Inelastic behaviour of reinforced concrete frames*

F. A. Noor

Contribution by Professor K. O. Kemp

University of London, University College

Dr Noor has presented needed information on the ultimate loads of reinforced concrete frames with several redundants and the most interesting (but unproven) conclusion he draws is that the frames behaved in accordance with rigid plastic theory except where poor detailing occurred.

The conclusions which are drawn about internal moments and moment–rotation characteristics, however, appear to be highly suspect. The frames were all statically indeterminate with seven redundants and only one of these was an external reaction measured experimentally. Dr Noor describes a method of deducing the value of the internal moments by an iterative process using three equilibrium equations 1, 2 and 3. In fact, only two of these equations are independent since equation 3 is merely a summation of equations 1 and 2 and must be satisfied if the measured reactions satisfy over-all moment equilibrium. There are then eight unknown internal moments to be determined from two equations, which is consistent with the six unmeasured redundants. There is, of course, an infinity of solutions for the internal moments satisfying the two equations and the method of adjusting initial elastic values used in the paper is quite arbitrary and unscientific. The fact that there are eventually errors in equations 3 means that not even over-all moment equilibrium is satisfied.

Dr Noor also refers to satisfying the equilibrium equations at higher loads by an arbitrary process of redistribution "whilst keeping the moments at the critical sections as near to their ultimate calculated values as possible". But, surely, this is prejudging the issue, since one of the important objectives of such tests should be to determine whether any moment-rotation softening has occurred?

In view of these strictures, would Dr Noor agree that no reliance should be placed on the information presented on internal moments and, therefore, moment–rotation behaviour?

Contribution by J. Munro

University of London, Imperial College of Science and Technology

Dr Noor, in the opening paragraph of his Introduction, correctly draws our attention to the need for experimental verification of proposed methods of analysis and design of reinforced concrete frames. These procedures are concerned, inter alia, with determining the deformations for some specified load programme at or before incipient plastic collapse or they are concerned with designing structures such that, inter alia, the frame does not collapse and that limiting deformations are not exceeded before some load state along some specified load path is reached. The methods cited are based on elastoplastic behaviour and it can be inferred from his Introduction that the principal objective of Dr Noor's paper is to compare such theories with the actual behaviour of his frames in controlled laboratory conditions. Such experiments are of great importance and I began reading the paper with some anticipation. Unfortunately the experiments were conducted in such a way

---

*Pages 209 to 224 of *MCR* 97.

that little insight could be gained into the validity of the previously proposed theories.

The first load sequence (Frames F2, F4, F5 and F8) appeared to induce the left-beam collapse mechanism and then to induce a sway collapse mechanism. If the loading associated with the first mechanism is maintained, *post-collapse* deformations result. These deformations can increase to large (finite) magnitudes in the case of perfect plasticity or until brittle failure in the case of limited ductility. It should not be possible to maintain the first mechanism loading while the second loading is being introduced. What, then, is the author's criterion for the formation of a mechanism?

Also, the post-collapse deformations are largely irrelevant with respect to the elasto-plastic frame analysis procedures discussed earlier. The only useful pieces of information that can be extracted from these experiments are the magnitudes of the plastic deformations when brittle failure of critical sections occur. However, since these are encountered in this unrealistic way, it is difficult to see how the results are of more value than those obtained from the kind of simplified experiments mentioned in the second paragraph of Introduction. In any case, these results are to do with *material properties* and are not concerned with frame analysis per se.

Similar criticisms apply to load sequence 2 (Frames F3, F6 and F7), whilst the proportional load path of sequence 3 (Frame F9) seems to be the only one for which a meaningful verification of elasto-plastic analysis could be attempted. However, only a somewhat arbitrary modified elastic analysis is mentioned. Why was no elasto-plastic analysis reported?

Some results were given for a form of limit analysis using the models indicated in Figure 17a. Presumably the broken lines indicate the collapse *modal* deformations. If so, why, for example, are there deformations indicated in the right bay for mode 1 and the right beam for mode 2? The indicated mode 4 does not even satisfy mechanism compatibility. Apart from its kinematic incorrectness, it is impossible for a plastic hinge to form at section 16 when there is no load at that point — unless, of course, the section becomes critical because of a change of cross-sectional properties, and this does not apply in this case. It is not possible to check the plastic limit analysis since the properties of only three of the sixteen critical sections are given for each frame in Tables 2 and 3.

Four alternative flexibility release systems are given in Figure 17b. The frame has an indeterminacy number of seven and release systems 1, 3 and 4 have seven hinges. Release system 2 has only six hinges and so presumably one of the reactions (R_1, R_2, R_3) is to be considered an indeterminate force in system 2 but not in the other systems. However there is nothing in the Figure to indicate this. The only mention of Figure 17b in the text is in the section entitled "Application of rigid-plastic theory". Should the reference 12 cited in this section be reference 20? If so, the methods cited purport to tackle *elasto-plastic* frames and what is the significance of the 'rigid' in the heading of this section? Also, it is stated that system 3 gave the "most economical distribution of moments". Is the most economical design the one with the least total volume of reinforcement? Is Dr Noor saying that the economy achieved is a function of the subjectively selected release system used as a basis of the computation? This is certainly not the case for the optimal design procedures described, for example, in reference 1.

In conclusions, it would regrettably appear that despite the large amount of work that has been done by Dr Noor, an opportunity to check the validity of proposed theories for the elasto-plastic analysis and design of reinforced concrete frames has been largely lost.

REFERENCE

1. MUNRO, J. and SMITH, D. L. Linear programming duality in plastic analysis and synthesis. *Proceedings of the International Symposium on Computer-Aided Structural Design. University of Warwick, 1972.* Stevenage, Peter Peregrinus Ltd, 1972. Vol. 1. pp. A1.22–A1.54.

Reply by the author

Professor Kemp makes the obvious point that there are a large number of solutions to the three compatibility equations. This was discussed at some length in the original report (reference 12 of my paper) and a particular solution was obtained by applying the following constraints.

(1) The solution should be independent of the starting point of the iterative procedure.

(2) The bending moments at the critical beam sections should take account of the results of representative tests on simply supported beams. The flexural strengths of beams were, of course, modified to take account of the axial forces present in the frame members.

The moment–rotation curves of Figure 11 of the paper, which show the calculated experimental moments in a non-dimensional form, are not really affected by a small error in the value of these moments. The vital information presented in these curves consists of the measured rotations and here there is not much uncertainty.

It is a fact that there cannot be a neat solution to the problem of determining the experimental moments at the various critical sections of a frame with internal redundancies. One cannot be absolutely certain of either the modified elastic analysis used in my paper or the elasto-plastic analysis advocated by Dr Munro. It may be argued that, if this is the case, one should not test frames which have internal redundancies.

I disagree with this because, even if the exact value of the internal moments cannot be determined, much useful information can be gained by observing the over-all behaviour of the frames. These tests have now been followed up by further tests on a restrained beam model. The beams were designed in accordance with CP110 and brittle shear failures similar to those observed in the frames were seen in specimens with moment–shear ratios ranging from 1·5 to 3·0. It is unlikely that these failures would have occurred in similarly reinforced simply supported beams.

Coming to the question of proving the validity of the rigid–plastic theory applied to single-storey reinforced concrete frames, once again it is not necessary to know the exact value of the internal experimental moments. In general, both the pattern of cracking and rotational behaviour showed evidence of considerable deformation beyond yielding of the main reinforcement. Furthermore, the theoretical load calculated from the virtual work equations was, in most cases, below the experimental load. In the cases where the theory failed, the inelastic rotational capacity was affected by either insufficient shear reinforcement or inadequate detailing at joints and tension laps.

In Figure 17a, the deformations indicated by the broken lines reflect the measured displacements of the frame. With reference to Mode 4, the reason for a hinge at the top of the right beam is that the top reinforcement provided at mid-span of frames F6 and F7 was one-third of that at section 4. The reinforcement provided at all 16 critical sections is given in Figure 2, and further information on both the layout of the reinforcing bars and the instrumentation can be found in the original report (reference 12 of the paper).

It is true that the flexibility release system 2, shown in Figure 17b, was meant for a loading situation where there were only two vertical reactions. Dr Munro's point as regards optimal design procedures is accepted. An optimum design is not necessarily the one which gives the least volume of reinforcement. However, all is not lost and the opportunity to check the validity of the elasto-plastic analysis, favoured by Dr Munro, is still there. All of the data on loads and deformations are still available for anyone who wishes to use them. Although the idealized force–moment–curvature relationship (reference 20) may not give the exact experimental distribution of moments, it may be possible to obtain a qualitative simulation of the over-all behaviour of the frames.

In sequence 1, the left-beam mechanism was considered to have been formed when there were significant cracks at section 15, and a small increment of the load W_1 gave a relatively large increase in deflection. The effect of the sway load was gradually to increase the stiffness of the left beam and hence it was possible to increase W_1 on frames F4 and F5 before the formation of the sway mechanism. In the case of the other two beams subjected to this sequence of loading, there was some strain softening. An important part of the information required from these tests was to determine the behaviour of the plastic hinge under an increasing shear force, and this was the reason for adopting the loading sequence stated.

It is agreed that the loading sequence 2 does not occur in practice. It is analogous to a frame with no dead load and subjected to wind forces, which increase in proportion to the dead load. However, in theory, there is no reason why the elasto-plastic method cannot be used to trace the behaviour of the frame under this loading system.

The only loading stage for which this method may not be used is when the load W_3 was applied after the formation of the left-beam–sway mechanism. This is because of considerable weakening of the frames after undergoing large sway displacements.

It should be noted that the complete information on the force–moment–curvature relationship for every critical section in the frame is really required before an accurate analysis can be performed. This means a considerable amount of further experimental work on beams, columns and statically determinate assemblies of beams and columns. The problem of developing an accurate analytical model for simulating the behaviour of frames with internal redundancies is not an easy one, otherwise Dr Munro and his colleagues would have solved it long ago. Perhaps the way forward is to find suitable materials for modelling reinforced concrete for both the elastic and the inelastic range of loading. At Teesside Polytechnic, a beginning has been made in this direction.