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The strip method of slab design: unique or lower-bound solutions?*

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Contribution by P. G. Lowe, PhD, MICE

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Mr Fernando and Professor Kemp provide further insights into the strip method in their paper, and incidentally raise some interesting questions about the relationship between the strip method and other possible methods. The strip method as described in the paper constrains the whole domain of the slab to have at most two orthogonal reinforcement directions, in either one or both faces of the slab. If this constraint is relaxed and we attempt to design the slab for minimum reinforcement content, it has been shown elsewhere[1] that, in general, a design results in which the domain of the slab is composed of a prescribed series of adjacent strip reinforcement regions. The feature of these optimal reinforcement patterns is their local, one-dimensional, strip nature.

It seems significant that, in one or possibly two of the examples discussed by the authors, the solutions which give exact solutions in the terms defined in the paper also lead to features in the resulting yield-line pattern and relative distribution of the steel required which are quite reminiscent of the corresponding optimal designs. Thus Figure 4c of the paper contains a number of features in common with the corresponding optimum, but Figure 4b has many fewer. Also the chosen moment field concentrates the steel at the edges, and this too is an optimum feature. Presumably there would be even closer similarity if the ratio of q distribution were allowed to vary in some beneficial way over the slab area. Somewhat similar remarks may be made about the mechanisms shown in Figure 7. The central region could be thought of as equivalent to the spherical deformation regions in the corresponding optimal design. Or, if the orthogonal reinforcement directions are chosen to be the diagonal directions, the resemblance is yet more apparent.

The extent to which the strip method can usefully be thought of as a genuine relative of the optimal design procedures, and vice versa, will no doubt depend upon the extent to which practical considerations allow constraints on the strip method to be relaxed while selective constraints to meet practical requirements are applied to the optimal results. There does seem to be some value, however, in exploring this region between these two existing approaches to slab design, and the points made by the authors in their paper will probably provide useful guides when examining such intermediate cases.

Incidentally, in Figure 6a, should the bending moment on the lower section of the Figure, shown as $\alpha qL^2/8$, be $\alpha qL^2/2$?

REFERENCE

1. LOWE, P. G. and MELCHERS, R. E. On the theory of optimal, edge beam supported fibre-reinforced plates. *International Journal of Mechanical Sciences*. Vol. 16, No. 9. September 1974. pp. 627–641.

*Pages 23 to 29 of *MCR* 90.

Reply by the authors

We are grateful to Dr Lowe for his discussion of our paper and for drawing attention to the similarities between the strip method and minimum-weight solutions. There is no doubt that this similarity increases if the load distribution is allowed to vary over the slab. This is an aspect of the strip method we are currently studying.

This similarity is intriguing and we agree with Dr Lowe in suggesting that there is value in exploring the region between these two plastic methods of slab design. It may well throw light on the remarkable fact that, although only the lower-bound theorem requirements are satisfied, it seems always possible to find a strip solution which gives the unique collapse load (for distributed loading and supports at least), even though no more than one mechanism of collapse may be possible.

The maximum bending moment on the cantilever in Figure 6a should indeed be $\alpha qL^2/2$ and not $\alpha qL^2/8$ as indicated.

A rational theory for balanced failure of prestressed concrete beams in torsion and bending*

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Contribution by G. S. Pandit†
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The authors' attempt to define the balanced failure condition of a prestressed concrete beam in combined bending and torsion must be commended, particularly in view of the paucity of published work on this complex problem. Earlier attempts to define a balanced section in combined loading have been confined mostly to reinforced concrete, although the basic principles are the same for reinforced concrete and prestressed concrete beams. For instance, it is now recognized that, for both types of beam, there are four categories represented by the four regions in Figure 4 of the paper. To a designer, it is important to have clear rules to define a balanced section and the boundaries of the four possible regions of Figure 4 without ambiguity. I wish to commend the present paper for achieving this objective.

The definition of a balanced section given by the authors is very similar to one given by me earlier[1,2]. The only basic difference is that I defined a balanced section as one in which the entire longitudinal steel (near both the top and the bottom faces) yields simultaneously with the yielding of the transverse steel and the crushing of concrete. Tests at the University of Alberta[3] have confirmed that the redistribution of stresses as a result of plasticity in combined bending and torsion is not so complete as to make the distribution of the longitudinal steel within the cross-section inconsequential. A balanced section must, therefore, have a definite distribution of longitudinal steel near the top and the bottom faces, failing which only part of the longitudinal steel (either near the top or near the bottom) may reach the yield point. This distribution is evidently a function of the torque to moment ratio ψ, as the influence of flexure is only to modify the effective steel area at the top and the bottom in accordance with the so-called 'fictitious steel concept'. I feel that a complete definition of the balanced section must require yielding of the entire (top and bottom) longitudinal steel simultaneously with the yielding of stirrups and the crushing of concrete.

For simplicity, the authors have ignored the longitudinal mild steel. I have serious reservations about this. Almost all prestressed concrete beams have longitudinal mild steel to hold the stirrups, if for no other reason. Often the mild steel constitutes a significant part of the total longitudinal reinforcement. Also because, unlike high-tensile steel, mild steel has a well defined and extended yield plateau, longitudinal mild steel is responsible for a substantial part of the ductility of a prestressed concrete beam.

Torsion tests at the Malaviya Regional Engineering College on axially loaded concrete beams showed a spectacular improvement in ductility due to the provision of mild-steel reinforcement[4,5]. These tests revealed that the contribution of the mild-steel reinforcing cage to the ultimate torsional strength remains substantially the same as in a reinforced (non-prestressed) beam. It is, therefore, evident that even a modest provision of mild-steel reinforcement would substantially change the limits of the four regions defined in Figure 4. I do not think the resulting simplicity is adequate justification for ignoring the longitudinal mild steel unless it can be shown to be inconsequential. In my view, the influence of longitudinal mild-steel reinforcement is not inconsequential.

Another difficulty which a practical designer may have in the application of the proposed analysis is due to the fact that most of the commercially available high-tensile prestressing steel does not have a well defined yield point. This is often arbitrarily taken to correspond with the proportional limit, 0·1 or 0·2% permanent set or 0·7% total strain. It is evident that the yield strain, ε_y, will vary considerably with the choice of the yield point. The authors have not indicated how the yield point should be defined to arrive at the idealized stress–strain curves of Figure 2a or the extent to which the proposed analysis and the regions defined in Figure 4 are sensitive to the choice of the yield strain, ε_y.

*Pages 30 to 36 of *MCR* 90.

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Reply by the authors

We appreciate Professor Pandit's remarks on our paper. We realize that longitudinal mild steel may contribute significantly towards the behaviour and the strength of prestressed concrete beams in combined bending and torsion, and in fact this steel is taken into account in the general theory on which the paper is based (references 2 and 3). Discussion of the effect of mild steel was omitted from the present discussion only for the sake of clarity and to focus attention on other variables.

In the numerical results presented in the paper, we have assumed that the prestress steel yields at a total strain of 0·010. However, as noted on page 35 of the paper, for a fully balanced failure condition, the analysis depends only upon the difference between the total strain at yield and initial strain in the steel due to prestress. The results given in Table 3, therefore, show indirectly the effect of ε_{py} upon reinforcement indices for a balanced mode 1 failure when ε_{pe} is held constant.

We agree that the various points made by Professor Pandit are all quite relevant to the problem. All of them come within the scope of the general theory developed here, but time is required for their evaluation. The definition of terms such as 'balanced failure' is also open for discussion. The paper deals only with the initial stages of the problem and work is continuing at the University of New South Wales. Although the ultimate aim is the guidance of the designing engineer, the enunciation of specific rules would be premature at this stage.

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