldots\ldots\ldots(22)$$

For the assumed failure surface, the torque is resisted by tensile forces in the stirrups. These tensile stirrup forces, however, also cause an internal bending moment:

$$M = A_{sv}\sigma_{vy}\frac{x_1\cot\alpha}{s_v}(y_1 + x_1)\cot\alpha\ldots\ldots(23)$$

The function of the longitudinal bars is to neutralize this internal bending moment, thus making internal equilibrium possible.

$$M = \frac{y_1}{u_{cl}}A_{sl}\sigma_{sl}x_1 + 2\frac{x_1}{u_{cl}}A_{sl}\sigma_{sl}\tfrac{1}{2}x_1$$

$$M = \tfrac{1}{2}A_{sl}\sigma_{sl}x_1\ldots\ldots\ldots\ldots\ldots\ldots\ldots(24)$$

Therefore, the following condition must be satisfied:

$$M = A_{sv}\sigma_{vy}\frac{x_1\cot\alpha}{s_v}(y_1 + x_1)\cot\alpha = \tfrac{1}{2}A_{sl}\sigma_{sl}x_1$$

$$\ldots\ldots(25)$$

This gives:

$$\cot^2\alpha = \frac{A_{sl}\sigma_{sl}}{u_{cl}}\frac{s_v}{A_{sv}\sigma_{vy}}\ldots\ldots\ldots(26)$$

Substituting this equation into equation 22, we get:

$$\frac{T}{2A_{cl}} = \sqrt{\frac{A_{sl}\sigma_{sl}}{u_{cl}}\frac{A_{sv}\sigma_{vy}}{s_v}}\ldots\ldots\ldots(27)$$

The equations obtained in this way completely agree with the equations (11 and 12) derived by truss theory.

Likewise, it can be proved that, for the case of longitudinal reinforcement consisting of four corner bars, the Russian approach gives the same equations as the truss theory if the following restrictions are made.

(1) The boundaries of the failure surface in the back and top faces of the beam must have different inclinations, which are related to one another by the condition $x_1 \cot\beta = y_1 \cot\alpha$ (which follows from equation 6).

(2) The stirrup stresses in the top and bottom of the beam must be taken at x_1/y_1 times the stirrup stresses in the side of the beam (which follows from equation 7).

The assumption sometimes made(2-5), that the yield point of the steel is reached in all the reinforcing bars passing through the failure surface, will therefore only be valid if the longitudinal bars are evenly distributed along the four sides of the beam. Most test series, however, consist of beams with four corner bars.

The same conclusion holds for the assumption(10,11) that the inclination of the failure crack is the same at each side of the beam. Likewise, the assumption that the stresses in the transverse reinforcement crossing the failure surface are of the same magnitude(10,11) is dubious for beams only reinforced with a longitudinal bar in each corner of the stirrup.

Final observations

(1) In the literature, a certain lack of conclusive knowledge is noticeable with reference to the management of the 'ultimate equilibrium method'. Several authors have given an interpretation of the form to be assumed for the failure surface and of the values of the stresses in the reinforcements crossing this failure surface. In this paper, as an alternative, an attempt is made to reconcile the truss analogy and the ultimate equilibrium method, in order to obtain a basis for evaluating all these interpretations.

(2) It will be clear from the foregoing considerations that, if the ultimate equilibrium method is applied, the results obtained will depend entirely upon the assumptions that are introduced. In this respect it may be stated that calculations based on the truss analogy are preferable, because they rest on a more definite

basis with regard to the internal equilibrium of the beam as a whole.

(3) The theoretical conclusion that, in beams reinforced with four corner bars, the stirrup stresses in the top of the beam will be smaller than those in the sides of the beam agrees with experimental findings$^{(7,8)}$.

(4) It can be seen that, in the several equations derived for ultimate torque, the quality of the concrete plays no part. This is a consequence of the assumption that the concrete compression members have a width $2c$, irrespective of the concrete strength. All the equations, therefore, hold only for under-reinforced beams.

(5) Since the main purpose of this paper is to compare two methods of calculation, no attempt is made to verify the derived equations with test results.

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Incremental deformations of under-reinforced concrete beams

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SYNOPSIS

Existing ultimate strength theories for reinforced concrete structures are not sufficient to describe behaviour under cyclic loading. In particular, the phenomenon called 'incremental collapse' or its converse, 'shake-down', must be considered. An elastic–perfectly-plastic theory has been developed, but its applicability has been questioned. If the material properties show evidence of strain-hardening, the system will shake down under cyclic loads up to and greater than the perfectly plastic limit load. This paper applies strain-hardening concepts to the theoretical response of reinforced concrete structures and verifies the predictions so obtained by comparing results with experimental responses of a set of reinforced concrete continuous beams.

Introduction

Existing ultimate strength theories for the design of reinforced concrete structures(1,2) are based on responses to monotonically increasing loads. The behaviour of such structures under more complicated load histories, such as reversed loading cycles experienced during earthquakes, or the repeatedly applied overload frequently encountered by a bridge structure during its lifetime, cannot be explained adequately by this type of theory. If ultimate strength theories (and eventually limit design methods) are to be used for such cases with the necessary degree of confidence, the influence of arbitrary load histories upon the strength and stiffness of structures must be understood.

When the maximum stresses due to such load histories are within the elastic range, problems due to fatigue may arise. Fatigue of concrete has been extensively studied(3). If, on the other hand, cycles of overload occur which cause inelastic behaviour in parts of the structure, the problem of 'low-cycle fatigue' or 'incremental deformations' may arise. It has been shown(4) that statically indeterminate structures of elastic–perfectly-plastic material may suffer a build-up of permanent deformations under repeated overloads, so that after only a few cycles permissible deflections may be exceeded. This situation is termed 'incremental collapse'. The converse condition, when the deformations stabilize at a permissible level irrespective of the number of applied load cycles, is called 'shake-down'. The theory underlying this behaviour of elastic–perfectly-plastic structures has also been developed, but its applicability to actual structures has recently been questioned(5).

The response of reinforced concrete structures to cyclic overloads was first studied by Rasmussen(6), Bertero and McClure(7), and others. Gerstle and Tulin(8) found that, contrary to the predictions of Rasmussen, reinforced concrete beams would shake down under cyclic loads above the perfectly plastic shake-down limit, and Ruiz and Winter(9) attempted to reconcile the different conclusions.

In a recent paper, Gerstle and Meyer(5) concluded on the basis of a historical type of analysis that even a minor degree of strain-hardening would lead to shake-down under cyclic loads up to and above the perfectly-plastic limit load, and obtained good agreement with test results of steel structures. Eiklid, Gerstle and Tulin(10) showed that concrete beams under-reinforced with the newer types of high-strength reinforcing steel exhibited considerable strain-hardening which enabled them to carry monotonically applied loads above the limit load level.

According to these studies, it may be anticipated that properly designed concrete structures may, thanks to strain-hardening, exhibit measurable and effective resistance to incremental collapse under cyclic load histories. It is the purpose of this investigation to apply

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the strain-hardening theory presented earlier(5) to reinforced concrete structures, and to verify the predictions by means of tests on continuous concrete beams.

Accordingly, in the following sections, a simple strain-hardening analysis is outlined, and test results are presented to confirm the validity of the theory. Lastly, conclusions are drawn regarding the expected response of concrete structures to cyclically applied over-loads.

Analysis

The analysis which is proposed here is based upon the following assumptions, which have been found satisfactory in earlier studies(10,5):

(1) Inelastic action occurs only at discrete critical sections. On the analogy of the concept of 'plastic hinges' in conventional limit analysis, this inelastic action may be modelled by 'strain-hardening hinges' of appropriate rotational stiffness.

(2) The rotational stiffness of the strain-hardening hinges may be linear or consist of a series of linear regimes to represent the beam stiffness beyond the elastic limit.

(3) Under cycles of loading and unloading, the strain-hardening hinge is first activated when the yield moment, M_p, of the beam is exceeded. Reduction of moment due to unloading causes the hinge to lock. Reloading proceeds with the hinge locked until the highest moment previously achieved at the section is reached, at which time the spring is again activated.

(4) Total beam deformations are obtained by appropriate superposition of elastic beam curvatures and rigid-body rotations of the discrete hinges at critical sections.

(5) The beam is strong enough in shear for no diagonal tension or bond failure to take place. The ultimate load is reached by compressive crushing of concrete when the rotation capacity of a critical section is exhausted.

The consequences of assumptions 1 and 2 have been discussed in references 10 and 5. In reinforced concrete beams, the inelastic rotations are caused by cracking and yielding of steel at discrete sections, as shown in Figure 1. In this case, the curvature is not a continuous function along the beam, and the procedure employing the method of integration of curvatures which is so widely used in structural analysis may not be applicable. Earlier investigators(11) have introduced the concept of 'plastic hinge length' over which the inelastic curvature associated with the peak moment is acting. The total stiffness of the section may also be obtained by means of rotation measurements made as shown in Figure 1, and the inelastic (or strain-hardening hinge) stiffness calculated from the resulting moment-rotation curve (Figure 2) by taking the ratio of the increment of moment and the increment of inelastic rotation, from which the hinge stiffness k becomes

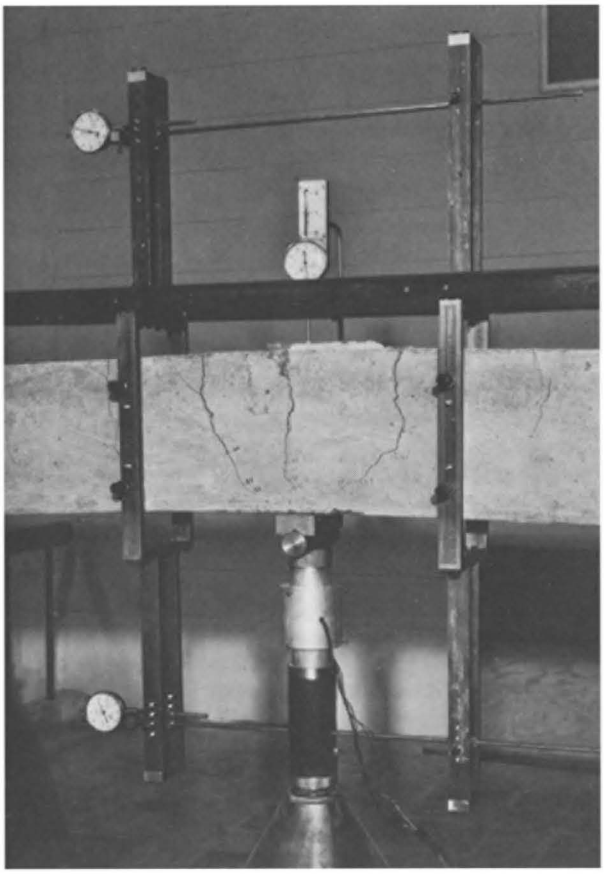


Figure 1: Hinging section of concrete beam.

$$k = \frac{\Delta M}{\Delta\theta_{\text{inelast}}} = \frac{k_1 k_2}{k_1 - k_2}$$

where k_1 = rotational stiffness in the elastic range, and k_2 = rotational stiffness in the strain-hardening range. It has been shown in reference 10 that a bi-linear moment-rotation relationship is adequate for the analysis of concrete beams under monotonic loading. The strain-hardening hinge stiffness, k, can conveniently be non-dimensionalized in the form kL/EI, where L is a characteristic span length and EI represents the elastic beam stiffness. The value of kL/EI can range from zero, representing the perfectly-plastic case, to infinity, in which case the beam is perfectly elastic.

With assumption 3, the strain-hardening analysis of statically indeterminate structures under any load history can be performed as a series of linearly-elastic solutions of the structure successively modified by the insertion or removal of strain-hardening hinges at critical sections, on the analogy of the step-by-step method used in perfectly-plastic analysis(12). For load histories involving unloading and reloading, it is necessary to keep track of the onset of strain-hardening hinge response. The calculation lends itself to graphical representation as discussed in reference 5, or it may be programmed for high-speed computer. In this case,

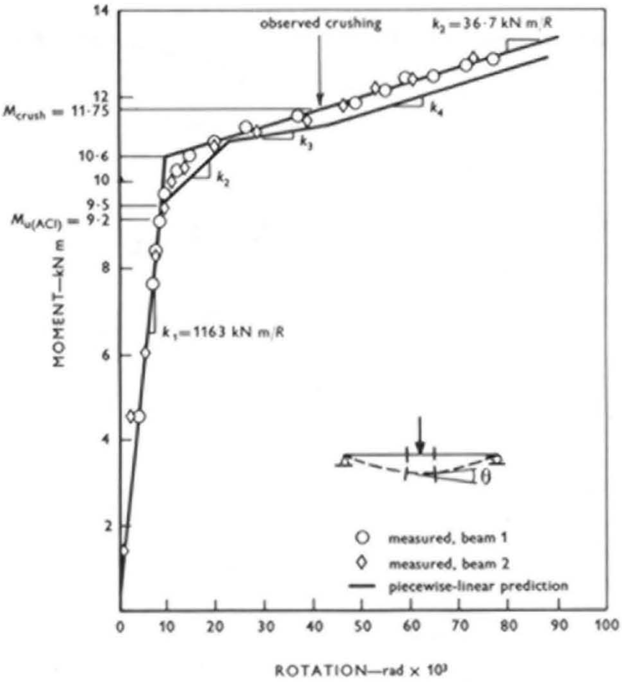


Figure 2: Moment-rotation relationship.

multi-linear moment-rotation relationships and spreading of plastic zones can be accommodated as well. However, because of the discontinuous nature of cracking of concrete and yielding of steel due to the bond between the two materials, it is not believed that this latter effect is as pronounced in reinforced concrete structures under point loads as it might be in steel beams. Figure 1, of a severely yielded section of one of the test beams, shows that inelastic action occurred in a relatively short plastic-hinge length. Similarly, Figure 3, of one of the test beams after removal of high overload, shows that the permanent rotations appear only in the regions immediately under the applied forces, the remaining portions returning to straight configuration, thus indicating the absence of any plastic action away from the hinge zones. It seems, therefore, that the insertion of a strain-hardening hinge is a reasonable way to describe the inelastic behaviour of a concrete member.

Experimental programme

To check the applicability of the proposed analysis in predicting the behaviour of reinforced concrete structures under cyclic overloads, an experimental programme, consisting of tests of two simply supported control beams and three two-span continuous beams under various cyclic load histories, was conducted.

Figure 4 shows the test specimens. They were all cast at the same time of ready-mixed concrete, with $f_c' = 27{\cdot}6$ N/mm^2 and reinforced with deformed bars of $f_y = 469$ N/mm^2 and distinct strain-hardening properties. Forms were stripped after 24 h, and this was followed by a period of moist curing of seven days. Testing took place at ages of 28 days or more.

The simply supported beams were tested in a universal testing machine. Instrumentation consisted of a dial gauge for determining mid-span deflection, and a rotation meter of the type shown in Figure 1 at mid-span, of 457·2 mm gauge length. The rotation measurements permitted determination of the stiffness properties of the beams, presented by means of the moment-rotation data of Figure 2. This illustration also contains matching idealized piecewise-linear relationships for use in the analysis. The calculated value of the ultimate moment M_p according to the American Concrete Institute is also indicated. This moment is seen to be quite conservative for these beams. Crushing of the extreme fibres of the compression concrete at mid-span, which took place under a total rotation of about 0·04 radians, is also shown.

With the elastic and strain-hardening properties of the simply supported beams, the mid-span deflections can be predicted by use of assumption 4, and are shown by the solid lines of Figure 5, along with the measured deflection values of the two specimens. Good correlation is indicated, whereas the limit load based on the ACI ultimate moment of 9·2 kN m is too low.

The continuous test beams were tied to steel tripods anchored to the structural test floor by yokes capable of resisting either upward or downward reactions, while maintaining freedom of rotation and axial extension by appropriate hinging mechanisms. The cyclic loads were applied through two jacks at mid-span,

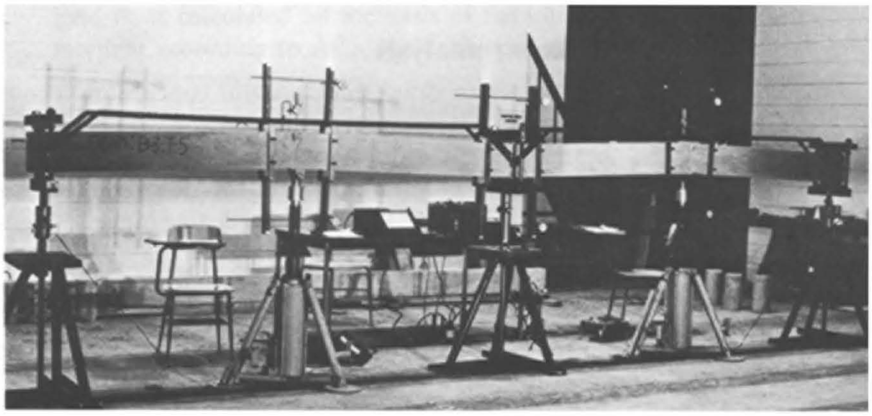


Figure 3: Two-span continuous test beam.

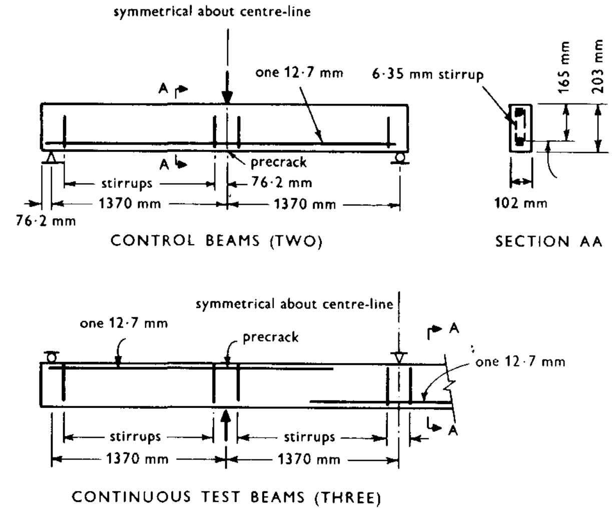


Figure 4: Details of test specimens.

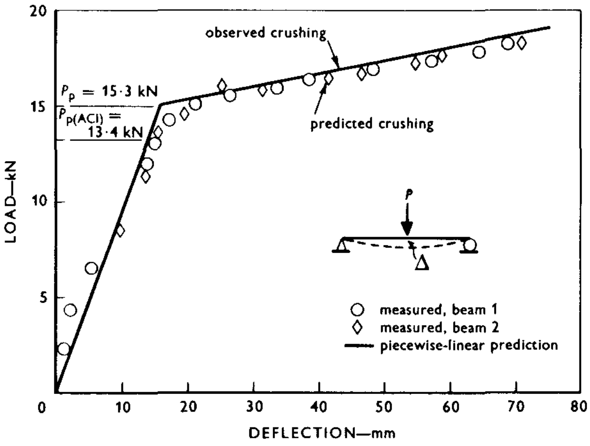
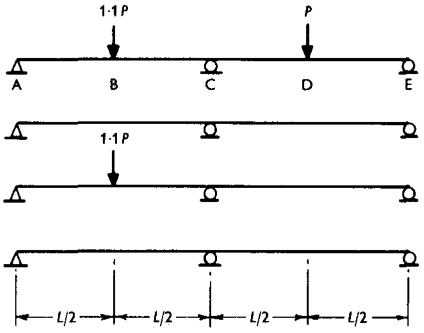
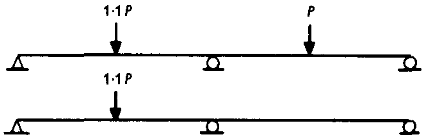


Figure 5: Load-deflection relationships for control beams.



(*a*) *Load history I*



(*b*) *Load history II*

Figure 6: Load histories.

capable of exerting upward forces against the test beam, and equipped with controls which permitted application and release of loads in accordance with the specified load history. Both supports and jacks contained load cells so that applied loads and reactions could be measured. These beams were also equipped with dial gauges at mid-span, as well as rotation meters at all critical sections. Figure 3 shows a typical test set-up.

Each load stage of the cyclic tests required several minutes to permit the various readings to be taken. During this time, the deflections were held constant, and the resulting load decrease due to relaxation was picked up during loading to the next stage. In this way, any creep deformations, which were not provided for in the theory, were minimized. A complete test required several hours.

Two types of load history, labelled I and II, were applied to the continuous beams. In the former, which involved complete unloading of the beam, each cycle consisted of the four loading steps shown in Figure 6a. The latter, which involved sequential loading and unloading of one mid-span load while the other remained on the beam, is shown by the two load steps of Figure 6b. After the tests were completed, a recalibration of the load cells indicated that one of the applied mid-span loads had been 10% larger than the other. This fact is considered in the analytical results presented later.

The applied load levels for the three continuous beams are shown in Table 1. These load levels are also expressed as ratios of the maximum to the perfectly-plastic limit load $P_p = 6M_p/L$ for two values of M_p: one calculated according to ACI of value 9·2 kN m, and the other used in the bi-linear analysis, of value 10·6 kN m.

The measured mid-span deflections of the continuous beams under cyclic loading are shown in Figures 7 to 10, along with predicted results. These results will be discussed in the following sections.

Bi-linear analysis

In order to draw some general conclusions about cyclic load effects and their comparison with measured beam response, an analysis was made on the basis of the simple bi-linear moment-rotation relationship shown in Figure 2. The strain-hardening hinge stiffness of this relation can be expressed by the non-dimensional factor kL/EI = 0·20. The analysis was performed for load history II at load levels of P/P_p = 0·92, 0·96 and 1·01. In addition, a sequence of loading consisting of a number of cycles at a load level of P/P_p = 0·96, followed by an increase to P/P_p = 1·01,

TABLE 1: **Applied load levels for the three continuous beams.**

Beam No.	Load history	Load levels								
		1			2			3		
		P (kN)	$\frac{P}{P_{p\,ACI}}$	$\frac{P}{P_{p\,exp}}$	P (kN)	$\frac{P}{P_{p\,ACI}}$	$\frac{P}{P_{p\,exp}}$	P (kN)	$\frac{P}{P_{p\,ACI}}$	$\frac{P}{P_{p\,exp}}$
1	I	18·9	0·94	0·82	19·4	0·96	0·84	19·9	0·99	0·86
2	I	20·4	1·01	0·88	21·2	1·05	0·92			
3	II	20·4	1·01	0·88	21·2	1·05	0·92			

was analysed in order to investigate the effects of various loading histories. The solid curves of Figure 7, indicating the mid-span deflection as a function of the number of load cycles, show the results of these analyses. The corresponding experimental deflections obtained from test beam 3 are also shown.

A study of the analytical results of Figure 7 leads to the following conclusions.

(1) Initial deflections during the first load cycle are very sensitive to the load level. A 4% increase of load from P/P_p = 0·92 to 0·96 results in a 100% increase in deflection.

(2) The rate of incremental deformations is not sensitive to the load level below the plastic collapse load. It is somewhat more sensitive when the maximum load exceeds the limit level.

(3) The effect of prior cycling at lower inelastic load levels is to reduce the rate of incremental deformations due to subsequent higher load cycles. This phenomenon had already been observed in earlier tests(8).

(4) Incremental deformations under a given cyclic history become asymptotic to a specific value for all load levels, that is, the structure will shake down.

The following points may be made on the basis of a comparison of the analytic and experimental plots:

(5) The inelastic deflections occurring under the first cycle of load cannot be predicted with accuracy by use of the bi-linear approximation. This follows from Conclusion 1. However, if the fully plastic load P_p is calculated on the basis of the ultimate moment according to ACI, the analytical prediction will be conservative.)

(6) Subsequent incremental deformations can be calculated with good accuracy by the bi-linear theory. Experiments confirm the above conclusions 2, 3 and 4.

(7) Under-reinforced concrete beams possess rotation capacity under cyclic loadings which can be predicted on the basis of the ultimate hinge rotation as controlled by ultimate compression strain, if shear failure can be prevented.

(8) A large number of inelastic cycles leads to deterioration of bond between concrete and steel which will cause larger incremental deformations than predicted by theory. There are also indications that this bond deterioration may have an adverse effect upon the shear strength of the beams.

Improved analysis

The preceding discussion has shown that analysis of incremental deformations based upon assumed bi-linear moment-rotation relationships at discrete hinges can represent the important features of beam response, but cannot give quantitative information about the total beam deformations due to loads near the perfectly-plastic collapse load. The reason for this is the relative inaccuracy of the bi-linear moment-curvature representation in the vicinity of its transition stage. For example, the experimental data of Figure 2 indicate inelastic action beginning near M = 9·2 kN m, but M_p according to the bi-linear assumption is 10·6 kN m.

This defect can be improved by a refined, multi-linear moment-hinge rotation assumption, as shown, for instance, by the curve consisting of four straight-line segments in Figure 2. This curve indicates first inelastic action under a moment of 9·5 kN m, and leads progressively to strain-hardening of the section. Analysis on the basis of this assumption is beyond the capability of hand calculation, and, accordingly, a computer program was formulated which was able to carry out the steps outlined in the section "Analysis". The results of this computer analysis, which are presented in Figures 8 to 10, along with corresponding experimental data, are used to verify the appropriateness of the refined analysis.

Figure 8 shows results of continuous beam specimen 1 subjected to five cycles of load history I at load levels of P/P_p = 0·94, 0·96 and 0·99. P_p has been calculated on the basis of the ACI moment of 9·2 kN m. The agreement between test and theory is considered satisfactory. Failure occurred after five cycles of a load of 99% of the perfectly-plastic limit load by crushing of the compressive concrete at the critical mid-span section under a rotation which was considerably in excess of the rotation capacity of 0·04 radians observed in control tests.

Figures 9 and 10 show results of continuous beam specimens 2 and 3 tested under two different cyclic

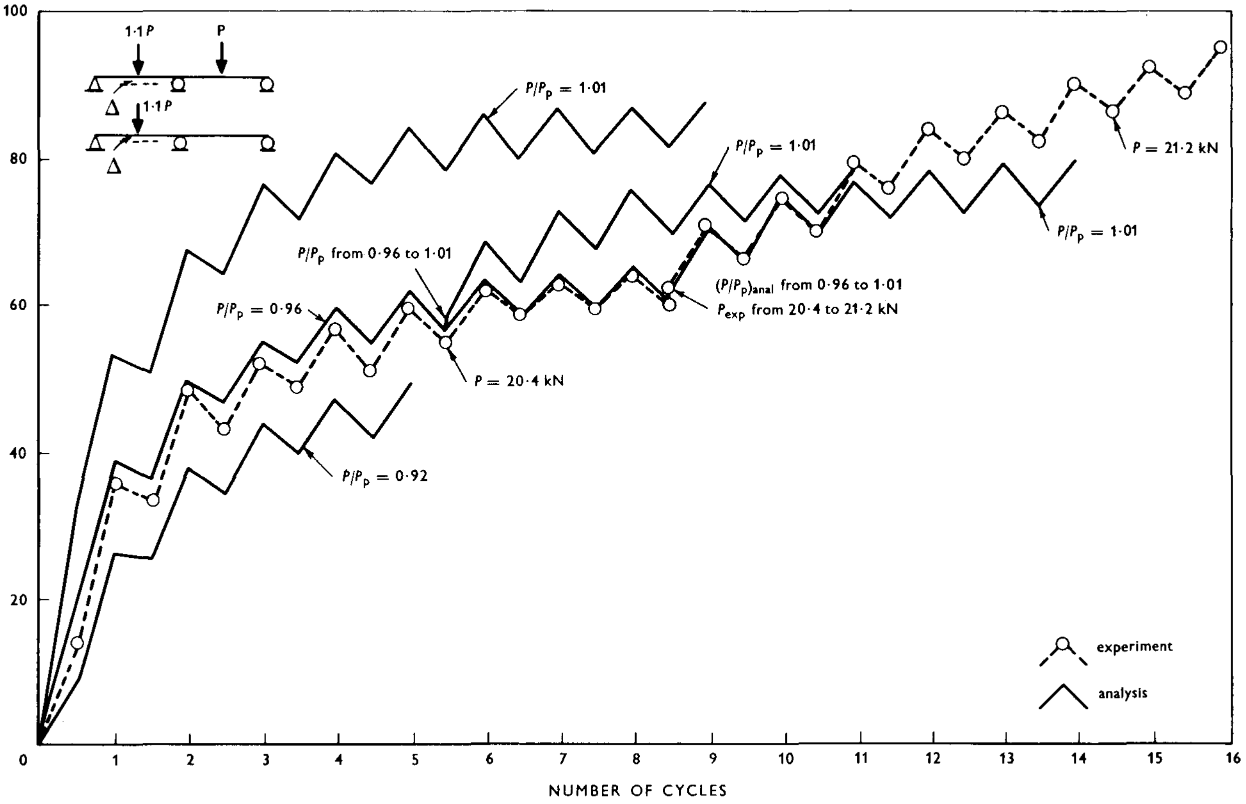


Figure 7: Bi-linear analysis and experimental results.

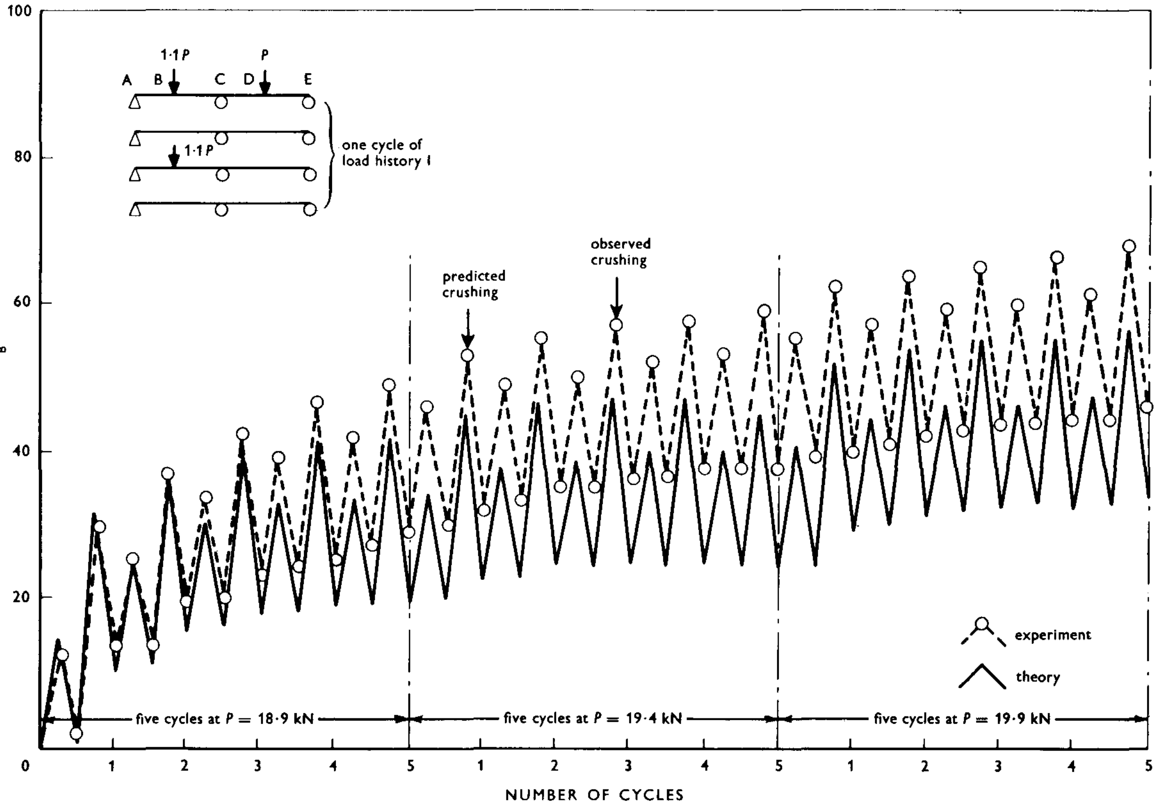


Figure 8: Deflections under cyclic loading—beam 1.

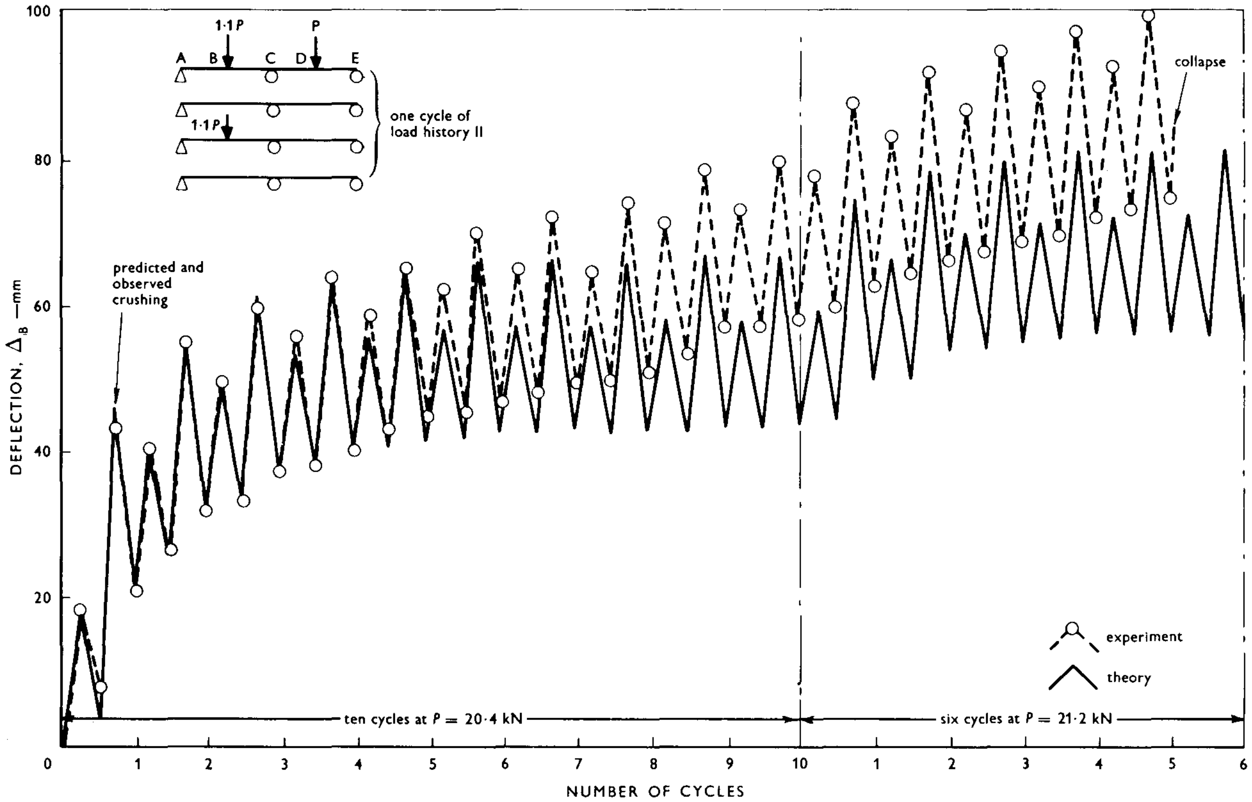


Figure 9: Deflections under cyclic loading—beam 2.

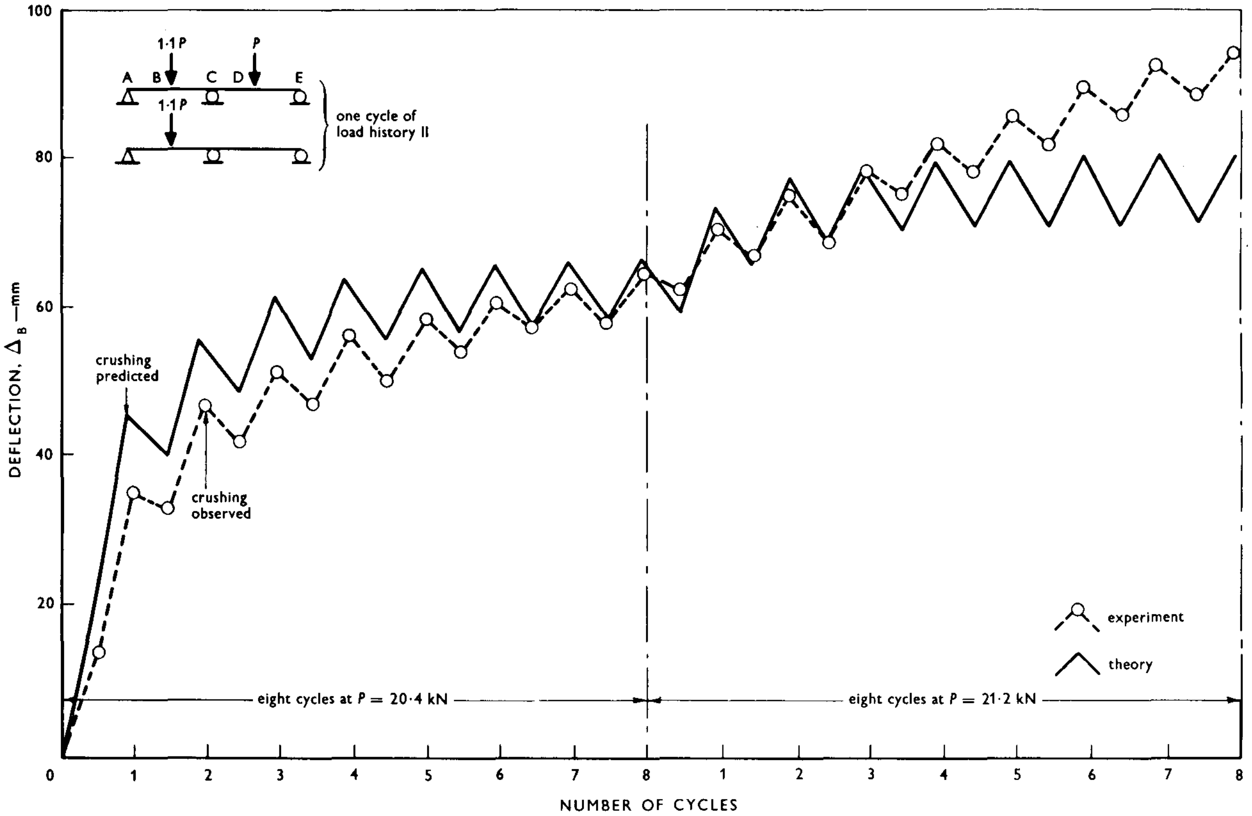


Figure 10: Deflections under cyclic loading—beam 3.

load histories, I and II, under identical maximum load intensities of $P/P_p = 1{\cdot}01$ and 1·05. In the beam subjected to load history I, excellent agreement between prediction and test results obtained for the first six cycles. Under subsequent loadings, a gradual drift toward larger deflections is observed. The growing discrepancy between predicted and experimental deformations may be attributed to the gradual deterioration of bond between steel and concrete under cyclic loading. This behaviour, however, becomes evident only after the ultimate beam rotation of 0·04 radians had been exceeded during the first cycle of loading. The observed crushing occurs close to the predicted value.

Analytical and experimental results for beam specimen 3, which was tested under load history II, show good agreement between predicted and actual deflections and occurrence of crushing. A comparison of the curves of Figures 9 and 10 also shows that in this case, the deflections at shake-down appear independent of the sequence of cyclic load applications, but depend only upon the extreme load values. This observation, however, cannot be generalized. Figure 7, for instance, shows a distinct lowering of the deformations at shake-down due to prior cycling.

Conclusions

On the basis of the results presented, the following conclusions may be drawn.

(1) Strain-hardening must be included in any realistic prediction of the flexural response of under-reinforced concrete structures to inelastic cycling. This strain-hardening can be simulated by the assumption of piecewise-linear inelastic moment-rotation relationships at discrete critical sections.

(2) For cyclic loadings at levels less than the perfectly-plastic collapse load, the incremental deflections will be of the order of magnitude of those occurring during the first load cycle. If premature shear and bond failure can be prevented, shakedown will occur. The ultimate rotation capacity at critical sections is that obtained from the beam response under monotonic loads.

(3) Deflections at shake-down depend primarily upon the level of applied load. Prior inelastic cycling will accelerate shake-down under subsequent higher load cycles.

(4) After a sufficiently large number of load cycles above the perfectly-plastic limit load, concrete crushing and bond deterioration may lead to incremental deformations larger than those predicted from the simple strain-hardening theory. Such bond deterioration may also trigger the propagation of diagonal tension cracks.

These points suggest that, because of the beneficial effects of strain-hardening, it may be possible to disregard the effects of possible cycles of overload upon the flexural strength of under-reinforced concrete beams. However, the effects of such repeated overloads upon bond and shear failure warrant additional study. Examination should also be made of the response to cyclic overloads of beams with more nearly balanced steel ratios.

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The plastic flow law for reinforced concrete beams under combined flexure and torsion

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SYNOPSIS.

The possible application of plastic analysis to reinforced concrete structures, the members of which are subjected to both bending and twisting moments, depends not only upon their ductility but also upon a knowledge of the associated plastic flow law. An experimental investigation to determine the flow law for rectangular reinforced concrete beams under combined flexure and torsion is described in this paper. Bending and twisting rotations measured during the pseudo-plastic stage occurring near maximum load are reported for 80 beams, which included both symmetrically and unsymmetrically placed reinforcement. Typical interaction curves with their associated incremental rotation vectors are presented and the corresponding flow law is stated. The complete interaction curve is considered and the implications of this and the plastic flow law are discussed in relation to the applicability of current methods of plastic analysis to concrete structures.

Introduction

Considerable research effort, both theoretical and experimental, has been directed towards the determination of interaction curves for reinforced concrete members under the combined action of bending and twisting moments. Comparatively little attention has been paid to the moment-rotation characteristics of such members although these are of considerable importance in relation to the feasibility of extending plastic analysis, now widely accepted for mild-steel structures, to reinforced concrete structures in which the members are subjected to primary bending and twisting moments.

A knowledge of the torque-flexure interaction diagram, whilst necessary, is not sufficient to enable a plastic analysis of this type of structure to be carried out. Adequate ductility (rotation capacity) of the members in the region of plastic hinges must be assured and, equally important, the associated plastic flow law, defining the relationship between the ratio of the bending and twisting moments and the direction of the axis of rotation at a hinge, must be known$^{(1)}$.

An experimental investigation to determine the moment-rotation characteristics and in particular the plastic flow law for reinforced concrete beams is described in this paper. Results are presented for symmetrically and unsymmetrically reinforced rectangular concrete beams under the combined action of flexure and torsion, with no shear.

Notation

A_{sL}	area of longitudinal bottom bar
A_{sL}'	area of longitudinal top bar
A_w	area of one leg of stirrup
b	over-all width of beam
f	yield function
f_c'	compressive strength of 305 × 152·5 mm dia. concrete cylinder
f_t	indirect concrete tensile strength ('Brazilian' test)
f_w, f_y, f_y'	yield stress for stirrups, bottom longitudinal and top longitudinal steel respectively
h	over-all depth of beam
M	bending moment
M_f, M_y	bending moments at failure and at yield respectively
M_p	sagging yield moment in pure flexure
M_p'	hogging yield moment in pure flexure
$\bar{M}$	$= \dfrac{M_y - \left(\dfrac{M_p - M_p'}{2}\right)}{\left(\dfrac{M_p + M_p'}{2}\right)}$
s	spacing of stirrups
T	twisting moment

T_y	twisting moment at yield
T_p	yield moment in pure torsion
T_p'	yield moment when $M_y = (M_p - M_p')/2$
$\bar{T}$ =	T_y/T_p'
u	compressive strength of 152 mm concrete cube
φ	flexural rotation
$\Delta\varphi$	incremental flexural rotation
θ	torsional rotation
$\Delta\theta$	incremental torsional rotation
φ_{pL}, θ_{pL}	flexural and torsional rotations respectively at the proportional limit
φ_p, θ_p	flexural and torsional rotations respectively between the proportional limit and failure

Experimental details

TEST SPECIMENS

A total of 80 rectangular reinforced concrete beams were tested. Concrete mix proportions were water:ordinary Portland cement:sand:19 mm irregular gravel:: 0·58:1·00:1·93:2·67 by weight; air-dried aggregates were used. Specimens were cured under wet hessian for 7 days and then in air, in the laboratory, until tested between 14 and 20 days after casting. Steel exhibiting a marked yield stress was used for both longitudinal and transverse reinforcement.

Typical reinforcement details are shown in Figure 1, and the cross-sectional dimensions, reinforcement and material properties for each beam are given in Tables 1 and 2.

TEST RIG

The general layout of the test rig is indicated in Figure 2. This arrangement was adopted to permit the simultaneous application of both bending and twisting

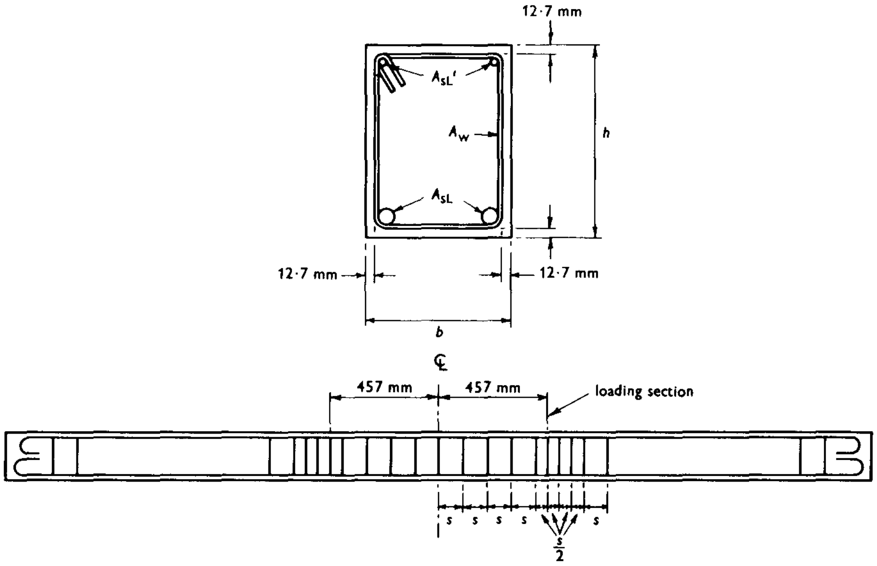


Figure 1: Details of reinforcement of typical beam.

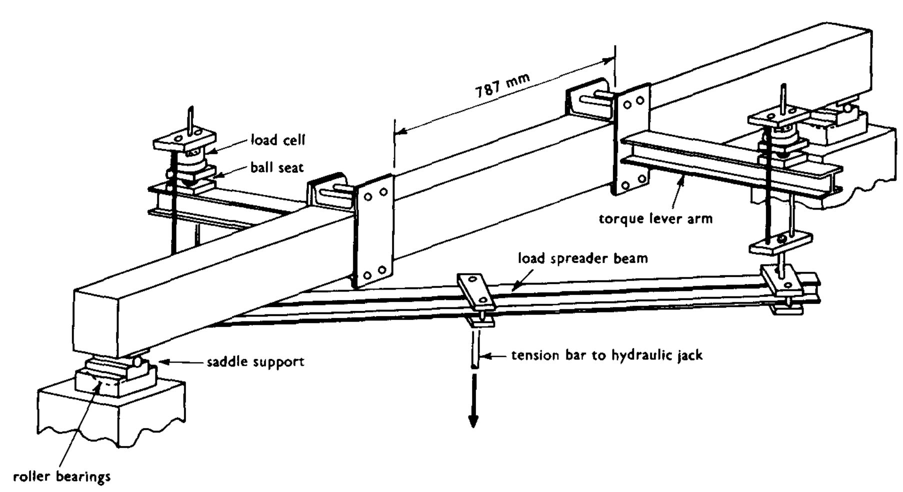


Figure 2: Isometric view of test set-up.

TABLE 1: **Details of symmetrically reinforced beams.**

Beam No.	Dimensions (mm)		Concrete properties (N/mm²)			Reinforcement details (per bar)						
	b	h	f_c'	u	f_t	A_{sL} (mm²)	$A_{sL}f_y$ (kN)	A_{sL}' (mm²)	$A_{sL}'f_y'$ (kN)	A_w (mm²)	A_wf_{wy} (kN)	s (mm)
A5-1	102	203	28·9	34·4	2·7	127	61·9	127	61·9	32	12·1	127
A5-3			26·8	32·4	2·5						11·6	
A5-5			29·1	34·8	3·1						12·1	
A5-8			28·6	35·0	2·4						12·1	
B4-1	102	152	29·9	33·8	2·2	127	57·8	127	57·8	32	12·1	102
B4-3			29·3	34·7	2·5						12·1	
B4-5			27·9	35·1	2·7						12·1	
B4-8			29·2	32·9	2·8						11·8	
C4-1	152	203	28·8	—	2·9	127	61·9	127	61·9	32	12·1	102
C4-2			28·6	36·6	2·7		61·9		61·9		12·1	
C4-3			28·7	33·6	2·9		61·9		61·9		12·1	
C4-4			30·0	38·8	2·8		59·6		59·6		12·1	
C4-5			29·8	33·9	3·0		59·6		59·6		12·1	
C4-6			29·8	35·0	2·3		58·7		58·7		10·5	
C4-7			30·5	34·8	2·3		58·7		58·7		12·1	
C4-8			29·2	33·9	2·8		58·7		58·7		12·1	
D4-1	152	152	30·7	35·9	2·5	127	58·7	127	58·7	32	12·1	102
D4-2			30·8	35·8	2·3		59·6		59·6		12·1	
D4-3			30·7	36·7	2·6		59·6		59·6		12·1	
D4-4			27·7	33·6	2·5		59·6		59·6		12·1	
D4-5			30·9	34·1	2·7		59·6		59·6		10·2	
D4-6			27·4	—	1·8		59·2		59·2		10·2	
D4-7			28·9	34·8	2·7		59·6		59·6		10·2	
D4-8			27·6	31·3	2·4		59·6		59·6		10·2	
E4-1	203	152	28·5	34·9	2·7	127	59·6	127	59·6	32	12·1	102
E4-2			28·8	37·2	2·5		59·6		59·6		12·1	
E4-3			29·1	35·9	2·5		58·7		58·7		10·2	
E4-4			35·3	45·2	3·8		58·7		58·7		10·2	
E4-5			30·1	34·5	2·5		58·7		58·7		10·2	
E4-6			29·4	32·6	2·5		58·7		58·7		10·2	
E4-7			29·7	37·7	2·6		58·7		58·7		10·2	
E4-8			28·3	34·5	2·7		58·7		58·7		10·2	
F3-1	203	102	26·7	29·1	2·5	127	58·7	127	58·7	32	10·2	76
F3-2			26·6	31·5	2·6		59·6		59·6			
F3-3			28·9	36·5	2·6		59·6		59·6			
F3-4			23·2	29·2	2·5		59·6		59·6			
F3-5			26·3	30·5	2·3		59·6		59·6			
F3-6			24·7	—	—		59·6		59·6			
F3-7			24·7	29·1	1·5		59·6		59·6			
F3-8			25·6	29·7	2·3		59·6		59·6			
G2-1	152	203	29·6	36·2	2·9	82	21·2	82	21·2	28	8·4	51
G2-3			32·9	37·0	2·9		21·2		21·2		8·4	
G2-5			31·4	37·7	3·1		20·2		20·2		8·8	
G2-6			30·9	36·9	2·8		20·2		20·2		8·8	
G2-7			32·5	38·0	2·8		20·2		20·2		9·8	
G2-8			30·2	35·8	2·0		21·2		21·2		9·2	

TABLE 2: **Details of unsymmetrically reinforced beams.**

Beam No.	Dimensions (mm)		Concrete properties (N/mm^2)			Reinforcement details (per bar)						
	b	h	f_c'	u	f_t	A_{sL} (mm^2)	$A_{sL}f_y$ (kN)	A_{sL}' (mm^2)	$A_{sL}'f_y'$ (kN)	A_w (mm^2)	A_wf_{wy} (kN)	s (mm)
AU-1	102	203	27·6	33·3	2·6	127	52·1	71	31·6	33	12·1	127
AU-3			28·8	32·7	2·7							
AU-5			24·8	28·7	2·5							
AU-8			27·4	32·4	2·8							
CU-1	152	203	28·9	35·5	2·4	127	52·1	71	31·6	33	12·1	102
CU-2			35·4	41·5	2·8						12·5	
CU-3			33·5	39·6	3·6						12·1	
CU-4			28·5	35·4	2·6						12·5	
CU-5			28·2	34·1	2·4						12·1	
CU-6			28·9	33·7	2·9						11·0	
CU-7			29·7	36·4	3·0						11·7	
CU-8			26·2	31·6	2·7						11·2	
DU-1	152	152	32·3	37·2	3·0	127	52·1	71	31·6	33	10·8	102
DU-2			31·3	35·7	3·1						10·8	
DU-3			29·4	36·5	2·8						11·9	
DU-4			31·3	35·8	3·0						11·9	
DU-5			33·1	37·9	2·8						10·9	
DU-6			31·8	35·8	3·0						10·9	
DU-7			29·6	37·6	2·9						11·7	
DU-8			25·6	34·1	2·7						11·7	
EU-1	203	152	28·2	35·1	3·2	127	52·1	71	31·6	33	12·1	102
EU-2			30·8	35·3	2·9		52·4		32·2		11·7	
EU-3			28·9	35·1	3·2		52·1		31·6		12·1	
EU-4			30·3	34·8	2·9		52·1		31·6		11·7	
EU-5			30·4	35·8	2·9		52·1		31·6		11·7	
EU-6			30·6	36·2	2·8		52·1		31·6		11·0	
EU-7			30·8	39·1	2·8		52·1		31·6		11·7	
EU-8			29·2	36·1	2·8		52·1		31·6		12·1	
GU-1	152	203	29·8	36·7	2·7	82	21·2	28	8·8	28	8·4	51
GU-3			28·9	34·4	2·9		21·7		9·4		9·0	
GU-5			34·0	39·4	3·4		20·3		8·8		8·9	
GU-6			30·5	34·0	2·9		21·2		8·8		8·9	
GU-7			30·4	38·8	2·6		19·8		8·8		9·2	
GU-8			31·1	37·4	3·3		22·0		9·1		9·2	

moments, the desired ratio of these moments for any given test being obtained by a suitable choice of shear span and the position of the loads on the torque arms.

TEST PROCEDURE

All beams were tested with their trowelled surfaces uppermost. Load was increased to failure in approximately thirty increments, the magnitude of these depending upon the stage of loading. Before cracking, rotation measurements were taken immediately the required load had been attained. After cracking, the beams exhibited time-dependent behaviour and a period of about three minutes was allowed for conditions to stabilize before rotations were measured. The load was noted both before and after the rotation measurements and the average load recorded. Beyond the yield load (see Figure 3), the load was recorded at approximately equal increments of rotation until the test was discontinued at a point beyond appreciable concrete crushing or when an appreciable drop in load had occurred. The total duration of each test was about two hours.

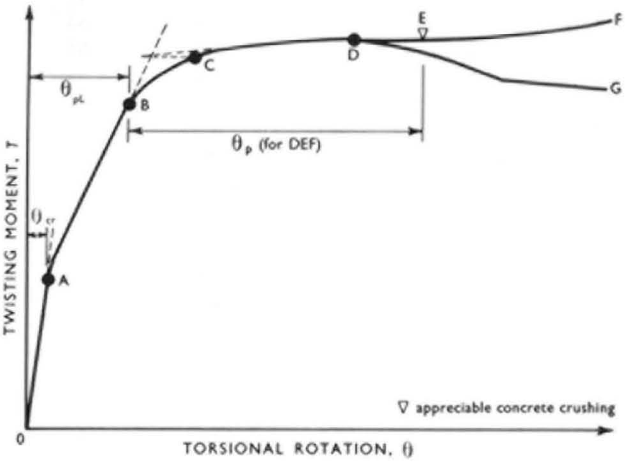
ROTATION MEASUREMENT

Torsional rotations and vertical displacements, at the main supports, at mid-span and at sections 381 mm each side of mid-span, were measured by using dial gauges. These gauges, reading directly to 0·01 mm, were located 500 mm apart at the ends of transverse arms attached to the beam at each of the five sections (Figure 4). Torsional rotations over the central 762 mm of the beams were obtained directly from the rotations measured at the ends of this central length. Flexural rotations over the same length were derived from the measured vertical displacements, as described in the Appendix.

Test results

Typical moment-rotation curves are shown in Figures 5 and 6. The flexural rotations for beam C4-8 have been omitted for clarity, as these were negligible and similar to those shown for beam G2-8.

The bending and twisting moments at yield and failure as defined in Figure 3, the flexural and torsional rotations over the central 762 mm and the directions of the associated resultant rotation vectors (axes of rotation) for all beams are given in Tables 3 and 4. Flexural rotations, φ_p, appear in some cases (marked ‡ in Table 4) to be inconsistent with the values of φ_p obtained for other beams in their respective series. As discussed in the Appendix, however, some doubt must exist as to the reliability of the flexural rotations for the pure torsion tests and in particular the rotations φ_p which are associated with extensive cracking. It is suggested that, as the associated rotations φ_{pL} are in every case negative, more reliance might be placed on



A: cracking
B: proportional limit
C: yield load
D: failure load for DG
E: failure load for DEF

Figure 3: Idealized moment-rotation curve.

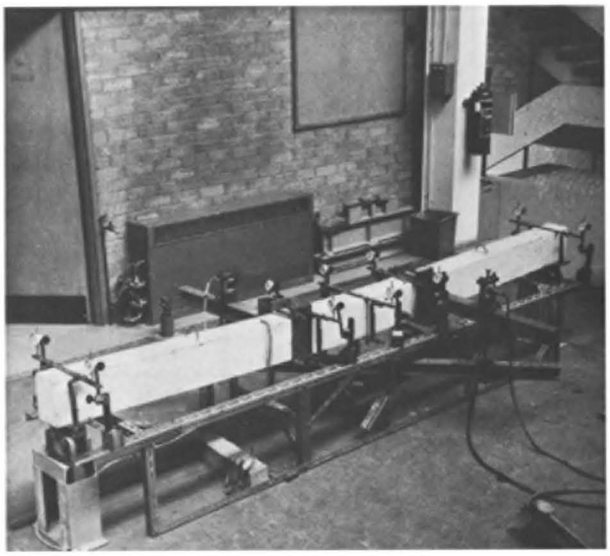
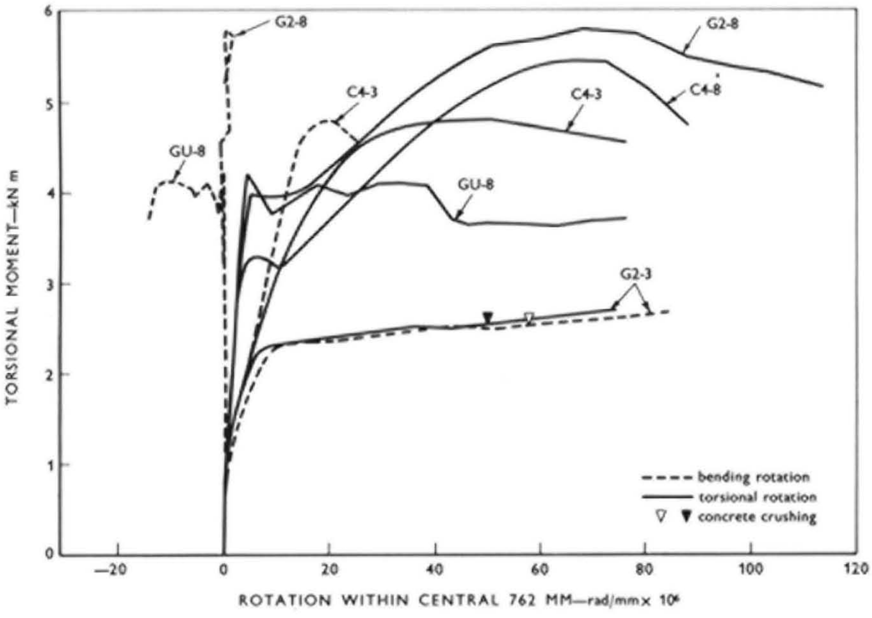


Figure 4: View of test rig.



Beam No.	*T/M*
C4-3 *G2-3*	0·4
C4-8 *G2-8* *GU-8*	∞

Figure 5: Moment-rotation curves (rotations within central 762 mm).

TABLE 3: **Test results for symmetrically reinforced beams.**

Beam No.	Applied T/M	Experimental moments* (kN m)			Measured rotations over central 762 mm—(rad/mm × 10^6)				Axis of rotation (degrees) $\tan^{-1}\theta_p/\varphi_p$
		T_y	M_y	M_f	θ_{pL}	θ_p	φ_{pL}	φ_p	
A5-1	0	0	20·23	20·68	0	0	20·9	85·8	0
A5-3	0·4	2·41	6·92	7·29	31·9	72·0	8·7	4·0	87
A5-5	0·8	2·54	3·66	†	48·8	72·4	4·7	2·0	88
A5-8	∞	2·42	0·12	†	25·6	89·4	0	0	90
B4-1	0	0	13·00	13·45	0	0	24·8	83·1	0
B4-3	0·4	1·74	5·07	5·19	24·8	35·8	9·8	4·7	83
B4-5	0·8	1·67	2·48	†	23·6	39·4	7·1	2·4	86
B4-8	∞	1·90	0·09	†	35·4	36·2	0	0	90
C4-1	0	0	21·24	21·75	0	0	19·7	82·7	0
C4-2	0·2	3·57	18·77	†	13·0	54·7	18·1	36·2	56
C4-3	0·4	4·68	12·95	13·25	20·1	31·1	12·6	8·3	75
C4-4	0·6	4·97	9·59	10·60	27·6	58·3	12·6	5·9	84
C4-5	0·8	5·19	7·12	†	41·3	39·0	7·9	3·2	85
C4-6	1·0	4·54	5·21	5·41	34·2	50·4	6·7	2·4	87
C4-7	1·5	5·31	3·96	†	53·5	48·0	5·5	1·2	89
C4-8	∞	5·47	0·18	†	35·8	37·8	0	0	90
D4-1	0	0	13·00	13·50	0	0	27·2	84·6	0
D4-2	0·2	2·32	12·40	13·24	13·8	63·4	25·2	47·6	53
D4-3	0·4	3·30	9·23	†	17·7	31·5	17·7	13·4	67
D4-4	0·6	3·42	6·74	6·90	20·1	51·6	15·4	9·4	80
D4-5	0·8	3·33	4·68	4·81	24·8	34·6	12·2	3·5	84
D4-6	1·0	3·16	3·70	3·75	26·8	41·3	9·4	2·8	86
D4-7	1·5	3·10	2·40	2·41	27·2	49·6	5·9	1·6	88
D4-8	∞	3·16	0·13	†	25·2	44·5	0	0	90
E4-1	0	0	14·24	14·86	0	0	21·3	145·3	0
E4-2	0·2	2·53	12·58	14·16	9·8	53·2	23·2	68·1	38
E4-3	0·4	3·55	10·38	†	13·8	48·2	18·1	29·6	59
E4-4	0·6	4·56	8·90	†	20·1	50·8	18·5	22·8	66
E4-5	0·8	5·11	7·03	†	24·4	39·4	12·2	5·9	81
E4-6	1·0	5·20	5·86	†	26·0	58·3	13·8	7·5	83
E4-7	1·5	5·29	3·32	†	19·7	54·7	10·2	3·2	87
E4-8	∞	4·86	0·18	†	24·4	42·5	0	0	90
F3-1	0	0	7·68	8·36	0	0	56·3	147·2	0
F3-2	0·2	1·33	7·36	8·32	8·7	52·0	33·5	113·8	25
F3-3	0·4	2·12	6·18	6·80	18·1	92·1	28·7	87·8	46
F3-4	0·6	2·46	5·04	5·20	26·0	81·1	24·0	47·6	60
F3-5	0·8	2·94	4·14	4·23	29·9	82·7	21·3	23·6	74
F3-6	1·0	2·88	3·36	†	27·6	64·6	19·7	15·0	77
F3-7	1·5	2·81	2·18	†	37·0	69·3	18·9	7·9	84
F3-8	∞	2·71	0·12	†	24·0	61·8	0	0	90
G2-1	0	0	7·74	8·07	0	0	7·9	42·1	0
G2-3	0·4	2·35	7·12	7·53	4·7	45·3	6·7	51·2	41
G2-5	0·8	3·72	5·31	5·51	8·7	99·6	6·7	50·4	63
G2-6	1·0	4·29	4·96	4·99	8·7	102·3	4·7	41·3	68
G2-7	1·5	4·70	3·56	3·60	17·7	62·2	6·3	20·1	72
G2-8	∞	5·79	0·18	†	36·6	42·1	0	0	90

* Total moments (including self-weight and weight of test rig)
† $M_f = M_y$

TABLE 4: **Test results for unsymmetrically reinforced beams.**

Beam No.	Applied T/M	Experimental moments* (kN m)			Measured rotations over central 762 mm—(rad/mm × 10^6)				Axis of rotation (degrees) $\tan^{-1}\theta_p/\varphi_p$
		T_y	M_y	M_f	θ_{pL}	θ_p	φ_{pL}	φ_p	
AU-1	0	0	15·90	16·40	0	0	8·3	80·3	0
AU-3	0·4	2·78	7·85	†	42·9	39·0	7·9	1·2	88
AU-5	0·8	2·54	3·66	†	31·5	50·2	4·3	1·1	89
AU-8	∞	2·49	0·12	†	51·2	80·3	−2·0	2·7‡	88
CU-1	0	0	17·12	17·68	0	0	16·4	96·6	0
CU-2	0·2	2·79	15·04	15·83	8·3	63·4	14·2	51·2	51
CU-3	0·4	4·48	12·44	12·63	13·4	37·4	13·4	10·2	75
CU-4	0·6	5·45	10·38	10·49	20·5	36·6	10·2	6·3	80
CU-5	0·8	5·20	7·15	7·26	29·8	43·7	8·6	2·4	87
CU-6	1·0	5·38	6·04	†	32·3	39·0	5·5	1·6	88
CU-7	1·5	5·20	3·89	4·00	22·0	52·4	3·7	0·8	89
CU-8	∞	4·92	0·18	†	25·2	48·0	−2·4	3·9‡	85
DU-1	0	0	12·09	12·60	0	0	28·0	71·6	0
DU-2	0·2	2·16	11·60	12·05	9·1	51·6	19·7	60·7	40
DU-3	0·4	3·16	8·89	9·40	19·3	57·5	18·9	28·4	64
DU-4	0·6	3·66	7·14	†	22·0	18·5	15·0	3·5	79
DU-5	0·8	3·76	5·21	†	21·3	35·4	11·8	3·5	84
DU-6	1·0	3·49	4·02	†	17·3	49·2	9·8	4·3	85
DU-7	1·5	3·62	2·75	†	15·8	37·8	6·3	1·2	88
DU-8	∞	3·57	0·13	†	44·5	46·1	−7·5	−3·4	94
EU-1	0	0	12·09	12·77	0	0	18·5	91·3	0
EU-2	0·2	2·24	12·24	12·80	6·3	52·0	19·3	81·5	33
EU-3	0·4	3·57	10·16	11·06	12·2	68·1	18·5	45·7	56
EU-4	0·6	4·63	9·01	9·11	15·0	69·3	16·2	33·1	64
EU-5	0·8	5·24	7·20	†	20·9	28·7	15·0	7·5	75
EU-6	1·0	5·45	6·10	6·35	19·7	34·6	13·8	7·1	78
EU-7	1·5	5·79	4·13	4·38	20·1	29·9	10·6	1·1	88
EU-8	∞	5·11	0·18	†	37·4	52·4	−6·2	2·2‡	88
GU-1	0	0	7·68	8·08	0	0	7·9	47·2	0
GU-3	0·4	2·42	7·29	7·58	4·9	32·9	7·5	36·2	42
GU-5	0·8	4·09	5·76	†	10·2	82·3	7·3	38·8	65
GU-6	1·0	4·11	4·78	4·82	10·6	63·0	6·3	26·8	67
GU-7	1·5	4·52	3·44	3·51	12·2	57·9	5·5	14·2	76
GU-8	∞	4·11	0·18	†	4·7	33·9	−0·2	−12·4	110

* Total moments
† $M_f = M_y$
‡ See text

Beam No.	*T/M*
CU-2 *EU-2*	*0·2*
CU-4 *EU-4*	*0·6*

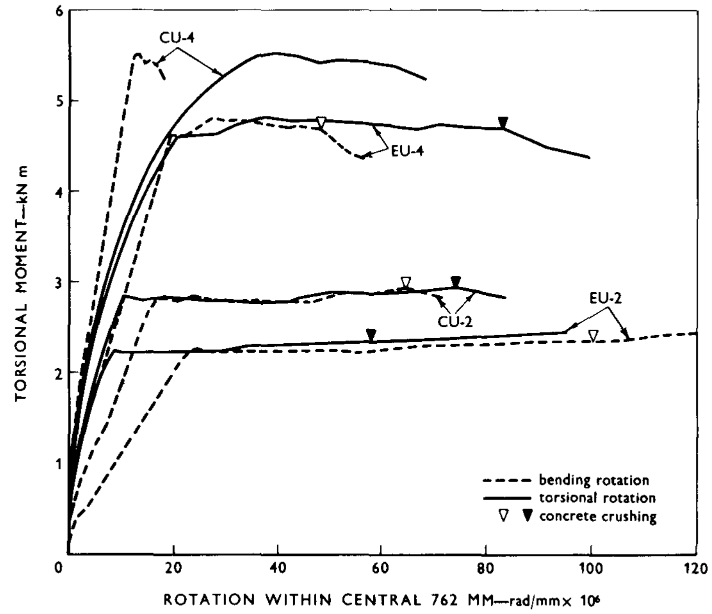


Figure 6: Moment-rotation curves (rotations within central 762 mm).

the negative rather than the positive, and apparently inconsistent, values of φ_p.

Typical interaction diagrams showing values of bending and twisting moment at the proportional limit, at failure and, where this does not coincide with the latter, at yield for beams in series C4, G2, CU, EU and GU are shown, together with experimental interaction curves for yield, in Figures 7 to 11 respectively. Incremental rotation vectors are shown at various stages beyond the proportional limit for each beam, each vector representing the incremental rotation occurring between it and the preceding vector.

The yield moments for all beams are plotted in non-dimensional form in Figure 12, the non-dimensional parameters M and T being so chosen that points $(-1,0)$, $(0,1)$ and $(1,0)$ are common to all beams. When these yield moments were being plotted, an allowance was made for individual torsional strengths for all beams for which differences from the average strength of the remainder of the beams in their respective series were estimated to exceed 6%. These were beams C4-6, D4-2, D4-3, D4-4 and E4-2, for which torsional strengths were estimated to be -10%, $+12\%$, $+12\%$, $+10\%$ and $+10\%$ in relation to averages for their respective series.

Discussion

ROTATION CAPACITY

Rotations θ_p and φ_p occurring beyond the proportional limit (see Tables 3 and 4) are in general at least comparable with, and in many cases appreciably greater than, their associated pseudo-elastic rotations, θ_{pL} and φ_{pL} respectively. It is possible that these rotations are sufficient to permit the required redistribution of moments within a structure, following the formation of the first plastic hinge(s), for the subsequent development of a collapse mechanism. Further investigation to determine the required rotation capacities is necessary before any more detailed and worthwhile discussion of the adequacy of the available rotation capacities is possible.

AXES OF ROTATION AND THE PLASTIC FLOW LAW

The directions of the axes of rotation during the pseudo-plastic stage, namely $\tan^{-1} \theta_p/\varphi_p$ in Tables 3 and 4, would seem to indicate that the use by many investigators of the term '*the* axis of rotation', when discussing failure surfaces in combined torsion and flexure, is incorrect. Concrete crushing may indicate '*an* axis of rotation', but it is apparent that component rotations about some other axis must also be present to produce the resultant axes of rotation observed in this investigation. It should be noted that the possibility of large rotations about some other axis is not precluded by the absence of any other visible crushing, for considerable rotations of under-reinforced concrete beams in pure bending, for example, occur before any signs of concrete crushing are apparent.

The incremental rotation vectors beyond the proportional limit (Figures 7 to 11), whilst showing small directional changes with increasing load, are seen to

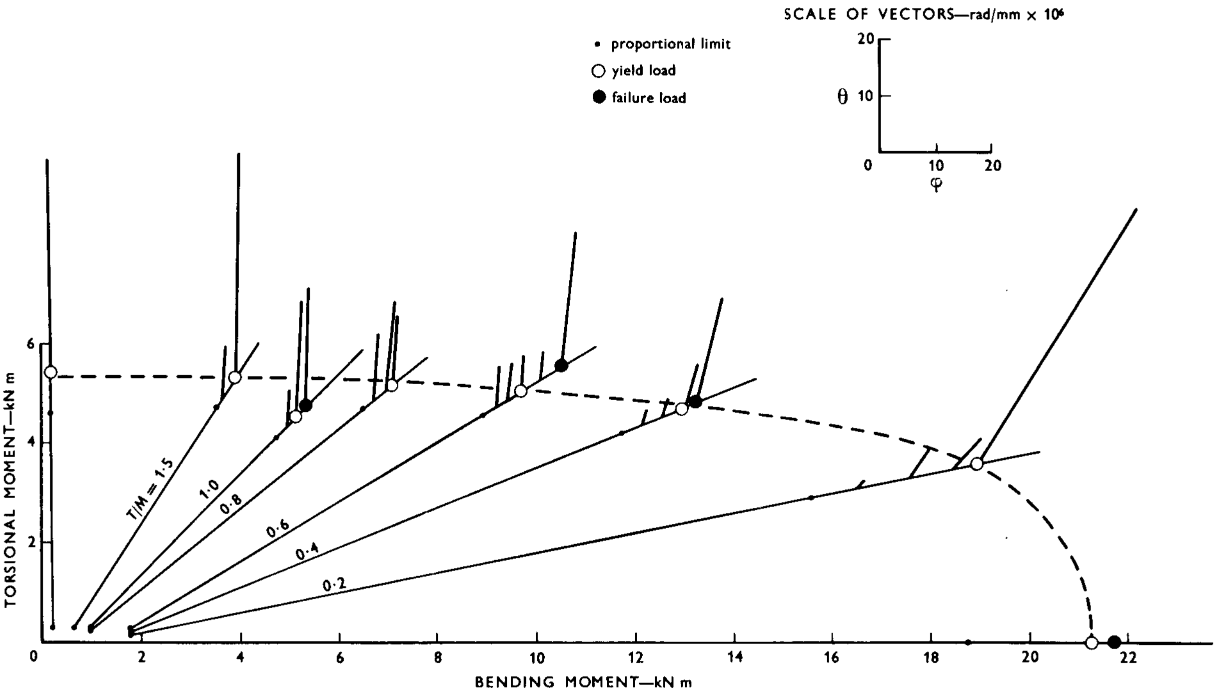


Figure 7: Interaction diagram (series C4).

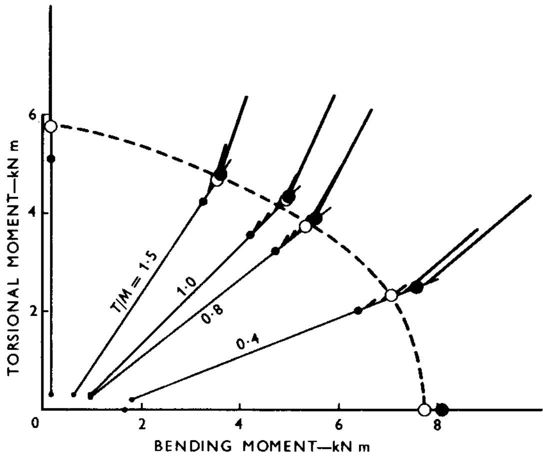


Figure 8: Interaction diagram (series G2).

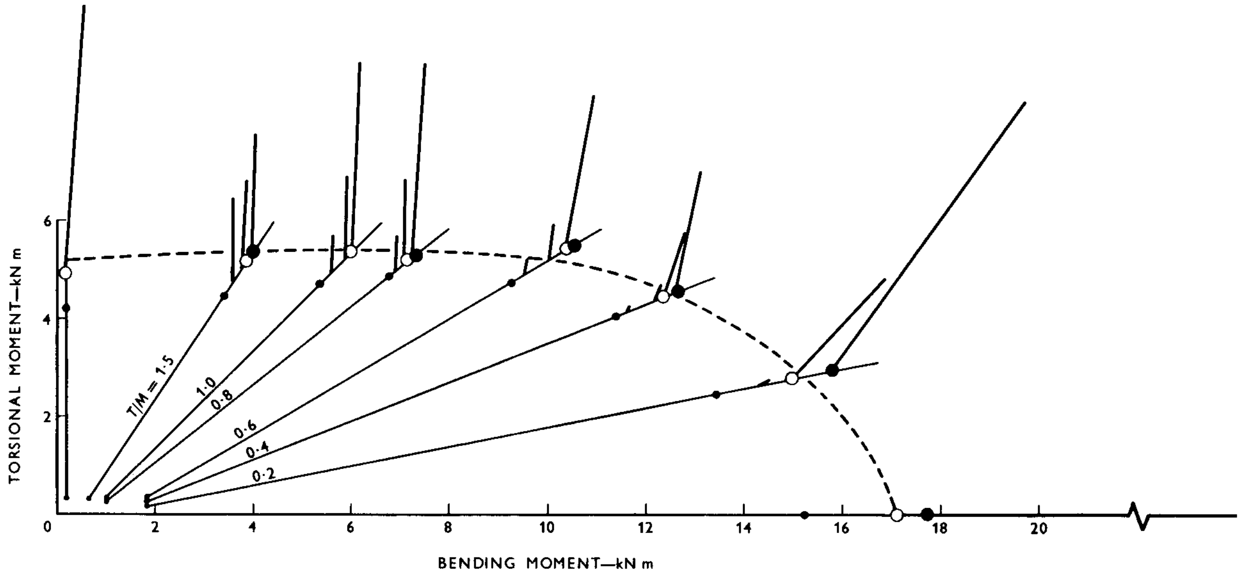


Figure 9: Interaction diagram (series CU).

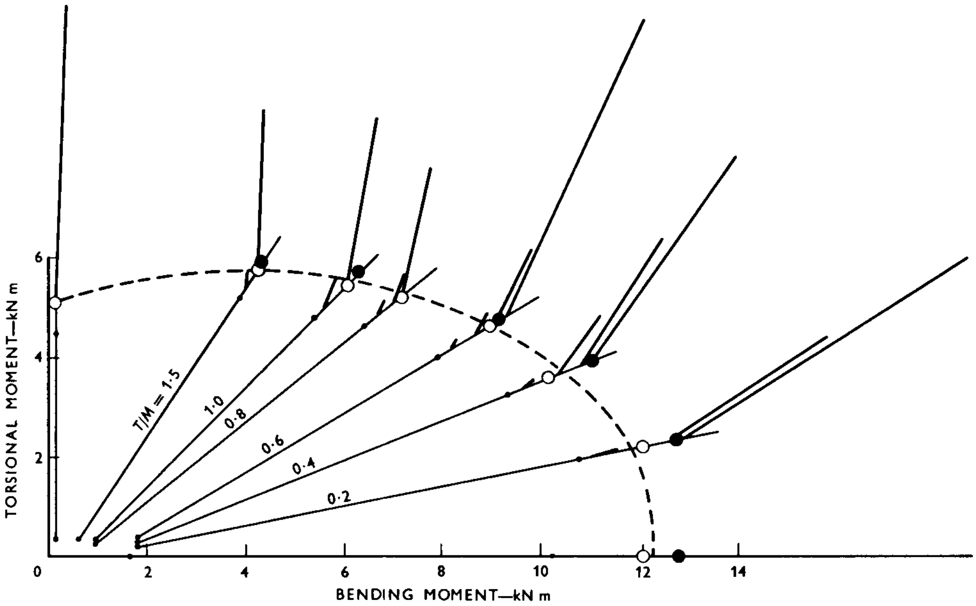


Figure 10: Interaction diagram (series EU).

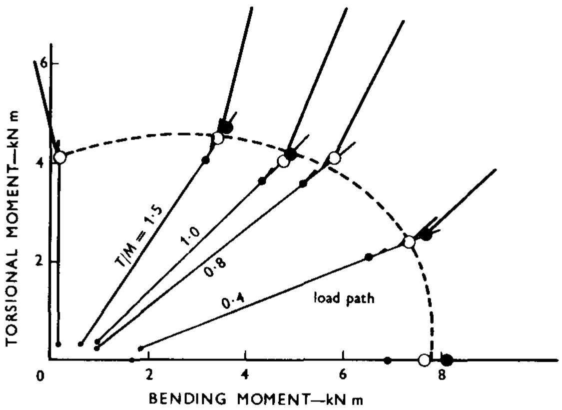


Figure 11: Interaction diagram (series GU).

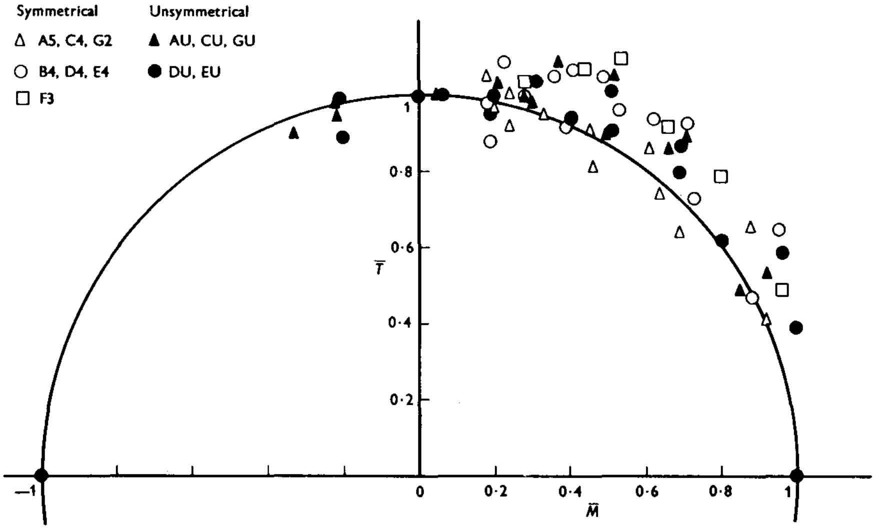


Figure 12: Non-dimensional interaction diagram.

be essentially normal to the interaction curves at the points corresponding to their associated yield moments. This 'normality criterion', more generally known as the plastic potential flow law, may be stated

$$\begin{aligned}\Delta\varphi &= \lambda \frac{\partial f}{\partial M}\\ \Delta\theta &= \lambda \frac{\partial f}{\partial T}\end{aligned} \qquad \ldots\ldots\ldots\ldots(1)$$

where $\Delta\varphi$ and $\Delta\theta$ are the incremental flexural and torsional rotations at a plastic hinge, f is the yield function, $f(M,T) = 0$, defining the interaction curve, and λ is any arbitrary positive scalar.

THE COMPLETE INTERACTION CURVE

For symmetrical sections, the complete interaction curve will be symmetrical about both the T and the M axes. Furthermore, for such sections under combined loading, it is not envisaged that any increase in twisting or bending moments beyond their respective values in pure torsion and flexure will occur. The fact that observed values of T_y exceed the pure torsional strengths in some cases (Table 3) may be attributed to normal variations between nominally identical members. It follows that the associated interaction curves must be everywhere concave to the origin.

Comparisons between the yield moments for both symmetrical and unsymmetrical sections, plotted in non-dimensional form in Figure 12, show that all moments lie within the same band, a reasonable approximation to which is provided by the circular curve $\bar{M}^2 + \bar{T}^2 = 1$. This confirms earlier agreement with a similar curve reported elsewhere(2).

It may be concluded that the complete interaction curves for the unsymmetrical as well as for the symmetrical beams considered in this investigation might be expected to be everywhere concave to the origin.

This leads to the important conclusion that not only are the directions of the rotation vectors uniquely prescribed at all points on the interaction curve by the plastic flow law but also there exists a unique relationship between the directions of the rotation vectors and the associated bending and twisting moments causing yield.

THE PLASTIC ANALYSIS OF STRUCTURES

Whilst the plastic analysis of a structure may not depend exclusively upon the applicability of either the plastic potential flow law or the uniqueness of the relationship between rotation vectors and yield moments discussed above, the complexity of any analysis in which either of these is not applicable must be considerably increased. Current methods of analysis(3,4), including the well-known upper- and lower-bound theorems of limit analysis, depend implicitly upon the applicability of both the above relationships and it is encouraging to find that both appear to be applicable to reinforced concrete members under combined flexure and torsion.

In an actual structure it is inevitable that shear also will be present at a plastic hinge, and further research into the moment-rotation characteristics of members subject to combined flexure, torsion and shear is necessary before the general applicability of normal methods of plastic analysis to reinforced concrete structures can be verified.

Conclusions

For the range of sections tested in this investigation, the following conclusions may be drawn.

(1) The complete interaction curve for both symmetrically and unsymmetrically reinforced sections is approximated to closely by an ellipse or, in non-dimensional form, a circle.

(2) The incremental plastic rotation vectors at all points on the associated interaction curve are in the directions of the outward normal to the curve. This is in accordance with the plastic potential flow law.

(3) Fundamental assumptions which form the basis of normal methods of plastic analysis of structures are satisfied by reinforced concrete members under combined flexure and torsion.

APPENDIX

Measurement of flexural rotations

Vertical displacements at sections a, b, c, d, and e (see Figure 13) were obtained for each load (or torsional rotation) increment by using dial gauges as described earlier. At any stage, the incremental displacements at a, b, d and e enable the incremental rotation $\Delta\varphi_s$ of chord ab relative to chord de to be determined. Whenever an increase of load occurs, this will be associated with a change of curvature along ab and de and, in consequence, $\Delta\varphi_s$ will be greater than the actual incremental flexural rotation $\Delta\varphi$ of the central portion bd. To determine the necessary correction factor, relating $\Delta\varphi$ with $\Delta\varphi_s$ beyond the proportional limit when the assumption of circular bending within the central portion is less acceptable, the following procedure was adopted.

In the early stages, before cracking, and later during initial cracking, it was assumed that essentially circular bending occurred within the central portion bd. The relative rotations of the ends of this central portion, φ_c, based on circular bending were obtained directly from the vertical displacements at b, c and d, whilst the relative rotations φ_s of chords ab and de were obtained from the displacements at a, b, d and e. A typical plot of maximum moment M against $(\varphi_s - \varphi_c)$ is shown in Figure 14. Before any cracking occurs within the shear span, $(\varphi_s - \varphi_c)$, which represents the effect of curvature along ab and de was directly proportional to the moment M, as indicated by 0–1. A similar essentially linear relationship was obtained following cracking within the shear span as indicated by 1–2 in Figure 14. Beyond point 2, as yield within the central portion was approached, the shape of this curve depended upon the location, within the critical central portion, of the larger cracks in relation to section c. The slope of the M versus $(\varphi_s - \varphi_c)$ plot along 1–2 was used to determine the required correction factor for any further change in moment beyond point 2.

Thus, for any moment increment ΔM, the corresponding incremental flexural rotation $\Delta\varphi$ was obtained from

$$\Delta\varphi = \Delta\varphi_s - \Delta M \tan\alpha$$

where α, the angle of inclination of 1–2 to the vertical, is as shown in Figure 14.

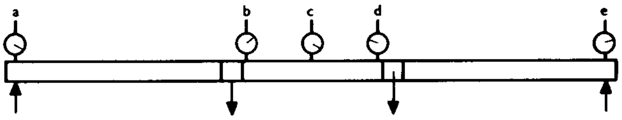


Figure 13: Location of displacement gauges.

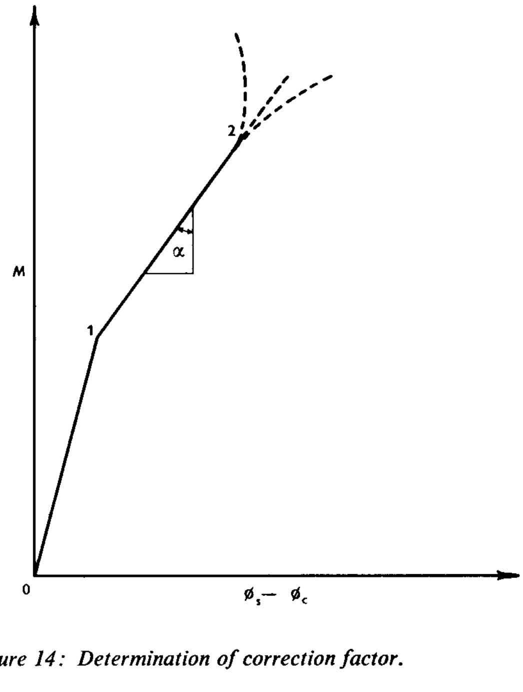


Figure 14: Determination of correction factor.

It should be noted that, for large values of T/M, no cracking occurred within the shear span, and yield within the central portion occurred beyond point 1 on the curve. For these tests, the slope of portion 0–1 was used to obtain the required correction factor.

For tests in pure torsion ($T/M = \infty$), flexural rotations were obtained by assuming circular bending throughout, no overhanging ends of the beam having been provided. Where negligible flexural rotations occurred, this approach was satisfactory but, for the unsymmetrically reinforced sections, where these rotations were not always negligible, the adequacy of the assumption of circular bending was very dependent upon the location of the larger cracks within the central portion.

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The apparatus and instrumentation for an investigation of the strains in mass concrete due to fluid pore pressure

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SYNOPSIS

The paper describes the apparatus and instrumentation that is being used to investigate the effects of triaxial stresses, including pore pressure, upon mass concrete. The specimens are hollow cylinders approximately 1·5 m high × 0·7 m outside diameter × 0·23 m inside diameter, of concrete with a 19 mm maximum aggregate size and a 28 day cube strength of approximately 28 to 42 N/mm².

Introduction

It is generally accepted(1) that the mortar matrix of any concrete incorporates two distinct types of pore—namely the gel and the capillary pores—in addition to the aggregate interface microcracks. The discrete gel pores, between 15 and 20 Å in mean diameter, are much smaller than the irregularly connected capillary pores, which are about 5000 Å in diameter. Because these pores are both numerous and interconnected, cement gel has a porosity of approximately 28% but, owing to their extremely small dimensions, the permeability(1) of the gel is only about 10^{-13} cm/s. The number of capillary pores depends upon the original water/cement ratio and the degree of cement hydration and, in theory, if the original water/cement ratio is 0·38, the gel volume on complete hydration is just sufficient to fill all the capillary pores(2). Higher water/cement ratios result in permanently open continuous capillary pores, and the introduction of fluid under pressure into these pores obviously affects the stresses within the concrete skeleton. It is evident that concrete aggregates also show pore-pressure effects, to varying degrees, depending upon their pore structure.

The importance of the magnitude of uplift due to the pore pressure acting on some 'effective area' has long been recognized by the designer of mass-concrete gravity dams(3). The present work aims both to verify previous results and to clarify the general triaxial strain behaviour, when pore pressures are included in the pattern of applied stress. The intuitive assumption that the ratio of the effective area to gross area (n) is numerically equal to the volumetric porosity(4) is disproved by experiment.

Important contributions to the understanding of pore-pressure uplift have been made by various workers, notably McHenry(5), Terzaghi(6), Leliavsky(7), Carlson(8,9) (reporting tests by Davis(10), and others) and Serafim(11,12), and their results, in general, indicate that n is in the region 0·8 to 1·0, although Serafim reports some values as low as 0·4. His explanation that these low figures were due to the immobility of adsorbed molecules of water is doubted by Powers(13). Leliavsky, with specimens 0·58 m high × 0·14 m outside diameter, Davies (0·76 m × 0·38 m o.d.) and Serafim (0·2 m × 0·05 m o.d.) used the type of apparatus represented diagrammatically in Figure 1.

The tensile axial component of stress, induced by the pore-water pressure in the concrete, was generally inhibited by the axial compressive stress. Leliavsky's tests terminated with the tensile failure of the specimen, from which the effective area at the moment of failure was deduced. His specimens were subjected to different values of applied axial compressive stress before the pore-water pressure was increased to failure. He maintained that, for design purposes, it was necessary to have information at the failure condition and he concluded from his results that the average value of n was then 0·91, independent of the concrete.

In the tests by Davis, the specimens were subjected to an axial stress of 1·4 N/mm² followed by a pore pressure of 1·4 N/mm². Since the values of n deduced from these results generally lie between 0·9 and 1, the specimens were consequently subjected to zero stress or a small resultant compressive stress.

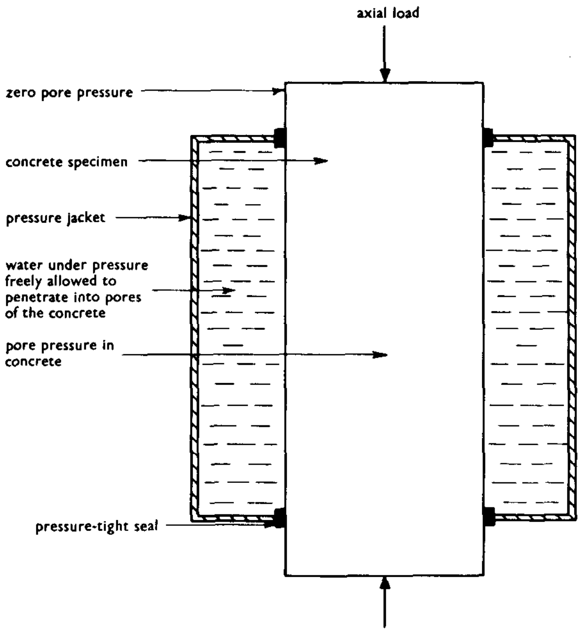

Figure 1: Diagrammatic representation of test configuration used by previous workers.

Serafim applied a pore-water pressure of approximately 0·5, 1, 2 or 4 N/mm^2 to specimens already subjected to a constant axial stress of approximately 4 N/mm^2. Using nitrogen as the pore fluid, he carried out three series of tests and concluded that n decreased with the subsequent change in resultant compressive stress. A similar conclusion can be drawn from Skempton's(14) discussion of 'effective stress' and contact area ratio, based on a Mohr-Colomb consideration of shear failure results.

Other authors(15,16) have suggested that n is either unity or near enough to unity for practical design purposes, and Moffat(3), accepting this premiss, has submitted to closer examination the pore-pressure distribution likely to occur during the lifetime of a dam.

The apparatus to be described has been designed to verify the previous conclusions on a somewhat larger scale, and in addition to extend the appreciation of the strain history into three dimensions.

Apparatus

A diagrammatic cross-section is given in Figure 2 and a general view of the apparatus is shown in Figure 3.

Because it was desired to carry out long-term tests under the adverse condition of high water pressure, a waterproof and stable means of measuring strain was needed. In addition, because it was proposed to use aggregate of 19 mm maximum size, it was necessary to have a minimum gauge length of 80 mm in order to measure average, rather than local, strain. For these reasons, 80 mm BRS type vibrating-wire gauges were chosen, to which a special waterproofing treatment (see later) was given.

The further need to achieve a uniform pore-water pressure in as short a time as possible seemed to dictate the use of a hollow cylinder in order to reduce the path length. Such an arrangement also permits the implementation of a pore-water pressure gradient through the specimen. The consideration of these factors, together with the requirement that measurements were to be taken in three orthogonal directions, led to the choice of a specimen size of approximately 1·5 m high × 0·7 m outside diameter × 0·23 m inside diameter.

Economics and the availability of a 2500 kN hydraulic press dictated the maximum pore-water pressure (7 N/mm^2) and a maximum axial stress of 7·5 N/mm^2. It was also necessary that the inner and outer pressure jackets should incorporate removable and separately pressurized seals, and Figure 4 shows the arrangement finally adopted. Separately pressurized seals were chosen to enable the strain response of the concrete specimen under the action of the seals to be measured and also to allow the restraint effect of the jacket to be examined. Some initial success was achieved by bedding the seal directly onto the concrete as cast but, after later difficulties had been experienced, it was decided to form rebates in the concrete, when the specimen was being cast, into which bands of polyester resin were subsequently cast. This procedure has minimized the troubles associated with the seals, especially in regard to the leaching action which was consequent upon a small leak between seal and concrete. The effect of seal pressure was tested by measuring strains in the specimen for different seal pressures. It was found that, under a high seal pressure, radial strain at level C (Figure 2) near the seal could be as high as 30 × 10^{-6}, but strains at levels A and B were too small to be detected. The seal pressure is recorded at each stage of the main tests to enable a correction to be made if required.

To estimate the restraint effect of the pressure jacket upon the specimen, tests were carried out in which the relationship between axial strain and load was measured firstly for a jacketed specimen with different seal pressures and secondly with the jacket removed. The strain behaviour was found to be independent of the presence of the jacket or the seal pressure. If any significant restraint effect had been observed, it had been the intention to submit the seals to short periods of reduced pressure during each combination of test pressures, allowing leakage and movement of the specimen relative to the jacket. In view of the results of the proving test, this was unnecessary.

The three sources of pressure required—for the operation of the hydraulic press (oil), the jacket (water) and the seals (water)—are provided respectively by two single-stroke Madan airhydropumps and one airhydropower unit, all operated by compressed

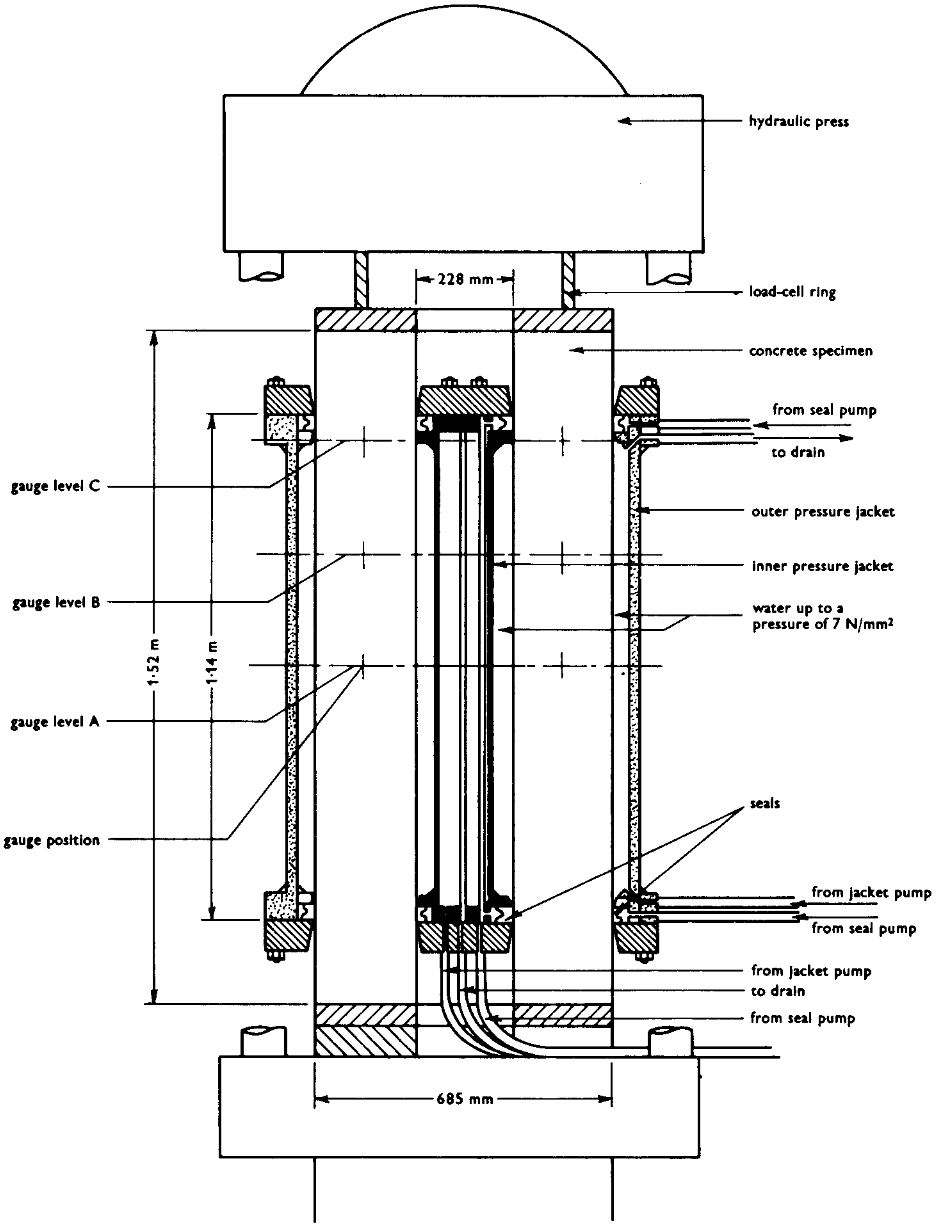

Figure 2: Diagrammatic cross-section of the apparatus.

air. These units automatically maintain the required pre-set pressures.

Assembly and curing the specimen were major factors in the design of the system as a whole. In order to avoid the incorporation of lifting devices in the specimen (approximately 1100 kg), it is first lifted by means of the mould and then positioned on a base-plate which has removable lifting bolts in its sides. A lifting cage is then used to place the concrete in a curing tank, and this is again utilized for positioning the specimen in the press. There are obvious difficulties in the positioning of the seal-retaining flanges, the closely toleranced jacket and the centre plug, and for these an overhead crane is necessary.

Instrumentation

For the reasons given above, 80 mm BRS type vibrating-wire gauges have been used to measure the changes in concrete strains in three orthogonal directions. Since up to eighteen of these gauges per specimen have been used and since they need to be scanned at pre-set time intervals, an automatic logging system was necessary, and it was therefore decided that, for simplicity, the remaining instrumentation would also be based on the vibrating-wire principle.

The axial load is measured not only by the oil pressure but also be a 3000 kN load-cell, in order to eliminate doubts about press friction, and this incorporates six such wires and exciter units.

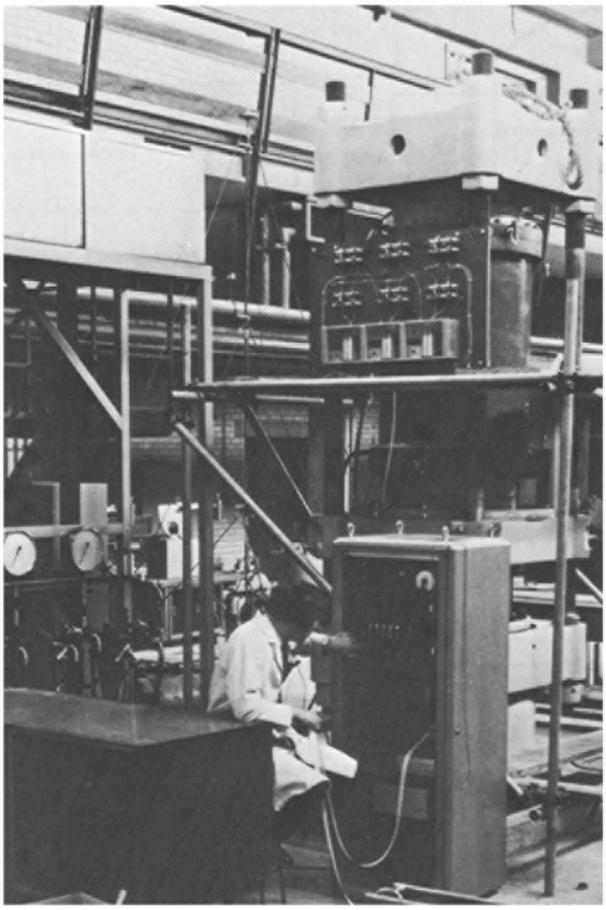

Figure 3: General view of apparatus.

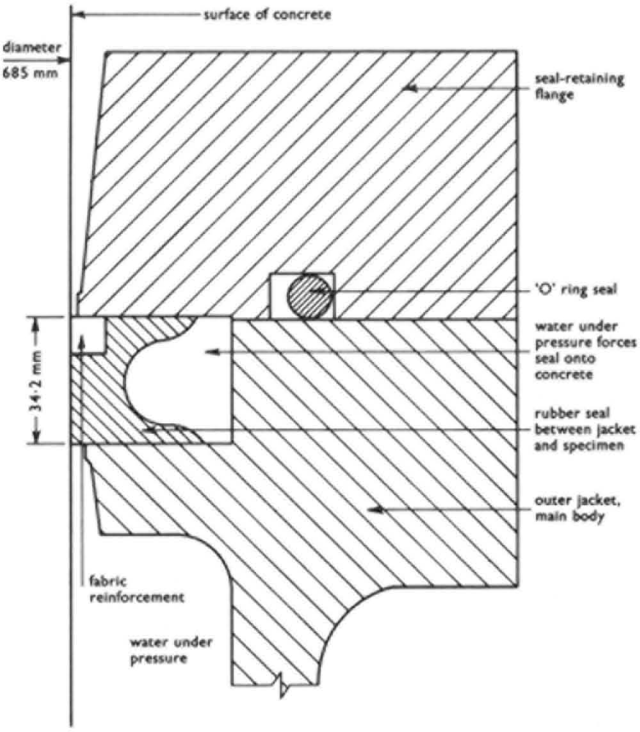

Figure 4: Detail of outside pressure jacket in the region of the seal.

Pore-pressure transducers are cast in the specimen but these have been less successful. They consist of sintered bronze 'probes' (approximately 22 mm diameter × 10 mm thick) coupled by 3·0 mm diameter high-rigidity plastic tubing to external beryllium copper pressure-sensitive diaphragms, the deflection of which is monitored by a vibrating wire. These worked well in the first specimen but those in the later specimens became blocked, not withstanding the double tube system necessary for de-airing. The rate of flow into the measuring system is small, dependent as it is upon the low permeability of concrete, but it was observed, in the first specimen, that an incremental jacket pressure produced full response in the pore-pressure transducer in less than one minute. In view of the difference in size between the capillary pores and cracks and the much smaller gel pores, it seems doubtful whether the pore-pressure measuring system, even when functioning correctly, measures a representative over-all pore pressure or the pressure in those larger pores which are intersected by the probe. It is thought probable that these two are different until steady-state conditions are reached. A similar diaphragm transducer connected to the pressure supply is used to measure and thereby log the water pressure in the jacket.

The strain gauges were modified and waterproofed to enable them to operate under the maximum water pressure of 7 N/mm^2. A new cable entry, incorporating a silicone rubber gland, has been provided in order that small-diameter (approximately 2 mm) single-core screened cable can be used, because the use of normal cable (approximately 7 mm dia.) would have meant a severe departure from the concept of mass concrete, in addition to providing possible large leakage paths. The strain gauges were completely coated in a thin layer of epoxy resin. A comparison of calibration factors between coated and uncoated gauges was carried out by embedding them in 500 × 100 × 100 mm concrete specimens, loading axially and measuring external axial strains with surface-mounted vibrating-wire gauges. No change in calibration factor was observed. Tests on plain concrete specimens indicated that, for the type of concrete used in the tests, the inclusion of the gauges produced no change in axial behaviour.

If the dumb-bell-shaped strain gauges are immersed in a fluid under pressure, a small amount of gauge compression is observed. It can, however, be considered that, in view of the fact that the embedded gauge is supported on all sides by concrete, the fluid pressure effect could be neglected. For each main specimen, a small cylindrical specimen was cast, of the same concrete, with a single embedded strain gauge. This was then subjected to a hydrostatic pressure in order to measure the 'intrinsic compressibility'. The value of the intrinsic compressibility as measured by these tests obviously includes this small gauge effect. The use of these values in the analysis will therefore compensate automatically for this small effect.

Figure 5 shows the mould, before casting, with the strain gauges and pressure probes positioned in space by means of vertical and hoop piano wires, to which they were attached by means of a rapid-curing steel-filled epoxy resin. The three instrumentation levels are indicated in Figure 2. In general there are, on each level, two gauges for vertical strain measurement, two for radial strain and two for tangential strain, together with two pore-pressure probes. All gauges are positioned with their centres at the same radius, namely on the centre-line of the wall. Each specimen is cast continuously but lift by lift in order to eliminate damage or displacement to the various instruments. The slightly tapered centre-hole-former was removed, by crane, about five hours after casting.

Test programme

Table 1 summarizes the testing carried out in this programme. All specimens were made with concrete having an aggregate/cement ratio of 8 and incorporating aggregate of 19 mm maximum size. The aggregate is of a middle grading corresponding to a scaled-down typical dam grading.

The testing sequence has been similar for each specimen, beginning at an age of at least two months and continuing for a further three months. The specimen is subjected to a series of jacket pressures from 0 to 7 N/mm^2 and at each pressure the external axial stress is varied between a low value at which the axial strain is small compressive up to the capacity of the press, 7·5 N/mm^2. The specimen is never allowed to go into tension.

After each load or pressure has been applied, one day or more is allowed for the pore pressure (which may have risen on the application of axial load) to reach equilibrium with the jacket pressure, and during this time hourly instrument readings are automatically taken and recorded by the data logger. The time-dependent strain due to creep is estimated from the results of creep testing also carried out on each specimen. This is done by observing the time-dependent strains during the four days after application of an axial stress and the four days after its removal. An estimate was obtained of the time-dependent strains from other causes by observing the specimen strains before and after this test at zero stress. The creep test is carried out before and after the main test series.

In addition, during the main test series, the specimen strains are recorded for each jacket pressure and corresponding minimum axial stress at the beginning and end of each axial loading cycle. This enables the time-dependent deformation which has occurred between these stages to be observed directly. The intermediate creep values are then obtained from the creep test results by means of the principle of superposition.

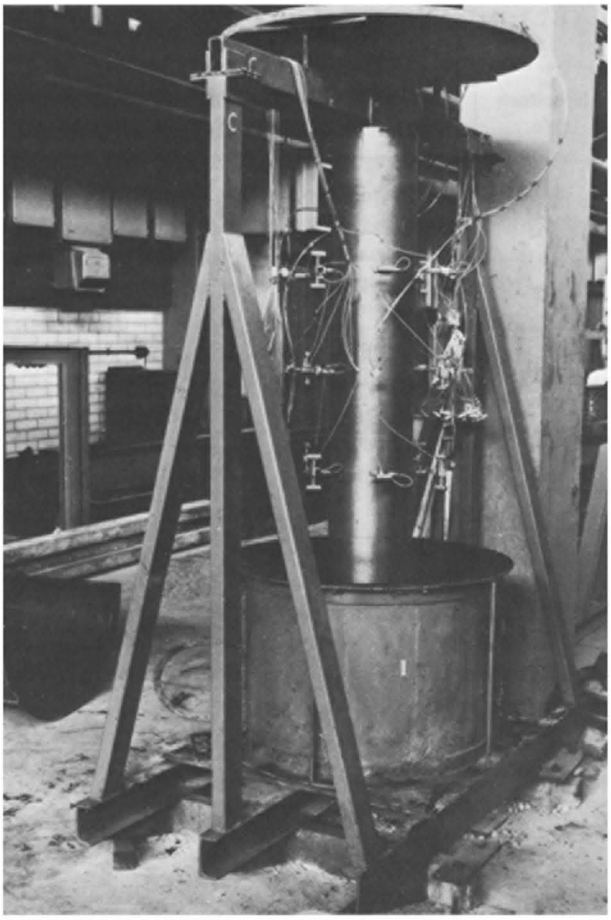

Figure 5: Specimen mould ready for casting, showing strain gauges in position.

An approximate analysis of the results indicates that the value of n is close to unity for the concrete tested. A more detailed examination of the results is in progress, including a continuous assessment of creep using a modified form of the principle of superposition, the intrinsic compressibility and the increase in modulus of the concrete with the ageing of the specimen during the test. If, as suggested by Serafim, n varies with the resultant compressive stress, this should be detectable from these results.

The apparatus can be readily adapted for other investigations. These could include measurement of strains under radial flow, examination of the effects and properties of construction joints and other discontinuities and inclusions, and the testing of specimens of variable cross-section in order to simulate a dam profile.

Conclusions

(1) A triaxial device for the testing of large cylindrical specimens of concrete has been constructed and is operating satisfactorily.

TABLE 1: **Summary of tests on precast hollow cylinders.**

Specimen No.	Type of cement	Water/cement ratio	Strength at 28 days (N/mm^2)	Number of strain gauges	Comments
1	Ordinary Portland	0·75	28	18	Specimen cracked during testing. All vertical strain gauges failed. All six pore-pressure probes operational.
2	Ultra rapid-hardening	0·75	39	8 (vertical only)	All strain gauges satisfactory. All the pore-pressure probes blocked.
3	Ordinary Portland	0·75	28	18	Specimen just tested. All strain gauges operational, pore-pressure probes blocked.
4	Ordinary Portland	0·55	42	18	Specimen just being set up (March 1971).

(2) The measurement of internal strain by means of the 80 mm BRS type vibrating-wire gauge has been accomplished under the adverse conditions of 7 N/mm^2 pore-water pressure.

(3) Measurement of pore-water pressure within the concrete by means of porous probes and vibrating-wire type transducers has been only moderately successful owing, it is believed, to the clogging of the pores of the probes.

(4) In a saturated specimen, it has been observed that the pore-pressure probes indicate a rapid rise in pore pressure corresponding to an increment of jacket pressure, but it is not known whether this is a general rise or restricted to the large continuous pores and cracks.

(5) The results obtained so far indicate a value of n close to unity.

ACKNOWLEDGEMENTS

The authors with to acknowledge their indebtedness to Professor R. C. Coates for providing the laboratory facilities to carry out the work and to the Science Research Council for financing the project.

The assistance of a number of specialist engineering manufacturers was received but particular acknowledgement is due to Aiton & Company Limited, Derby, who made the pressure jacket.

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Contributions discussing the above paper should be in the hands of the Editor not later than 31 March 1972.

UDC 624.953 : 666.94

Improvement in long-term bulk storage of cement for a concrete laboratory

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SYNOPSIS

The paper describes a simple and economical system which has been successfully used to preserve in good condition cement stored in a silo for a long period. The system also enables any leaks which may occur in the silo to be detected rapidly.

Introduction

A system for the long-term bulk storage of cement was described by Lunt and van der Meulen* in 1965. Since then an important improvement in the system for the preservation of the cement has been made, which it is felt will be of interest to other users of cement stored in bulk for long periods of time.

After several years of successful operation of the original storage system, it became evident that the steel silo was no longer watertight because certain welds had cracked. All the remaining cement therefore had to be rejected. In order to prevent the recurrence of such a loss, the air supply system was modified in such a way as to reduce the chances of non-detection of a leak through the silo walls. Routine maintenance was also thereby simplified.

Description of modification

Originally, the air entering the silo while cement was drawn off passed through a filter containing soda lime, calcium chloride and silica gel. This system was satisfactory for maintaining a dry, carbon-dioxide-free environment within the silo (as long as the silo was air-tight) although replacement of the silica gel and calcium chloride twice a week was a tedious chore. When it was first installed, the silo was checked for leaks, but no routine checking was done to ensure that it remained air-tight because it was expected that it would do so. In the event, leaks developed, permitting outside air to be sucked directly into the silo as a result of the nightly sharp drop in temperature. The situation was further aggravated by the sudden cooling of the outside air and the silo during rainstorms. This was only realized after deterioration of the cement had been discovered. It was accordingly decided to develop a system that would eliminate the need for regular replacement of the filter elements and enable any leaks in the silo to be detected rapidly.

The basis of the new system is the application of a small gas over-pressure to the silo. By using high-purity nitrogen for this purpose and making it the sole supply of gas to the silo (to replace the volume of cement drawn off) a dry, inert environment for the cement is ensured. The modified arrangement is shown schematically in Figure 1. The pressure in the silo is kept just above atmospheric pressure by means of a sensitive pressure-control valve together with the ordinary pressure gauge and regulator used on high-pressure gas-bottles. A simple mercury manometer is used to indicate the line pressure and an ordinary gas meter in the supply line indicates the volume of gas used. Gas consumption is noted daily. An irregular pressure drop or any sudden increase in the volume of gas used will give an immediate indication of the presence of a leak in the silo.

An additional advantage of the over-pressure in the silo is that, if a leak does develop, the risk of air or water entering is extremely small and only a small volume of low-cost gas will be wasted.

Comments on operation

The modified system has now been in use for over three years. During this time about 120 m^3 of nitrogen

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*Magazine of Concrete Research. Vol. 17, No. 51. June 1965. pp. 103–105.

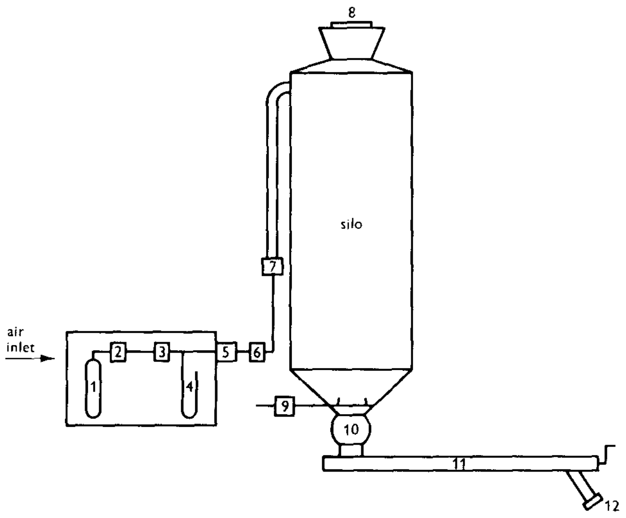

Figure 1: Diagrammatic layout of the bulk storage system.

- *(1) Nitrogen gas supply*
- *(2) Ordinary pressure gauge and regulator*
- *(3) Sensitive pressure-control valve*
- *(4) Mercury manometer*
- *(5) Ordinary gas meter*
- *(6) Stop valve*
- *(7) Silo filler-cap*
- *(8) Top sealing lid*
- *(9) Valve to fluidizing nozzles (not used in normal operation)*
- *(10) Rotary-vane feeder*
- *(11) Screw conveyor*
- *(12) Air-tight cap on outlet*

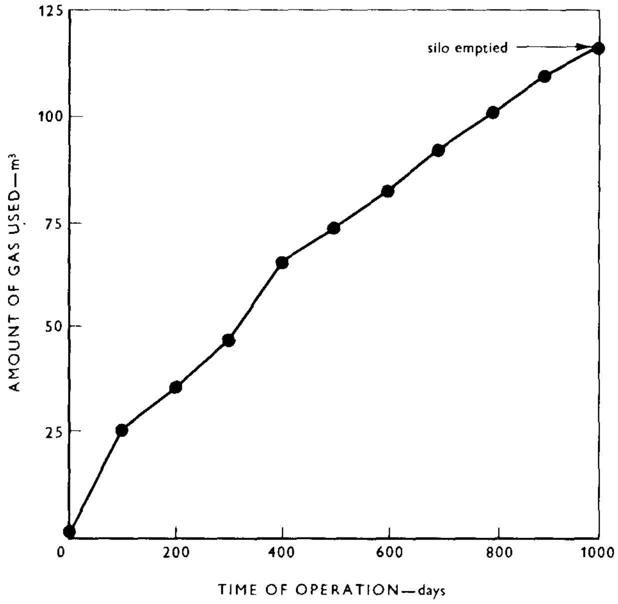

Figure 2: Amount of gas used during operation.

at atmospheric pressure have been used. At the time this system was adopted, it was feared that the loss of gas while cement was being drawn off from a nearly empty silo might become excessive. However, this has not been the case, as may be seen from Figure 2, which shows the gas consumption over a period of 33 months up to the time of emptying the silo. During this period there was a regular and fairly constant use of cement.

The reason why the loss of gas was constant when cement was drawn out of the silo was that the cement had become rather well compacted, by its own weight, during the long storage, thus allowing only a very slow migration of gas through the cement. In addition the last two tons of cement were drawn off in one batch and transferred to bins, in order to receive a new supply in the silo for new projects.

During the storage period, control tests on the cement were carried out at intervals. The results of these tests are given in Table 1. It can be seen that no appreciable changes in the cement took place.

Conclusion

After more than three years of use, it is clear that the nitrogen supply system described is very reliable and requires very little maintenance. The amount of gas used involves such a small outlay that this method of storage and preservation of cement over long periods can be highly recommended.

Contributions discussing the above paper should be in the hands of the Editor not later than 31 March 1972.

TABLE 1: **Properties of cement stored in the silo.**

Time after arrival of cement at which sample was taken (days)	Compressive strength of water-cured specimens (N/mm²)			Setting times (min)	
	150 mm concrete cubes tested at 28 days	70·6 mm mortar cubes tested at 3 days	7 days	Initial	Final
0	30·1	19·0	33·8	175	238
6	28·5	18·2	31·0	184	234
13	28·1	19·3	31·0	150	250
27	28·5	20·3	31·0	165	245
51	29·9	17·6	29·7	165	260
112	33·3	18·6	28·6	140	230
223	31·9	19·0	30·0	180	230
709	31·2	20·5	29·3	175	258
929	32·3	18·0	32·1	170	245
1035	30·3	22·8	31·6	190	250

Discussion on a paper published in

Magazine of Concrete Research

Volume 22, Number 70: March 1970

Destructive tests on rationally designed slabs*

M. A. Muspratt

Contribution by G. I. N. Rozvany, PhD

Civil Engineering Department, Monash University, Clayton, Victoria, Australia

The experiments described in the paper under discussion formed part of the ARGC project No. 65-15897 and were carried out in the Civil Engineering Department, Monash University. For this project I was the Chief Investigator.

Membrane action

In explaining membrane action, Mr Muspratt gives in his Figure 13 a cross-section of the slab showing compressive and tensile zones and lever arm as indicated in Figure Ia of this discussion. This diagram is based on one in a paper by Taylor(1) but Taylor's diagram appears to have been misinterpreted. Such a stress pattern would arise in a homogeneous beam cross-section under flexure and the moment capacity of the beam would be proportional to the lever arm indicated. However, the same does not apply to two-way slabs. The diagram by Taylor(1) (see Figure Ib) considers the stresses along yield lines on a rigid segment T. The resultant of these stresses is a couple that balances the moment about the edge support of the external loads on segment T. The neutral axis can be determined from considerations of plastic potential(2) and is not horizontal in general. Instead of the 'centroid of the tensile zone', the centroid of the tensile reinforcement determines the lever arm. Naturally, if more reinforcement is concentrated along the central band of the slab (as in Hillerborg's solution and in the torsionless solution), the membrane action is more pronounced owing to the greater lever arm. This statement has been verified by the above experiments.

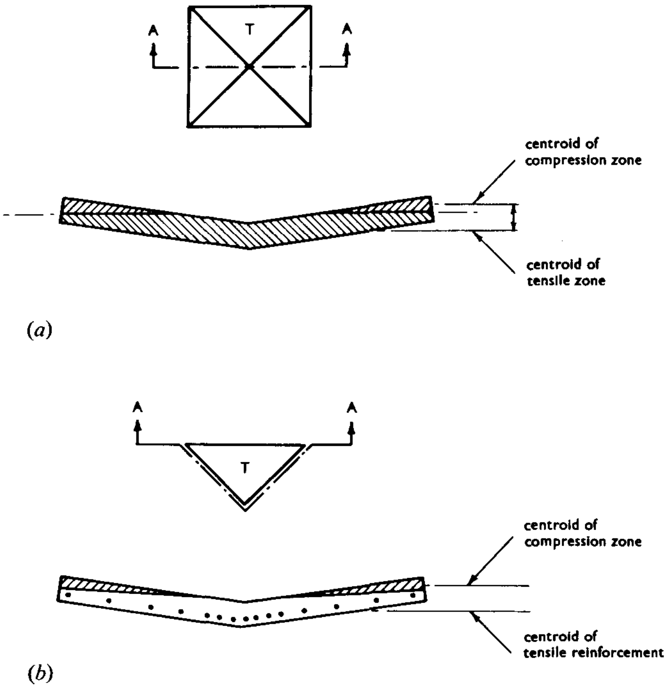

Figure I

The experimental points shown in Figure 12 of Mr Muspratt's paper do not seem to agree with the measurements recorded during the experiment.

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*Pages 25 to 36 of *Magazine* No. 70

'Monte Carlo approach'

In an earlier paper(3), Mr. Muspratt concludes that 'pre-yield' behaviour does not depend upon the reinforcement layout and almost any reinforcement pattern 'develops' at yield. On the basis of these conclusions, he recommends(3) the 'Monte Carlo approach' and refers to the paper under discussion. The Monte Carlo approach turns out to be a random distribution of reinforcing bars.

In actual fact, a random reinforcement may result in an entirely unserviceable and unsafe structure. If the reinforcement content is low in areas of high elastic moments, local yield and large cracks may occur at service loads. One cannot rely on the tensile strength of the concrete, because in most practical slab design the cracking load is smaller relative to the ultimate load than in the above experiments. Further, shrinkage may decrease the cracking load capacity.

Considering various reinforcement layouts of the same volume, the ultimate load capacity varies considerably and depends on the reinforcement distribution used. Hence a random layout may result in a failure at an unexpectedly low load value*.

Over-all yield

Mr Muspratt in item (3) under the heading 'strains' claims that 'Rozvany's assumption of general yield' is 'inexact'. There appears to be some misunderstanding here, because I have never claimed that simultaneous yield throughout the slab would occur in optimized slabs. In fact, in describing earlier experiments on optimized slabs in 1966(4) reported quite simple failure mechanisms.

Mr Muspratt's criticism might be a result of some misconception about the usual lower-bound approach, in which the design moment capacities are made either greater than or equal to some statically admissible moment field. Naturally, simultaneous yield throughout the slab would never occur, because of the finite steps in the reinforcement and the unintentional variation of the concrete strength and thickness over the slab.

Deflections at yield

Mr Muspratt lists deflection values corresponding to the formation of the yield-line mechanism. These values are somewhat unreliable and arbitrary, because the deflection/load slope is almost infinite at that stage and the deflection increases rapidly in time. It would be better to compare deflections at a lower load level of a given value.

The listing of these values seems to be a result of misinterpreting a statement by Kemp(2). The latter did not suggest that a deflection/depth ratio of 0·6 is 'generally acceptable' for ensuring 'satisfactory service behaviour under working load'. Kemp(2) and Taylor(1) correctly note that the load capacity increases monotonically with the deflection after flexural failure and hence it is necessary to define the 'limit load' in terms of a given deflection. This concept is not related to service behaviour.

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*Naturally, a random distribution of a large number of very thin steel fibres is likely to result in an essentially isotropic reinforcement. However, Mr Muspratt proposed a random layout of a few bars.

Reply by the author

Membrane action

Dr Rozvany has misunderstood Figure 13. Section AA was intended—hence the centroid of the tensile zone is correct, rather than that of the tensile reinforcement, in view of the uncracked concrete along this section. This Figure was constructed from experimental data and similarity to Taylor's figure was merely for convenience.

Regarding Figure 12, Kemp's theory assumes rigid plastic behaviour up to yield, i.e. negligible deflections. All experimental deflections after yield were thus taken relative to the deflection just prior to the theoretical yield point. If we take as an example, $W_0/d = 0{\cdot}5$, for $d_t = 2{\cdot}25$ in., $W_0 = 1{\cdot}12$ in. From Figure 4, the load corresponding to this deflection measured relative to the deflection at yield is about 300 lb/ft^2. This figure includes the self-weight of the slab of about 50 lb/ft^2. Therefore

$$\text{True load} = 250 \text{ lb/ft}^2$$

and so

$$\frac{P}{P_y} = \frac{250}{220} = 1{\cdot}14$$

Monte Carlo approach

Dr Rozvany is referred to reference 17 for more details of this approach. Concrete strengths and thicknesses vary stochastically, as do load distributions, and so the deterministic system definition is highly idealized. The method for including spatial cross correlations between load and strengths to minimize the probability of any slab section being subject to overload and under-strength will be published elsewhere. This strategy will, except for conditions lying in the very extreme tails of load and strength distributions, give safer designs than the deterministic approach.

Over-all yield

If design moment capacities are made greater than some statically admissible moment field, the design is no longer optimal. The whole point of optimal design as proposed by Dr Rozvany is simultaneous collapse of all sections of the slab—this discrepancy again highlights the unavoidable practical perturbations existing in levels of variables used for system definition, and reinforces my proposal for stochastic design.

Deflections at yield

This comment seems irrelevant—time dependence of deflections occurs before as well as after yield. The yield point is a definite transition in the slope of the load-deflection curve and so is an easily reproducible characteristic.

Dr Rozvany's interpretation of "service" is an error in semantics. 'Overload' as well as 'working load' service conditions are acceptable in view of contemporary looseness in the use of this terminology and the tremendous latitude in interpretation.

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Discussion on papers published in

Magazine of Concrete Research

Volume 22, Number 73: December 1970

Limit-state design for shear in rectangular and T beams*

P. E. Regan and A. Placas

Contribution by H. K. Sen, BCE(Hons), MTech, PhD, DIC, CEng, MICE

University of London, Imperial College of Science and Technology, Department of Civil Engineering

Dr Regan and Dr Placas are to be commended for discussing at length, for the first time, the problem of shear transfer across the vertical interface between the flange and web of a reinforced concrete T beam. This problem can be more acute in composite steel-concrete beams, and Parts I and II of CP 117(1), the Code of Practice for composite beams in building and bridges, give clauses stipulating: (1) the amount of transverse reinforcement required, (2) the minimum amount of transverse reinforcement which must be provided, and (3) the maximum amount of longitudinal shear in the concrete which may be permitted irrespective of the amount of transverse reinforcement. The requirement of transverse reinforcement in CP 117, Part I, is based upon the assumption that at failure the longitudinal shear is resisted by the cohesion in concrete ($= 2{\cdot}8\sqrt{u_w}$) and a frictional resistance equal to the force at yield in the transverse reinforcement.

The CP 117 value for cohesion ($= 2{\cdot}8\sqrt{u_w}$) can be obtained by substituting $f_c' = \frac{2}{3}u_w$ in the ACI-ASCE recommendation(2) that the ultimate diagonal tension in an unreinforced web should not exceed $3{\cdot}5\sqrt{f_c'}$ where f_c' is the cylinder strength. However, the analogy between the transfer of longitudinal shear in the flange of a composite beam and of vertical shear in the web of a rectangular reinforced concrete beam is not direct.

Dr Regan and Dr Placas assume that the resistance to longitudinal shear is provided by cohesion ($= 0{\cdot}1u_w$) and friction equal to the force in the transverse reinforcement at yielding. The value of cohesion ($= 0{\cdot}1u_w$) taken by them will be greater than the CP 117 value ($= 2{\cdot}8u_w$) for all values of u_w greater than 784 lb/in^2 and, in the light of results of experiments on composite beams with varying amounts of transverse reinforcement now available(3), it seems that a higher value may be justifiable. In this respect the higher value used in the paper ($= 0{\cdot}1u_w$) looks attractive and I would be grateful if the authors would produce some evidence in support of this value.

A major difference between the CP 117 approach and that used in the paper is that Dr Regan and Dr Placas consider both shear and in-plane bending across the shear plane, and take cohesion to exist only where there is compression in concrete due to in-plane bending, whereas CP 117 neglects in-plane bending and assumes cohesion over the entire shear-span.

It appears that equation 23, which is the condition for longitudinal equilibrium, is erroneous. The correct form is:

$$K_c\left(c + Kr_t f_{yt}\frac{K_s}{K_c}\right) \geqslant \frac{F}{at}\ldots\ldots\ldots\ldots(23)$$

Similarly, equation 25 should be:

$$(r_t f_{yt})_{\min} = \frac{F}{at}\,\frac{1}{(1 - 0{\cdot}5K_c)}\left(1 - \frac{cat}{F}K_c\right)\ldots.(25)$$

Consequently, one of the two sets of curves in the

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*Pages 197 to 208 of *Magazine* No. 73.

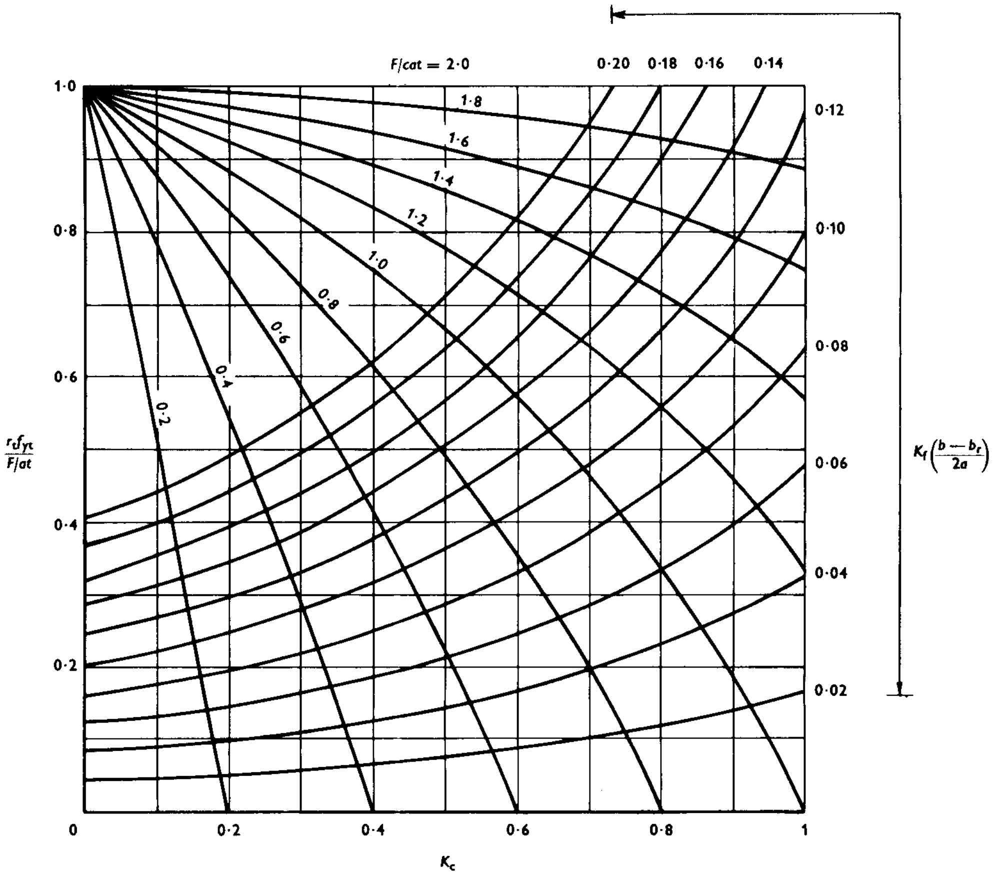

Figure I: Corrected design chart for transverse flange reinforcement.

design chart (Figure 10 of the paper) has to be redrawn; the corrected version is shown here (Figure I).

Although the set of curves corresponding to equation 24 in original Figure 10 is correct, the equation itself has a printing error. It should read:

$$(r_t f_{yt})_{\min} = \frac{F}{at} K_f \left(\frac{b - b_r}{a}\right) \Big/ \left(1 - \frac{K_c}{2}\right)^2 \; \ldots\text{(24)}$$

Reply by the authors

We wish to thank Dr Sen for his interest in our paper and his corrections to equations 23, 24 and 25 and to Figure 10. In each case, his version is correct and ours was in error.

The remainder of his contribution raises the problems of the magnitude of the 'cohesive' strength of concrete at cracked surfaces and the distributions of stresses along web-flange junctions.

The cohesion is, of course, not a true material property in the soil mechanics sense, but a property dependent upon a number of characteristics of the crack in question, amongst which the most important are probably the type of cracking and the width of the crack. With regard to the first, there is a basic distinction between cracking due primarily to shear and cracking due to direct tension. Data from tests(4) of specimens with transverse reinforcement and involving shear cracking are compared with our 'cohesion + friction' equation in Figure II, and it can be seen that the correlation is satisfactory, subject to an upper limit on shear resistance irrespective of the reinforcement provided. This is the third condition of CP 117 referred to by Dr Sen and is undoubtedly rational. However, in practice, it is extremely unlikely to be a governing criterion in reinforced concrete members. The assumption that cohesive strength is directly proportional to the concrete's cube strength is questionable but should not cause any serious errors within the practical range of concretes.

When the cracking is due to direct tension and a direct separation occurs, the post-cracking shear resistance is reduced, as can be seen from Figure III, which gives results of tests of specimens(4) similar to those of Figure II but pre-cracked along their shear

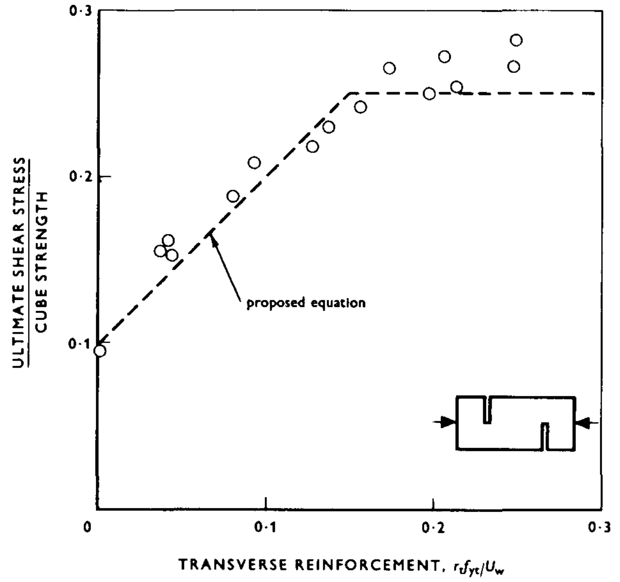

Figure II: Results of tests on specimens without initial cracks.[4]

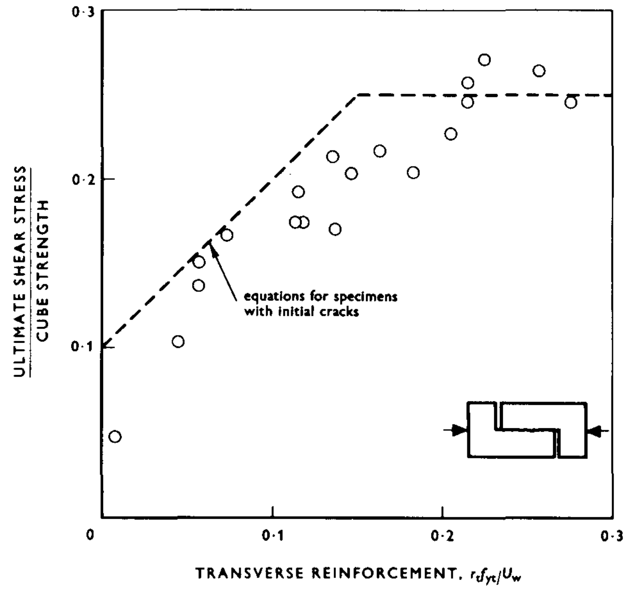

Figure III: Results of tests on specimens with initial cracks.[4]

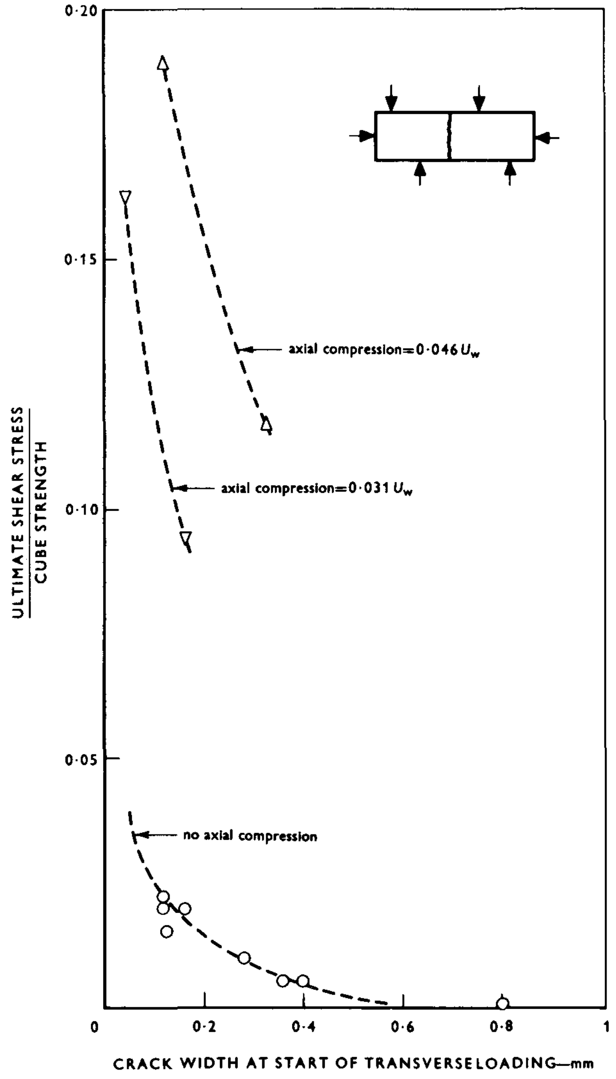

Figure IV: Results of tests on specimens with initial cracks and without transverse reinforcement.[5]

planes. The reduction is essentially confined to the cohesive component, whilst the friction effect is almost unchanged.

The influence of crack widths upon shear resistance cannot easily be studied in reinforced specimens because the crack width is a function of the behaviour of the reinforcement and cannot be controlled externally. Using unreinforced specimens of the type shown in Figure IV, pre-cracked in tension, Bara[5] obtained the results shown in the same Figure. In the tests with external compression, the axial load was applied before the shear loading was begun. It can be seen that shear resistance is markedly dependent upon the crack width at the start of the shear loading, from which stage the width remains more or less constant until failure.

Our approach does not take direct account of the effect of crack widths but assumes that, in the region of which the shear resistance is taken into consideration, the crack widths should be similar to those obtained in the tests of reinforced specimens which involved considerable variation in reinforcement details.

The design shear resistance is limited to the length of the junction subject to transverse compression because, once plasticity of the transverse reinforcement is assumed, there is no theoretical control over the widths of cracks along the remainder of the junction. The most reasonable view of the behaviour of the latter length is that the concrete is free from transverse stresses and that its shear resistance should correspond to Bara's results for specimens without axial loads. This would give ultimate shear strengths of $0{\cdot}02u_w$ with a crack width of 0·1 mm and only 0·005 with a width of 0·4 mm. Such resistances are negligible in comparison with that of the compressed region. The use of a relatively high cohesive strength in the compressed region is justified by the fact that it is also the region where the cracking is of the shear and not the tension type, and by Bara's and Mattock's results, which show that reduction in strength caused by tensile cracking is not very significant when the concrete is restrained by reinforcement or subsequent compression.

Finally, with regard to the provisions of CP 117 referred to by Dr Sen, it seems to us that, although a rather crude analysis is acceptable, it is illogical to consider only the shear and not the direct transverse forces for two reasons. Firstly, if the transverse effects are neglected, the force system is not in equilibrium and the errors involved can be appreciable. Secondly, as shown above, shear resistance is affected by transverse stresses in its cohesive as well as its frictional component.

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Magazine of Concrete Research : Vol. 23, No. 77 : December 1971

Application of mathematical programming to yield-line analysis*

Andrew C. Palmer

Contribution by A. B. Templeman and R. C. Slater

University of Liverpool: Department of Civil Engineering

Dr Palmer's paper deals with an interesting problem in yield-line analysis: how may the pattern of yield lines which maximizes the ultimate moment be found? Mathematical programming methods can be of great assistance in solving such problems, but we believe that the use of Powell's method in the way suggested in the paper cannot be recommended as a general procedure.

Powell's method is an unconstrained method and cannot handle geometric constraints. Dr Palmer correctly formulates the first example which consists of minimizing equation 1 subject to the constraints 2. He then proceeds to ignore the constraints and minimizes equation 1 by using Powell's method. He is very fortunate that the results obtained in this way happen, coincidentally, to satisfy the constraints which he has ignored.

A similar procedure has been followed in the second example which consists of maximizing equation 7 subject to the constraints 4 and 6. Dr Palmer ignores the constraints and simply uses Powell's method to maximize equation 7. Fortunately again his results also satisfy the constraints, but there is no a priori reason why they should.

The question immediately poses itself: what should the engineer, who has programmed his specific yield-line problem for Powell's method as is suggested in the paper, do if his results do not satisfy geometric requirements? Dr Palmer's answer, towards the end of his paper, is that he must then use some other method such as SUMT or Rosenbrock's method, in which event the time spent on Powell's method will have been wasted.

The fact that Dr Palmer's procedure requires an almost trivially small amount of computer programming effort in addition to Powell's sub-program is of little account if the results obtained from it do not satisfy geometric requirements. This procedure is unsound and, in cases where constraints may or may not be active, they should always be included ab initio in whatever mathematical programming method is being used. We therefore suggest that any method of constrained optimization is in all cases preferable to Powell's method.

The object of the paper, however seems to be to emphasize how useful mathematical programming can be in yield-line analysis and we support this entirely. We have used constrained optimization techniques in investigating far more complex yield-line problems than those discussed by Dr Palmer. The slab in question formed a bridge deck, curved in plan, containing four variable steel areas: top longitudinal, bottom longitudinal, top transverse and bottom transverse. The slab could carry three loading cases corresponding to different combinations of HA and HB loading. For a given depth of slab it was required to minimize the total volume of reinforcement

For a particular set of reinforcement areas, several different types of yield-line pattern were examined for each loading case. In fact, for the three loading cases, a total of eleven different general patterns were investigated. For each failure pattern, the yield lines were generally specified and a mathematical programming procedure was used to vary the pattern until the ultimate moment was maximized, i.e. a procedure similar to that outlined in Dr Palmer's paper. Each pattern of the eleven, therefore, produced a maximum ultimate moment, the maximum of which was then taken as the slab ultimate moment. Thus, for each set of reinforcement, a slab ultimate moment was obtained.

A second optimization routine was then carried out by varying the four reinforcement variables until the steel areas which gave the minimum volume of reinforcement were obtained. The lower constraint on the individual steel areas was the minimum acceptable area of reinforcement as specified by the Draft Unified Code for Structural Concrete; the upper constraint was generally defined so that an excessive area of steel would not be used.

Mathematical programming techniques may therefore be used to great effect in yield-line analysis and can provide solutions to a much wider range of more complex problems than those outlined in the paper.

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*Pages 227 to 231 of *Magazine* No. 73

Reply by the author

In their criticism of my paper, Dr Templeman and Mr Slater assert that I "ignore" the geometric constraints on the yield-line patterns. Their assertion is wholly mistaken. In my example 1, for instance, equation 1 defines the collapse load in terms of the parameters describing the yield-line pattern, and inequalities 2 define the constraints. It is surely natural to examine first the corresponding unconstrained problem, that of minimizing the function defined by equation 1 without any constraints, and then to see whether or not the minimizing values of the parameters satisfy the constraints. If they do satisfy the constraints, the problem is solved; if they do not, then the constraints must be taken into account in the minimization process itself, and several ways of doing this are mentioned in the paper.

The point at issue can perhaps be made more plain by a simple example detached from yield-line theory. Consider the elementary problem of minimizing the function

$$f = x^2 + y^2 - 2x + 4$$

subject to

$$x + y \leqslant 2$$

The most straightforward procedure is to attack the unconstrained problem first, to observe that f is convex (so that only one stationary value exists), to evaluate $\partial f/\partial x$ and $\partial f/\partial y$, to set these derivatives separately equal to zero, and finally to solve the resulting equations for x and y. The unconstrained minimum is at $x = 1, y = 0$, but must now be checked against the constraint. By inspection, these values do satisfy the constraint, and the problem is solved. If, of course, the minimizing values for the unconstrained problem had failed to satisfy the constraint, the problem would not have been solved, and one would have to resort to a constrained minimization technique, such as the use of Lagrangian multipliers. The implication of Dr Templeman and Mr Slater's comment is that the simple procedure outlined above is somehow improper, and that because there is a constraint one ought to bring in Lagrangian multipliers from the beginning, without pausing to ask whether or not the constraint is operative. To me, this seems to complicate the approach quite unnecessarily.

Dr Templeman and Mr Slater also assert that, if the problem turns out to be constrained, "the time spent on Powell's method will have been wasted". This, too, is mistaken. If one is going to use sequential unconstrained minimization, or any equivalent penalty function method, one still has to have a means by which the successive unconstrained minimizations can be effected, and Powell's method is quite suitable. Suppose, for instance, that, in Example I of the paper—minimize

$$f = 24\,\frac{1 - 2v + \left(1 + \dfrac{m}{M}\right)\dfrac{uv}{u - v}}{1 - 2uv^2} \quad \ldots\ldots\ldots(1)$$

subject to

$$\left.\begin{array}{l} 0 \leqslant u \leqslant 1 \\ 0 \leqslant v \leqslant 0{\cdot}5 \\ u > v \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots(2)$$

—the unconstrained minimum had turned out to violate the constraints. It would then be simple to modify the function f by adding to it a penalty function which becomes large as the constraints are approached, thus

$$f^* = f + r\left(\frac{1}{u} + \frac{1}{1 - u} + \frac{1}{v} + \frac{1}{0{\cdot}5 - v} + \frac{1}{u - v}\right) \quad \ldots\ldots\ldots(3)$$

where r is a positive multiplier, and then to use Powell's method to carry out a sequence of unconstrained minimizations of f^*, starting with a large value of r and examining the sequence of minimizing values of u and v for successively smaller values of r. This is the SUMT technique, and is described in great detail in reference 11 of the paper. As far as a computer program is concerned, only five Fortran statements need to be added to modify the program to use this technique: it seems exaggerated to claim that the earlier effort was "wasted".

I thank Dr Templeman and Mr Slater for their interest, and am delighted to learn that they have successfully applied the method to more ambitious problems.

Comments on the long-term strength of plain concrete*

K. Komloš

Contribution by J. Bhargava, DrTech

Institution of Structural Engineering and Bridge Building, The Royal Institute of Technology, Stockholm

Dr Komloš has presented some very useful data for the tensile and flexural strength of concrete at various ages. I feel that the use of term 'long-term strength' to denote the strength at different ages can be misleading. The term is mostly used to denote the strength under sustained or 'long-term' loading, and has been used in this sense in connection with studies of the rheological behaviour of concrete.

It is not clear what the moisture condition of the specimens was at the time of the test, especially in the case of specimens tested at later ages. Were these specimens tested in a saturated surface-dry condition, or were they allowed to stand in the laboratory air (at a lower humidity) for some time before the test? This is of significance for the analysis of the results such as the ratio of tensile strength to cube strength at various ages.

Dr Komloš's results confirm the observation made by earlier investigators that there is a significant difference between the rate of deceleration of the development of various strengths of a concrete(1,2). It had been observed by Popovics(3) that the effect of change in water/cement ratio was relatively less for the flexural and tensile strengths than for the compressive strength; a possible mechanism, based on the difference between the slow crack propagation under tension and compression, was proposed by him(4). A similar trend can be seen in Dr Komloš's results for tensile strengths, although the difference in the case of flexural and compressive strengths is relatively much less.

Alkalis present in the cement have a significant influence upon the development of strength with age(5,6). From the chemical composition of the cement given by Dr Komloš, the cement appears to be alkali-free. I would like to know whether Na_2O/K_2O content was determined. It would also be of interest to know the specific surface of the cement used. The increase in the cube strength, between 90 and 360 days, was rather large in Dr Komloš's tests (see Table I). These values are substantially higher than the values generally obtained with modern Portland cements(1).

TABLE I: **Cube strength ratios.**

	Water/cement ratio		
	0·40	0·52	0·63
σ_{90}/σ_{28}	1·224	1·003	1·307
σ_{180}/σ_{28}	1·381	1·476	1·736
σ_{360}/σ_{28}	1·635	1·527	1·727

Contribution by C. D. Johnston and M. A. Ward

Associate Professors of Civil Engineering, University of Calgary

We would like to comment on several of the relationships and conclusions given in Dr Komlos's paper in the light of our earlier work(7-9). We feel that the impression given in his concluding paragraph, that the ratio of tensile to compressive strength is dependent only upon the age of the concrete, presents an incomplete picture of the changes in this ratio which can occur and the contributory factors. Dr Komloš's results show that the ratio decreases with increasing age, whilst Figure 7 of reference 7 shows that it decreases with decreasing water/cement ratio. Both these parameter changes are responsible for a major increase in the over-all strength of the concrete, and, on the basis of the combined data(7-9) shown in Figure I, it can be concluded that the ratio of tensile to compressive strength is mainly dependent upon the magnitude of concrete strength rather than any single parameter, at least for concretes with a similar aggregate grading. This conclusion appears justified for a general relationship in view of the wide range of parameter variation represented in the Figure, age, water/cement ratio and aggregate/cement ratio in Komloš's data, water/cement ratio, aggregate/cement ratio and aggregate type in Ward's(8) data and water/cement ratio,

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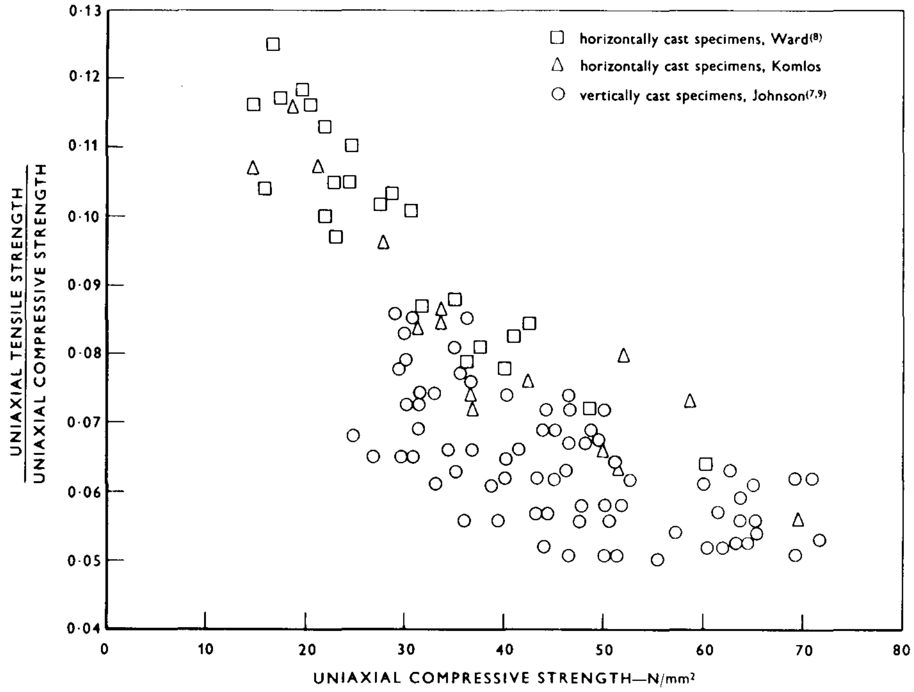

Figure 1: Relationship between compressive strength and the ratio of tensile to compressive strength.

aggregate maximum size, grading and type in Johnston's(7,9) data. However, the ratio is not solely strength-dependent, because Figure 7 of reference 7 and Ward's(8) results for mortars show that a decrease in the mean aggregate particle size, a parameter change responsible for a minor increase in strength(9), results in a greater rather than a lesser value of the strength ratio. In addition, whilst it is not strictly correct to compare Komloš's values with the other data because his compressive strengths are based on cubes rather than prisms, it should be noted that any correction would tend to raise each data point slightly and would further distinguish the trend for the horizontally cast specimens of Komloš and Ward(8) to give values of the ratio noticeably higher than those for the vertically cast specimens of Johnston(7), particularly at lower levels of compressive strength. Thus, the influence of direction of casting and mean aggregate particle size are certainly separate and distinguishable from that of strength alone.

Turning now to the relationship between splitting tensile strength and uniaxial tensile strength, we would like to point out that, whilst the relationship shown in Figure 7 of Dr Komloš's paper holds true for his particular mixes, in which the aggregate maximum size and grading appear to have remained constant, it would probably not be applicable to mixes containing much smaller or much larger aggregate. Figure 8 of reference 7 clearly shows that the difference between splitting tensile strength and uniaxial tensile strength for specimens similar to those of Dr Komlos increases as the mean aggregate particle size increases. In other words, the slope of the relationship given in Figure 7 of Dr Komloš's paper can be expected to increase as the proportion and maximum size of the aggregate in the grading increase.

In conclusion, we would like to state that caution should be exercised in developing general correlations between the results of different strength tests, as it has been proved that, whilst the relationships obtained by different investigators are qualitatively similar, they can differ quantitatively due to differences in materials, mix parameters and casting and testing techniques.

CORRIGENDA

notified by A. B. Lingam, BSC, BE and G. Ramakrishna, BE

Engineering Research Laboratories, Hyderabad

The second and following lines on page 238 should read as below, and corresponding corrections apply to Figures 6 to 9 on the previous page.

cube strength and direct tensile strength:
$f_{\mathrm{t}} = 1{\cdot}2568 + 0{\cdot}0445 f_{\mathrm{c}};\ r_{\mathrm{tc}} = 0{\cdot}921$
cube strength and splitting tensile strength:
$f_{\mathrm{s}} = 1{\cdot}1750 + 0{\cdot}0477 f_{\mathrm{c}};\ r_{\mathrm{sc}} = 0{\cdot}992$
cube strength and flexural strength:
$f_{\mathrm{f}} = 1{\cdot}7525 + 0{\cdot}1142 f_{\mathrm{c}};\ r_{\mathrm{fc}} = 0{\cdot}947$
direct tensile strength and splitting tensile strength:
$f_{\mathrm{s}} = 0{\cdot}9332 f_{\mathrm{t}} = 0{\cdot}2404;\ r_{\mathrm{st}} = 0{\cdot}943$
direct tensile strength and flexural strength:
$f_{\mathrm{f}} = 2{\cdot}3933 f_{\mathrm{t}} - 0{\cdot}9584;\ r_{\mathrm{ft}} = 0{\cdot}958$
splitting tensile strength and flexural strength:
$f_{\mathrm{f}} = 2{\cdot}4335 f_{\mathrm{s}} - 1{\cdot}1794;\ r_{\mathrm{fc}} = 0{\cdot}965$

Reply by the author

I am grateful to Dr Bhargava, and Dr Johnston and Dr Ward, as well as to Mr Lingam and Mr Ramakrishna, for their discussion of some of the assumptions and results of my paper.

I agree with Dr Bhargava that the term 'long-term strength' is mostly used in connection with studies of the rheological properties of concrete under sustained loading. Nevertheless, this term was used to avoid the term 'strength of concrete at various ages', which seemed to me a little bit too clumsy.

The specimens were not allowed to stand in the laboratory air for some time before the test. They were tested immediately after they had been taken out of the curing chamber.

It is a well-known fact that the kinetics of strength increase of concrete are to a high degree dependent upon the kind of cement used. The cement used in our investigations was not a pure Portland cement and so not comparable with the cement mentioned in Popovics's paper. The specific surface of the cement applied in our tests was 3400 cm^2/g (according to the Blaine method). The Na_2O/K_2O content was not determined.

In connection with the contribution by Dr Johnston and Dr Ward, I would like to point out that I did not want to give in my paper the impression that the ratio of tensile to compressive strength is dependent only upon the age of the concrete. My aim was to show that the age of concrete is an important factor influencing this ratio. On the other hand, I agree with Dr Johnston and Dr Ward that the age of concrete is not the most important factor, but only one among many others. I am also of the opinion that the maximum particle size of the aggregate influences the relationship between splitting tensile strength and uniaxial tensile strength.

Dr Johnston and Dr Ward state in their conclusion that caution should be exercised in developing general

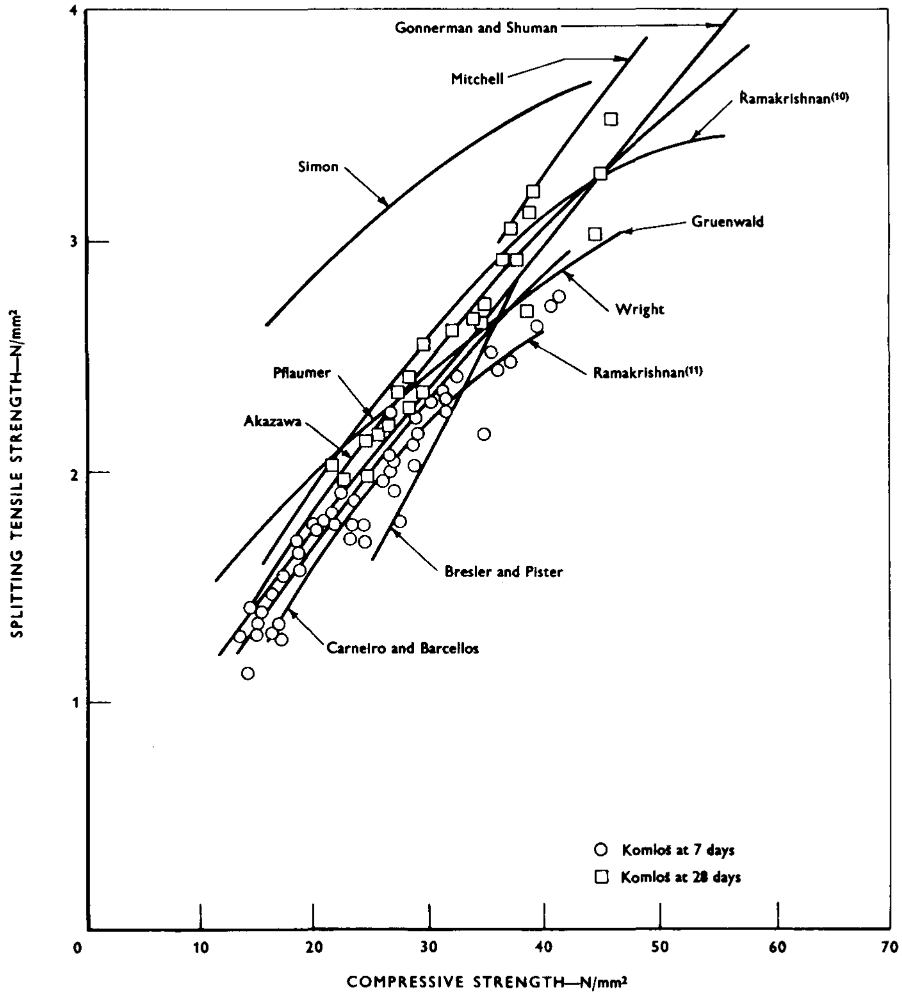

Figure 11: Relationship between splitting-tensile and compressive strength according to various authors, and results by Komloš.

correlations between the results of different strength tests. I agree with Dr Johnston and Dr Ward that the results differ owing to differences in materials, mix parameters and casting and testing techniques. However, I think that it will be very difficult to include all these factors in a general correlation formula$^{(10-12)}$. The relationships between strength characteristics of concrete have been studied by many investigators and the results of their investigations differ to a greater or lesser extent. As an example, I would like to show some relationships between splitting-tensile and compressive strength obtained by various authors:

Ramakrishnan (normal concrete) $f_s = \frac{f_c}{20} + 8$

Ramakrishnan (lightweight-aggregate concrete) $f_s = \frac{f_c}{15} + 6$

Akazawa $f_s = 0{\cdot}675 f_c^{0{\cdot}730}$

Carneiro and Barcellos $f_s = 0{\cdot}554 f_c^{0{\cdot}735}$

Vinayaka $f_s = 0{\cdot}880 f_c^{0{\cdot}716}$

Ramesh and Chopra $f_s = 2{\cdot}172 f_c^{0{\cdot}604}$

Sen and Bharara $f_s = 0{\cdot}2345 f_c^{0{\cdot}84}$

Sen and Desayi $f_s = 0{\cdot}628 f_c^{0{\cdot}73}$

Dewar $f_s = 1{\cdot}23 f_c^{2/3}$

ACI Building Code 318-63 $f_s = 5{\cdot}4 f_c^{0{\cdot}5}$

Evans $f_s = 0{\cdot}056 f_c + 7$

where f_s = splitting tensile strength, f_c = compressive strength.

Some other relationships can be found in Figure II. The above survey of formulae indicates that it will not be easy to find a common basis for a unified expression of the relationships.

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Book reviews

The chemistry of cement and concrete

F. M. Lea

Published by Edward Arnold (Publishers) Ltd, London. Third edition. 1970. 233 × 158 mm. pp. xiv, 727. Price: £12·00.

The engineer should not be deterred by the word 'chemistry' from reading this admirable book. The complicated processes of chemical attack on concrete or corrosion of reinforcement are dealt with in a manner that is perfectly comprehensible by the non-chemist and, as the book is planned as a work of reference, the designer may use it to look up the probable effects of an unfamiliar chemical upon concrete.

Whilst the emphasis is mainly upon Portland cement, other types, including high alumina, slag and pozzolanic cements, are also covered. The anhydrous cements, their hydration reactions and their behaviour in concrete are described, and there is a full chapter on concrete aggregates, in which the relationships between chemical constitution and physical properties such as creep and shrinkage as well as strength are discussed and all forms of chemical attack, whether by natural or artificial destructive agents, are explained.

This is the third edition of a book which was first published thirty-five years ago. This edition has been completely revised and re-set, and thus manages to accommodate about 30% more material than the 1956 edition in a book which is still of manageable size and readable typeface. Typographical errors, a feature to which readers are becoming resigned nowadays, are at least fewer than one would expect as a result of the re-setting.

The comparative amounts of new material in different chapters reflect the levels of interest in various aspects of cement research in the last 15 years. In spite of the necessity for re-drawing all the phase diagrams in Chapter 4, it seems clear that interest has shifted from pure phase or solution chemistry to the study of kinetics and mechanisms. The most extensively revised chapter is that on the setting and hardening of Portland cement, which includes a very lucid review of the theories of Rebinder and his colleagues in the USSR. Work on the mechanism of bonding and on the relative importance of morphology and chemical bonding is discussed, and the conflicting results of using water or nitrogen to measure surface area of cement pastes by gas absorption are considered.

Inevitably there is much new work making use of physical methods of analysis such as DTA, thermogravimetry (with DTG), electron-probe microanalysis, and x-ray fluorescence spectroscopy. Conduction calorimetry, however, though mentioned in the chapter on setting and hardening, is not to be found in the index, and it is not made clear that Tenoutasse's work on the effect of gypsum and calcium chloride upon the aluminate and ferrite hydration reactions was based on this powerful technique.

Chapter 9, on the hydration of Portland cement, has also been extensively revised, and the rival theories of 'solid state' or 'through solution' reactions are carefully evaluated. Work on the C-S-H system has been extended by studies of the effect of other ions, such as Al, S, Fe, and the alkalis. The C-A-H system has also been considerably revised and there is now a most useful Table (pp. 221–222) of the optical and crystallographic properties of the calcium aluminate and ferrite complex salts. This might perhaps have been stretched to accommodate thaumasite $\{Ca_6[Si(OH)_6]_2 24H_2O\}(CO_3)_2(SO_4)_2$, which has been detected several times in

Portland cement hydration products and, being isomorphous with ettringite, is probably frequently mistaken for it.

A good deal of new work on steam-curing is reported, both of ordinary Portland cement and Portland blastfurnace cement concretes, and there is also a reconsideration of the assessment of reactivities of slags.

In Chapter 17, on special cements and cement properties, there are new sections on "Expanding and non-shrinking cements" and on "Corrosion of steel in concrete". The former elucidates the chemistry of Type K (Klein's) and Type M (Mikhailov's) expansive cements, on both of which much misleading information is to be found elsewhere in print, but neglects Type S (a high-C_3A Portland cement with extra calcium sulphate, developed by the Portland Cement Association, Skokie) and the Japanese work on a type analogous to Klein's.

These are trifling criticisms and should not be allowed to detract from the general excellence of the book. It has been and remains the outstanding work of reference on its subject in English, and probably in any language.

The last four chapters, those dealing particularly with concrete, should be required reading for any engineer with responsibility for the durability of a concrete structure. A. E. MOORE

Creep of concrete

Plain, reinforced and prestressed concrete

A. M. Neville

Chapters 17 to 20 written in collaboration with W. Dilger.

Published by the North-Holland Publishing Company, Amsterdam, 1970. 230 × 158 mm. pp. xx, 622. Price: HFl. 108. Distributed in the United Kingdom by the Cement and Concrete Association, price: £12·60. Distributed in the USA and Canada by American Elsevier Publishing Company Inc., price: $30·00.

In this book, which seems obviously destined to become a standard work, the creep of concrete is, for the first time, dealt with in a comprehensive way covering all aspects of the subject clearly. The author (at present Head of the Department of Civil Engineering, the University, Leeds), who is well known as the writer of the text-book *Properties of concrete,* has published numerous papers on creep, among them one in the ACI Journal of September 1955 in which he was the first to distinguish the various basic theories dealing with the mechanisms of creep of concrete. He was for several years Chairman of the ACI Creep Committee which, with its many sub-committees, is making a thorough study of this problem. It should be noted that the existence of creep was first recognized in the USA as early as 1907 by Hatt and afterwards by MacMillan in 1915, as the author mentions in his introduction. The research work by Raymond Davis, in the USA, and Glanville and Thomas in this country in the 1920s, is fully appreciated in this book. Numerous investigations by other authors during the last thirty years in many countries have also been considered by the author; these include work done in the USSR which may not be known to many specialists.

The first sixteen chapters deal with creep of concrete in general and cover the various influences upon it, in Chapters 3 to 5, for example, those of cement and admixtures, aggregate, water/cement ratio, age and size, all of which are related to the composition of the concrete. Other influences, such as curing, humidity and temperature, are discussed in Chapters 6 and 7. The problem of creep recovery and that of irrecoverable creep in connection with the validity of the principle of superposition is the subject of Chapter 8. The effect of creep under different kinds of stress such as tension, torsion and bond, as well as the more usual case of compression, is dealt with in Chapter 9, which also covers creep under alternating loading and the very important problem of creep under very high stresses. Chapters 10 and 11 deal in more detail with the various theories of the mechanism of creep and the different creep hypotheses. In Chapter 12, the basic expressions for creep as presented by various authors are investigated, such as power expressions, exponential, hyperbolic and logarithmic expressions, some of which may result in rather complicated formulae, particularly if variable stress is considered.

A very important problem is that of "Prediction of creep" dealt within Chapter 13; this is, for designers, of greatest importance. Basic and drying creep are distinguished, and the composition of the mix, whether it is of normal-weight or lightweight aggregate, is considered and this should permit a general prediction; rather complicated calculations may, however, result. Also the relaxation of stress is briefly investigated and the relationship between stress and strain considered. Chapter 15 is devoted to rheological models and to the effect of damping. Simple models illustrate the behaviour of ideal materials, but very complicated arrangements are necessary to characterize the actual behaviour of concrete under all conditions and stages. In Chapter 16, various devices for the measurement of creep are described.

For Chapters 17 to 20, dealing with structural members, W. Dilger (who, before his present activity as Associate Professor at Calgary University, Canada, worked with Professor Leonhardt in Germany) co-operated with the author. These chapters, which are of particular importance to the structural engineer, cover the analysis of the effects of shrinkage and creep in uncracked and cracked reinforced and prestressed concrete members, including short- and long-term deformations. With regard to prestressed concrete, this is amplified by various conditions which may occur, such as the effect of multi-stage prestressing. One of the chapters is devoted to creep in arches, continuous beams, composite members and cylindrical shells. With regard to code requirements in various countries, reference is made to Chapter 13, "Prediction of creep", previously mentioned which also includes the CEP-FIP Recommendations.

To sum up, it should be stated that this excellent book which is illustrated with numerous clearly drawn diagrams, will be of very great importance not only to the research worker and concrete specialist for whom it is essential, but also to the structural engineer dealing with concrete, because it will enable him to recognize the various influences and to realize that it is vital to take them into consideration. It should, however, be appreciated that it is very difficult to predict the effect of creep at the design stage when particulars about the aggregates to be used, the mix, curing, weather and temperature conditions at casting and other variables are not fully known, but safe assumptions have to be made. Over-simplifications are just as bad as complicated, detailed computations, based on mere assumptions, which need not necessarily be correct. For the designer to have a sound knowledge of these problems is therefore vital, but unhappily this is often not appreciated.

P. W. ABELES

Adjuvants et traitements des mortiers et bétons

(Admixtures and treatments for mortars and concretes)

M. Vénuat

Published by the author, Chef du Service Technologie, Centre d'Etudes et de Recherches de l'Industrie des Liants Hydrauliques, 23 rue de Cronstadt, 75 Paris 16e. 1971. 246 × 162 mm. pp. 430. Price : 80F + 5F postage.

Dr Vénaut has set out to fill a gap in concrete technology: it is hard to find a good book on concrete admixtures and surface treatments. The effort has been worth while. The author is obviously well versed in the subject-matter and has managed to collect information from many sources, sift it carefully and present the result in a very readable form.

A basic problem in dealing with admixtures is to find the right expressions to convey the meaning without involving chemical terms which are largely incomprehensible to the average engineer. In this respect Dr Vénaut has succeeded very well, although he does not shirk the use of chemical formulae and names in the proper places. Some moderately detailed knowledge of chemistry on the part of the reader is implied.

Another problem is classifying admixtures which have more than one function. This is all explained satisfactorily in the introductory section of the book and will clarify the situation for many puzzled users of admixtures.

The first main part of the work is largely devoted to methods of test for admixtures. The coverage here is comprehensive—perhaps too comprehensive. In the author's attempt to mention all the procedures, the details of each are inclined to be sketchy. Although it is clearly stated that standard procedures have been omitted, more guidance here would have been an asset. For determining the chloride content of a product, for example, we are told that it can be carried out potentiometrically, gravimetrically or volumetrically, but very little else. It would have been appropriate to have been given full details of a procedure for measuring this and other active ingredients commonly found in admixtures, as well as to describe the methods for measuring other parameters of the concrete such as bond strength, shrinkage and so on, rather than whet the reader's appetite with an outline of the procedures that are available. The experience gained in using the methods can be of great interest to the engineer.

Having tested the products we are not always told what to do with the results. What limits should be applied in composition for consignments of a proprietary plasticizer? This is the sort of question which presents difficulties when a national Standard on this topic is being drawn up.

The chapter on concrete admixtures is the core of the book. It covers the normal water-reducing and air-entraining admixtures as well as accelerators, retarders and the more specialized products such as fungicides, pigments and so on. Lignosulphonate compounds, the almost universal base for water-reducing agents, are given separate detailed attention. This chapter includes data on fly-ash, pozzolanas and other powders which can be used to modify the properties of concrete at high addition rates, and ends with a few notes on additives used by the cement manufacturer.

Dr Vénaut then turns to release agents and paints and varnishes for concrete. In this chapter, too, there are signs that, in the attempt to make the coverage as comprehensive as possible, the finer detail has been omitted. In some places there is only room to list materials with a few brief notes on the important aspects. Determined to extend the scope to the limit, the author turns to efflorescence and stain removal, curing compounds, resin repairs and, in Part 4, the manufacture and placing of concrete and mortar, discussing equipment and apparatus such as admixture dispensers.

Finally, there is a comprehensive list of several hundred products which are available on the French market with their suppliers listed alphabetically. This list is impressive in its detail and so your reviewer was particularly disappointed to find that the French admixture he knows best was not included. The main active ingredients of the proprietary products are not given.

As a thoughtful gesture, a long glossary of terms has been appended in French, English, German and Spanish.

Yes, a good book, but perhaps the author tried to cover too much ground at one time. Although aimed specifically at the construction industry in France, it is well worth studying in other countries.

R. KEEN

Shorter notices

Les résines de synthèse dans la construction

(Synthetic resins in building construction)

RILEM Symposium, Paris, 4–6 September 1967
Paris, Editions Eyrolles, 1970. 160 × 250 mm.

Vol. 1. pp. xxxv, 475. Price: 132 F + 5 F postage.
Vol. 2. pp. xiv, 591. Price: 189 F + 5·30 F postage.

These two volumes contain seventy-seven papers presented on the occasion of the Paris symposium. There is also a general report and a very small amount of discussion on each of the main topics.

Vol. 1 deals with Topics 1a and 1b. Topic 1a is officially called "Concretes and mortars, improvement by adding resins". The general reporter, however, noting that most of the papers draw attention to restrictions, would have preferred to call it "Modifying the properties of mortars and concretes by adding resins". Topic 1b is "Concretes and mortars without cement". Vol. 2 covers Topic 2, "Structures, jointing, reinforcing", and Topic 3, "The role of resins in the protection and repair of structures".

The general reporters make a good job of classifying the diverse material they have to handle. Most of the papers are well supported with data, but inevitably the basis of comparison varies from country to country and from material to material.

Each paper is printed either in English or in French. (These are the two official languages of RILEM but are not necessarily the original language—the title of a paper from Germany about strengthening beams refers to "the sanitation of prefabricated bridge members"). On Topics 1a, 2 and 3, there are more than twice as many papers in English as in French; on Topic 1b, the position is reversed. The summary of each paper and the general report of each session are printed in both languages.

A.E.B.

Concrete for high temperatures

by A. Petzold and M. Rohrs

Translated by A. B. Philips and F. H. Turner

London, Maclaren and Sons, 1970. pp. 235. 241 × 174 mm.
Price: £6·00.

The authors of this book, originally published as *Beton für hohe Temperaturen* by VEB Verlag für Bauwesen, Berlin, have been prompted by the apparent lack of available information on refractory concrete and by the growing importance of this material in the construction of industrialized furnaces, to produce a comprehensive account of the present stage of development in the field of concretes for use at high temperatures. The greater part of the book is devoted to heat-resistant and fire-resistant concretes made from cements, these being the most common materials in this field, but mention is also made of concretes derived from special cements or waterglass with new binding agents, such as magnesia binders, phosphoric acid or phosphate. The problems of mix design, concrete mixing and placing are discussed very thoroughly with examples related to cement-bound concretes; similar problems regarding the newer types of concrete are suggested as subjects for future development.

The importance of 'high-temperature concretes' is demonstrated by the wide range of uses discussed in Chapter 6. Typical examples are given together with the results of experience gained; these can be seen to vary in different industrialized countries. The book also devotes a chapter to the economic aspects of refractory concretes and outlines their advantages in comparison with the conventional use of fired bricks.

The text, which is clearly written and easy to read, is illustrated with some 70 Figures.

P.V.M.

Papers and books on cement and concrete

received in the C&CA Library MAY-AUGUST 1971*

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