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Moment-curvature relationships for prestressed concrete in constant-moment zones*

by M. J. N. Priestley, R. Park and F. P. S. Lu

Contribution by Cheng-Tzu Hsu,† MEng, and M. S. Mirza‡, MEng, PhD, MEIC

McGill University, Montreal, Canada

Dr Priestley, Professor Park and Professor Lu have presented a very interesting paper on the theoretical formulation of moment-curvature relationships for prestressed concrete members subjected to constant bending moments, taking into account the contribution of concrete in tension between cracks. We would like to discuss several points as follows.

(1) The definitions of the 'initial concrete strain' and the 'initial steel strain e_{si}''' used in the derivation of equations 3 to 10 based on the stress and strain distributions in Figure 1 are correct, provided the concrete strains e_c and e_{cc} include the effects of both the prestress at transfer and the strains caused by the applied uniform bending moment.

(2) The concentric prestress causes uniform compressive strain across any beam cross-section and does not affect the curvatures produced in the beam due to the applied uniform bending moments (caused by externally applied loads). However, a prestress eccentric towards the bottom of the beam produces negative bending moments and negative curvatures at zero applied external load. Figures 12 and 13 indicate negative curvatures at zero moment but do not show the negative bending moments produced by the eccentric prestressing force. Did the authors neglect the negative bending moments caused by the eccentric prestressing force?

(3) The authors have assumed that, at first cracking, the bond stress decreases linearly from a maximum u_m at the crack to zero at a distance l_B, equal to the bond length, away from the crack. For the average condition of two cracks spaced $1{\cdot}5l_B$ apart, they have used the bond stress distribution shown in Figure 4 of the paper. This distribution suggests that, at a point distant $0{\cdot}75l_B$ from either crack, bond stress is a multi-valued function which can have any values from $+0{\cdot}25u_m$ to $-0{\cdot}25u_m$ along the vertical line. If one examines the equilibrium conditions at the steel-concrete interface and the bond stress distribution suggested by the ACI Committee 408[1], the bond stress at any section along the bar has a unique value and is zero at the point B (Figure I) for a constant-moment zone between the two cracks.

Assuming that the bond stress is zero at point B and that there is a linear variation of bond stress between the cracks, one obtains a bond stress distribution shown in Figure I which is given by

$$u = u_m\left(1 - \frac{l}{0{\cdot}75l_B}\right) \quad \ldots\ldots\ldots (27a)$$

At a distance $l \leqslant 0{\cdot}75l_B$ from a crack, the tensile stress in the steel can be calculated from equations 17 of the paper and equation 27a as:

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†Research Assistant and ‡Associate Professor, Department of Civil Engineering and Applied Mechanics.

$$f_s = f_{scr} - \int_0^l u_m\left(1 - \frac{l}{0{\cdot}75 l_B}\right)\frac{\Sigma 0}{A_s}\, dl$$

$$\ldots\ldots(28a)$$

$$= f_{scr} - \frac{u_m \Sigma 0}{A_s}\left(l - \frac{l^2}{1{\cdot}5 l_B}\right)$$

Then

$$u_{av(1)} = \frac{A_s(f_{sA} - f_{sB})}{0{\cdot}75 l_B \Sigma 0} \ldots\ldots\ldots\ldots(28b)$$

where f_{sB} is the steel stress at point B (see Figure I) away from the crack.

Also, since a linear bond stress distribution is assumed,

$$u_m = 2u_{av(1)} = \frac{2A_s(f_{sA} - f_{sB})}{0{\cdot}75 l_B \Sigma 0} \ldots\ldots(29a)$$

By substituting u_m from equation 29a into equation 28a, the steel stress at the distance $l \leqslant 0{\cdot}75 l_B$ from a crack is given by

$$f_s = f_{scr} - 2(f_{sA} - f_{sB})\left(\frac{l}{0{\cdot}75 l_B} - \frac{l^2}{1{\cdot}125 l_B^2}\right) \ldots(30a)$$

As the applied uniform bending moment is increased from zero just before cracking, the steel stress is uniform at all sections in the constant-moment zone, and therefore the bond stress is zero and the steel stress is f_{sB} (this can be calculated from equations 1 to 4). Just after the formation of the cracks (assumed to be at $1{\cdot}5 l_B$ centres), the stress at the point midway between the two cracks (Figure 4) remains constant while the stress at A changes to $f_{sA} = f_{scr}$. The stress at the middle point can now be checked by equation 30:

$$f_s = f_{scr} - 2(f_{sA} - f_{sB})\left(\frac{0{\cdot}75 l_B}{l_B} - \frac{(0{\cdot}75 l_B)^2}{2 l_B^2}\right)$$

$$= f_{scr} - 0{\cdot}9375(f_{sA} - f_{sB})$$

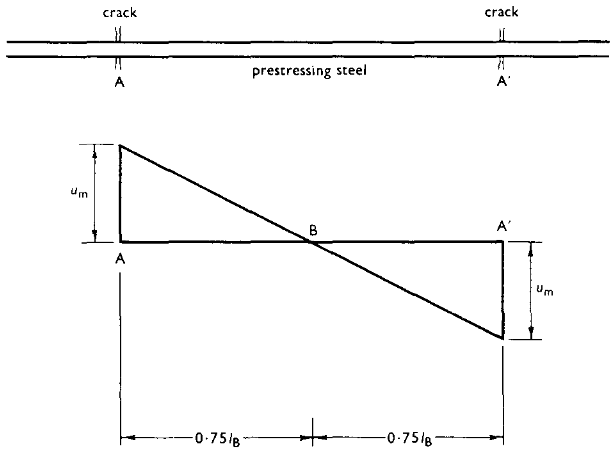

Figure I: Bond stress distribution between adjacent cracks in a constant-moment zone.

$$= 0{\cdot}0625 f_{sA} + 0{\cdot}9375 f_{sB}$$

$$\neq f_{sB}$$

This reflects the error in the assumed bond stress distribution shown in Figure 4. Also, if our equation 30a is used, the value of steel stress at section B (Figure I) is given by

$$f_s = f_{scr} - 2(f_{sA} - f_{sB})\left(\frac{0{\cdot}75 l_B}{0{\cdot}75 l_B} - \frac{(0{\cdot}75 l_B)^2}{1{\cdot}125 l_B^2}\right) = f_{sB}$$

which is satisfactory.

Our recent work on moment-curvature relationships in reinforced and prestressed concrete has indicated that the use of generalized stress-strain characteristics for steel and concrete can be handled conveniently in a computer program developed to evaluate the moment-curvature characteristics for any given reinforced, prestressed or partially prestressed concrete member. The authors are to be complimented on generating several new ideas in this field.

Contribution by B. V. Subrahmanyam, N. Lakshmanan and P. Srinivasa Rao

Indian Institute of Technology, Madras

We would like to congratulate the authors upon developing a method for accurately predicting the moment-curvature behaviour of prestressed concrete beams. In the method suggested, the calculation of the average curvature from the maximum curvature involves the consideration of a number of sections between any two adjacent cracks and thus is a little tedious.

A simpler method has been suggested by Dr Srinivasa Rao[2,3] for the average steel stress as

$$\varepsilon_{s,av} = \varepsilon_{scr} - \frac{K f_t'}{\mu E_s} \ldots\ldots\ldots\ldots(1)$$

where $\varepsilon_{s,av}$ = average steel strain

ε_{scr} = steel strain at a cracked section

$K = 0{\cdot}18 \dfrac{f_{sA}}{f_s}$

μ = reinforcement ratio

E_s = Young's modulus of steel

The maximum compressive strain in concrete would also vary between a maximum value at a cracked section and a minimum value between cracks. This difference can, however, be neglected. The average curvature is thus

$$\varphi_{av} = \frac{\varepsilon_c + \varepsilon_{s,av}}{d} \quad \ldots\ldots\ldots\ldots\ldots (2)$$

where ε_c = concrete strain.

The above method has been successfully applied to flexural and compressive reinforced concrete members[2,4]. We have now applied it to beam 1 discussed in the paper and the results are presented in Figure II. it can be seen from this Figure that both the authors' method and the method due to Srinivasa Rao predict the test results well. Rao's method, however, involves less computation.

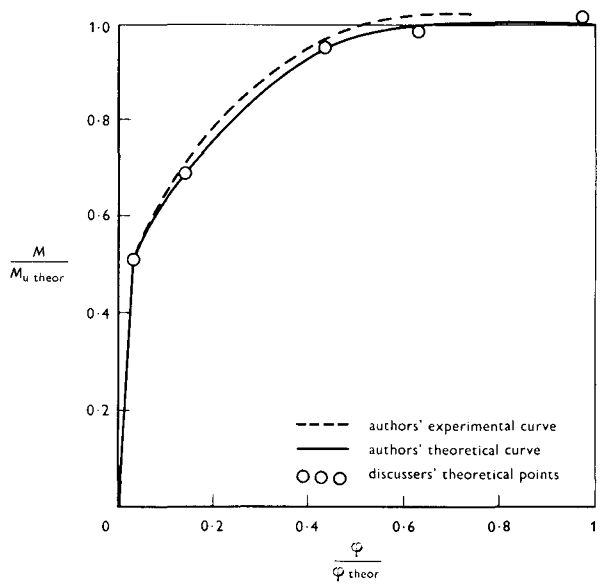

Figure II: Moment-curvature relationship for beam 1 of the paper

Reply by the authors

We thank Mr Hsu and Professor Mirza, and Mr Subrahmanyam, Mr Lakshmanan and Dr Srinivasa Rao, for their interest and comments. We reply first to the points raised by Hsu and Mirza in the order given.

(1) As used in the analysis, e_c and e_{cc} include the effects both of prestress and of applied external load.

(2) In Figures 10 to 13, the experimental and theoretical moments both relate to external actions, and are measured relative to zero applied external load. However, the curvatures are measured relative to zero concrete strain and therefore include any curvature due to prestress. Consequently, any negative bending moment induced by the eccentric cables is equal to the moment required to return the curvature to zero.

(3) We agree that the assumed shape of the bond stress distribution between cracks given in Figure 4 is approximate, and implies a multi-valued bond stress at the point midway between the cracks. However, this type of anomaly is well known. For example, what is the shear force under a point load on a beam? Precision of definition of the bond stress distribution at one point has been sacrificed for simplicity. Strictly, the bond stress distribution should be curved when approaching this point. Bell[5] has investigated the use of other bond stress distributions including the triangular distribution used by Hsu and Mirza.

Using our equation 30, Hsu and Mirza show that the stress in the steel immediately after cracking at the point B midway between two cracks spaced at the average distance of $1{\cdot}5l_B$ is higher than the steel stress there just before cracking begins (f_{sB}). They dispute this result and claim that these two steel stresses should be equal. They claim that our result is incorrect because our assumed bond stress distribution is in error, and state that their assumed bond stress distribution shown in Figure I leads to the correct result of the two steel stresses being equal. However, we feel that there is an error in their logic. There is no valid reason for assuming that the steel stress midway between cracks should be equal to that there just before cracking begins. Further, their modified equation (30a), which produced $f_s = f_{sB}$, would appear to lead to an impossible condition. Consider the situation after the *first* of two adjacent cracks form. The steel stress will be increased to greater than f_{sB} for a distance l_B on both sides of the crack. In particular, at a distance $0{\cdot}75l_B$ from the crack, the stress assuming a triangular bond stress distribution as calculated from our equation 30 will be

$$f_s = 0{\cdot}0625f_{sA} + 0{\cdot}9375f_{sB} > f_{sB}$$

The second crack may now form at any distance between l_B and $2l_B$ from the original crack. At the average spacing of $1{\cdot}5l_B$, bond stress at the new crack will be equal to u_m, decreasing in some fashion on either side of the new crack. If Hsu and Mirza's equation 30a is valid, the steel stress at $0{\cdot}75l_B$ from the first crack (and hence now midway between the cracks) would have to *reduce* from $f_s = 0{\cdot}0625 f_{sA} + 0{\cdot}9375 f_{sB}$ to $f_s = f_{sB}$ owing to the formation of the second crack. We cannot think of a valid bond stress distribution that would cause such a reduction, and feel that, if anything, f_s may increase slightly because of the second crack. Hence we prefer our assumed bond stress distribution, which does not lead to any change in steel stress within the distance $0{\cdot}75 l_B$ from the first crack when the second crack forms $1{\cdot}5 l_B$ away.

The simplified analysis suggested by Dr Srinivasa Rao appears to predict adequately the average behaviour in the example given. However, as the method we have proposed involves computer analysis, the extra complexity of the more exact analysis may be hardly significant. Further, although the effect of assuming constant maximum concrete compression strain between cracks appear to be small in the discussers' example, it is felt that in some cases significant errors might result. The difficulty with an empirical expression such as that suggested by Dr Srinivasa Rao is that it can only be applied with confidence to situations which are similar to the test data from which it was derived.

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