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Developments in the variable-stiffness approach to reinforced concrete column design*

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Dr Wood and Mr Shaw are to be congratulated for presenting a general but still simple design method for reinforced concrete columns subjected to uniaxial bending. An attempt has been made to predict out-of-plane buckling, due to the reduction of the column bending stiffness in the plane transverse to bending, to which I would like to add the following contribution.

It should be noted that the phenomenon of out-of-plane buckling is of great importance for the design of biaxially loaded concrete columns, because the most common approach to the design of these given in codes[1–3] is to reduce the biaxial problem to one or two cases of uniaxial bending. This approach can sometimes lead to very unsafe results if out-of-plane buckling is disregarded.

Dr Wood and Mr Shaw state that out-of-plane buckling is only possible with a very pronounced strong axis and that it is therefore sufficient, for simplified design purposes, to consider a constant moment in the plane of bending. Studies that I have carried out at Munich[4] have shown that this statement is incorrect, since many important cases of out-of-plane buckling were detected for columns presenting a high slenderness ratio ($\lambda > 70$) in the bending plane. For such cases, a solution based on the stiffness of a single cross-section for a constant moment is inadequate.

*Pages 127–141 of *MCR* 108.

A simple design procedure for concrete columns with general end supports subjected to a constant axial force N_x and any type of transverse loads T_y and T_z in the main planes of symmetry x–y and x–z respectively (see Figure I) has been presented in reference 4. There out-of-plane buckling was taken into account by applying a reduction factor F to the maximum transverse load resisted by the column in its most rigid plane of bending. The determination of this factor is summarized below.

For the column shown in Figure I with a given

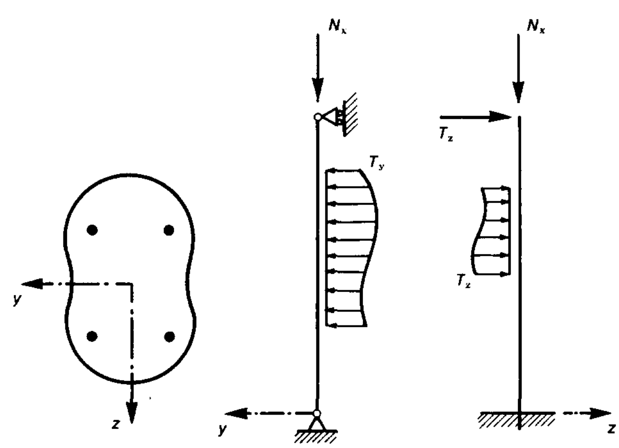

Figure I: Column with a constant doubly symmetric cross-section.

degree of reinforcement, the following parameters are defined.

α_y (or α_z) = a multiplier such that $\alpha_y T_y$ (or $\alpha_z T_z$) is the greatest transverse load acting together with N_x that the column can resist according to a second-order analysis, when displacements can occur only in the x–y plane (out-of-plane buckling is disregarded). For a column with loads applied at its ends only, this parameter may be determined by using the procedure developed by Dr Wood and Mr Shaw.

β_y (or β_z) = a multiplier such that $\beta_y T_y$ (or $\beta_z T_z$) is the greatest transverse load acting together with N_x that can be resisted by the column according to a geometric linear analysis. This parameter can be determined by using common design diagrams, such as those given in reference 2.

X_y, X_z = non-linearity factors for the x–y and x–z planes respectively, where

$$X_y = \frac{\beta_y}{\alpha_y} \text{ and } X_z = \frac{\beta_z}{\alpha_z}$$

It has been shown in reference 4 that the reduction factor F can be given approximately as a function of a single parameter Q ($Q = X_y/X_z$, where $X_y > X_z$) by the following equations.

For $Q \leqslant 3$:

$$F = 1$$

Therefore out-of-plane buckling does not occur.

For $3 < Q < 200$:

$$F = 0{\cdot}21 + 0{\cdot}27\,[\log(200/Q)]^{1{\cdot}8}$$

For $Q \geqslant 200$:

$$F = 0{\cdot}21$$

These equations have been obtained by means of a regression analysis based on 130 cases of out-of-plane buckling detected in a thorough study of an eccentrically loaded pinned column with a rectangular cross-section. They have been tested for columns with other supporting conditions, cross-sections and types of loading. The results are plotted in Figure II.

In order to avoid out-of-plane buckling, the maximum transverse loading in the most rigid plane of bending (say x–y) will be $F\alpha_z T_z$.

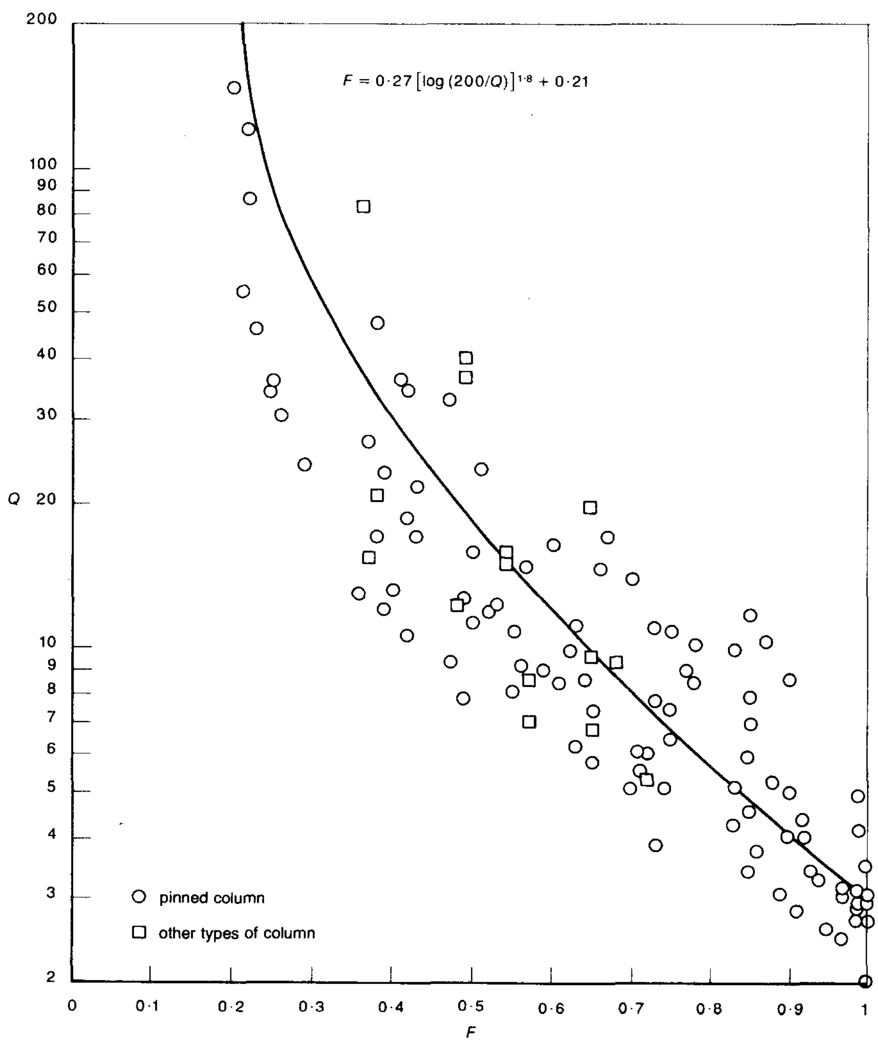

Figure II: The reduction factor F as a function of the parameter Q.

Reply by the authors

We wish to thank Dr Galgoul for his appreciation of this new variable-stiffness approach to column design. However, we cannot yet claim it to be a 'general' design method, for work is in progress to extend the method to other patterns of beam loading and end restraints.

By contrast, Dr Galgoul appears to have concentrated on transverse column loading (which will be a necessary extension to our method in future) within the context of pin-ended columns, which is unrealistic, and a few cases with 'other supporting conditions' unspecified. It is necessary, however, to master first the general case of applied beam moments, which can change sign from acting to restraining. In non-linear buckling, it is not feasible to rely on intuitive use of equivalent lengths of columns based on elastic critical loads.

Dr Galgoul is right to point out that out-of-plane buckling of the column does not necessarily imply a pronounced strong axis. To illustrate, in the spectacular case of out-of-plane buckling given in our text, such a strong axis would be necessary. Generally speaking, however, there will, of course, be a continuous interactive function between x–x and y–y bending, but we wished to produce a clear demonstration that the 'tangent' stiffness of what is still left elastic in the column controls buckling, whichever axis the bending arises from. With the high slenderness of Dr Galgoul's example ($L/h = 70$), it is almost impossible for the interactive function to have more than a mild effect. In the case of steel columns [Wood, reference 5 of the paper, now permitted by the ECCS (European Convention on Constructional Steelwork) rules, and the new draft British Standard]; buckling is almost entirely by buckling out-of-plane with predominant x–x bending, on account of the marked properties of the standard section, where the variable stiffness method is seen at its best. With reinforced concrete columns, however, the greater versatility with column shapes means that interactive behaviour between the two axes will have to be sought in future.

The present CP 110 with its added moments has no interactive effect of any sort, though there is no certainty that designs, which are usually very conservative, will always remain conservative. In this context, Dr Galgoul's suggestions have been noted with interest.

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