The effect of aggregate concentration upon the strength and modulus of elasticity of concrete*

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Contribution by D. W. Hobbs

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I found the paper by Dr Stock, Dr Hannant and Mr Williams particularly interesting and in this contribution I would like to offer a possible explanation for the observed change in tensile strength with aggregate volume concentration, V_a. Failure of concrete in tension is a progressive process[4], complete failure occurring after cracks have been initiated at or propagated from a number of points throughout a concrete specimen. Filure is therefore not determined by extreme values of stress and strain within the concrete and it is possible that a realistic estimate of the dependence of tensile strength upon V_a can be obtained by using an averaging technique.

If concrete is subject to uniaxial tension, a stress concentration is induced in the paste next to the aggregate particles. This means that failure of the paste phase will begin at or close to the aggregate rather than in the general framework of the paste and, if the number of aggregate particles around which failure occurs is large enough, the surrounding framework can no longer support the applied load and complete failure occurs. The prediction of the actual stress and strain at all points throughout a concrete specimen is impossible, since its internal geometry is so complex and because there are so many interacting particles. However, it can be argued[5] that increasing V_a reduced the average stress-concentrating effect of the aggregate particles so, except when V_a is small, tensile strength would be expected to increase with increasing V_a. Let us now see whether we can correctly quantity this increase using a simple idealized elastic model.

Let σ_a and σ_p be the *average* stresses in the aggregate and paste phases respectively acting in the same direction as the applied tensile stress. The tensile stress in the paste fraction acting in the same direction as the applied tensile stress will be greatest at the paste–aggregate interface, where it will be taken to be equal to σ_a (see Figure III).

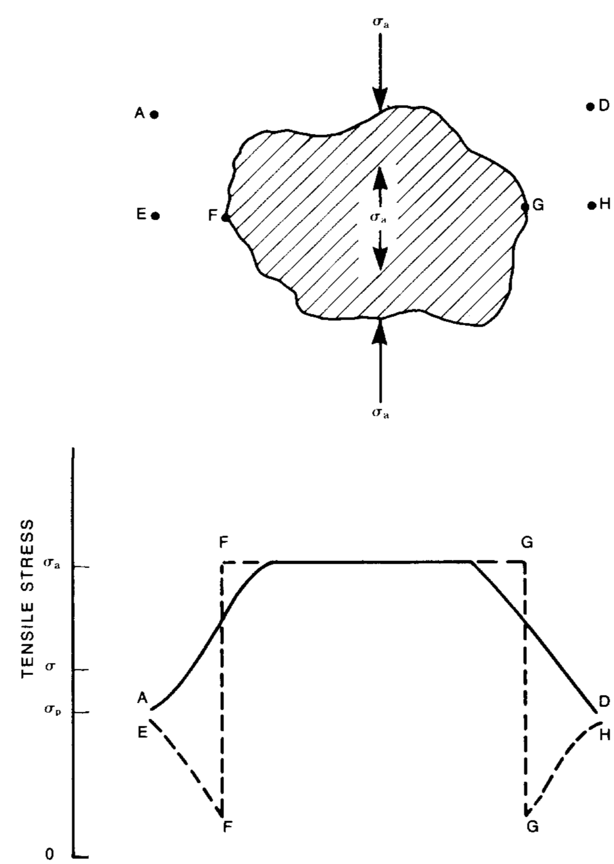

Figure III: Idealized tensile stress distribution along AD and EFGH.

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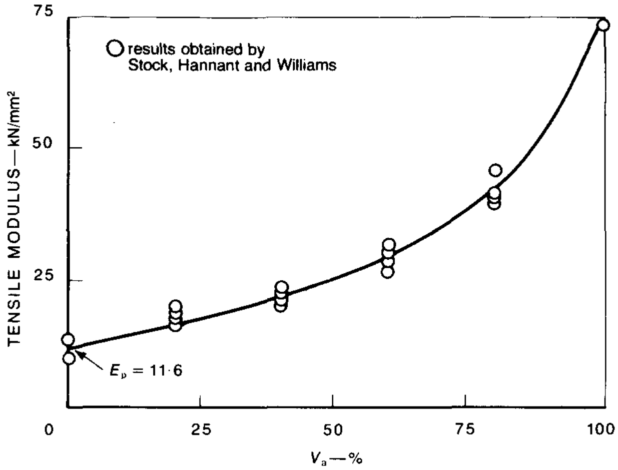

Figure IV: Relationship between tensile modulus and aggregate volume concentration.

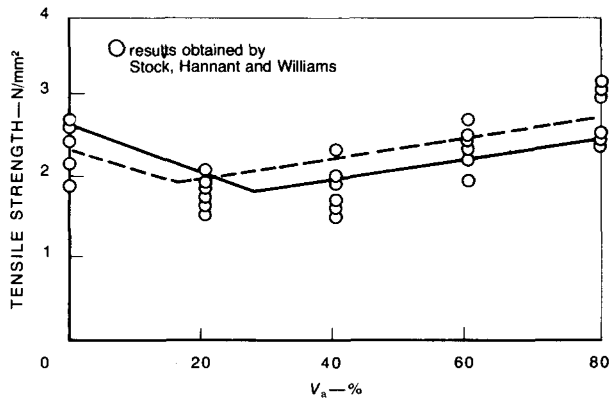

Figure V: Relationship between tensile strength and aggregate volume concentration.

Because the stresses and strains balance, we have

$$\sigma = (1 - V_a)\sigma_p + V_a\sigma_a \quad \ldots\ldots\ldots(1)$$

and

$$\frac{\sigma}{E_c} = \frac{(1 - V_a)}{E_p}\sigma_p + \frac{V_a\sigma_a}{E_a} \quad \ldots\ldots\ldots(2)$$

where σ is the applied tensile stress.

The Young's modulus of concrete in both compression[4, 6] and tension (see Figure IV) may be expressed by

$$E_c = E_p\left[1 + \frac{2V_a(E_a - E_p)}{E_a + E_p - V_a(E_a - E_p)}\right] \ldots(3)$$

From the above equations, we obtain

$$\sigma_a = \frac{2E_a\sigma}{(E_a - E_p)V_a + E_p + E_a} \quad \ldots\ldots(4)$$

Taking F_{ap} as the strength of the paste at or close to the interface and substituting f_{ap} into equation 4 and rearranging, we obtain for the tensile strength of concrete

$$f_t = \frac{f_{ap}}{2}\left(1 - \frac{E_p}{E_a}\right) V_a + 1 + \frac{E_p}{E_a} \ldots\ldots(5)$$

This equation gives the tensile strength at normal values of V_a.

At low values of V_a, we will take the tensile strength to be given by

$$f_t = \frac{f_p}{1 - V_a}$$

where f_p is the tensile strength of the cement paste specimens. Equations 5 and 6 are shown fitted to the author's data in Figure V for two values f_t and f_{ap}. It can be seen that there is approximate agreement between the predicted behaviour and the observed results when both f_t and f_{ap} are chosen to be equal to 2·6 N/mm². Thus, in this particular case, the strength of a cement paste specimen can be used to develop predictive equations for the tensile strength of the concretes tested. However, I agree with the authors that this may not generally be the case, since it is unlikely that bond strength or the strength of the cement paste fraction close to the aggregate particles will be the same as that of a cement paste specimen.

Finally, I would like to point out that, in my paper[4] to which the authors refer, I dod not report "that aggregate volume has little effect upon cube strength". I assumed in this paper that cube strength was proportional to $(1 + V_a)$. Thus, when V_a is increased from 0·4 to 0·8, the cube strength is assumed to increase by 29%, which is similar to the increase reported by the authors. In practice, the changes in V_a will be much smaller, perhaps 5%, so both the observed and predicted changes in strength will be small.

Contribution by Professor M. S. Akman

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The research by Dr Stock, Dr Hannant and Mr Williams dealing with the effect of aggregate concentration upon the strength and modulus of elasticity of concrete, bringing this topic back into the realms of investigation and discussion as it does, is thoroughly appreciated. The special care shown to determine the absorption of aggregate and to obtain an homogeneous concrete is of great merit.

In an earlier research programme of ours, the variations of some properties of concrete such as compressive and flexural–tensile strengths, the modulus of elasticity in compression, Poisson's ratio, discontinuity and loosening points were investigated as functions of gravel/sand ratio[1]. In the mixes, the free water/cement ratio was held constant at 0·50, whilst the cement content and the total aggregate volume concentration were varied from 482 kg/m³ to 227 kg/m³ and 0·55 to 0·79 respectively. The matrix of the concrete evaluated as a composite material is taken as the mortar made with the sand size fraction 0–3 mm, rather than the cement paste. The disperse phase consists of gravel or crushed stone between 15 and 30 mm. Consequently, the aggregate is gap-graded. Regarding the fineness moduli, the composition of the total aggregate is comparable to that of a graded one only when the gravel/sand ratio reaches unity.

We found that, until the coarse aggregate concentration reaches the value of 0·3, which corresponds to a gravel/sand ratio of 1·00, the compressive strength decreases with an increase in the proportion of coarse particles. At higher concentrations, this decrease stopped and even an increase was observed, depending upon the mineralogical structure. The flexural–tensile strengths and the moduli of elasticity in compression continuously increased. On the other hand, surface roughness and epitaxic properties of coarse aggregate affecting the bond between coarse particles and mortar, friability and modulus of elasticity of the coarse aggregate influenced this general behaviour.

In our opinion, the increasing effect of aggregate concentration upon strength and modulus of elasticity depends upon the interlocking of the coarse aggregate phase[2, 3]. In the type of investigation described in the paper with graded aggregate, this phenomenon cannot clearly be observed. As a matter of fact, the useful effect of aggregate concentration has not been observed in the mortar phase (references 9 and 11 of the paper). Therefore, it is more realistic to regard the concrete as a composite material consisting of mortar and coarse aggregate phases for this problem.

It would have been possible to make exact comparative evaluations if the authors had given information about the mineralogical structure and surface roughness of the aggregate used. In the case of deformable and weaker aggregate with high bond capacity, such as relatively soft crushed limestone, the internal restraint effect could no longer dominate, and the positive effect of aggregate concentration is lost above some concentration limit.

Contribution by Peter Bartoš

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Dr Stock, Dr Hannant and Mr Williams have presented results of interesting work which has potential practical applications. Increases in the price of cement are likely to remain much greater than the rises in the cost of aggregate and, as a consequence, the economically optimum proportions of cement and aggregate in current concrete mixes will tend towards higher concentrations of aggregate. However, at one or two points, the paper appears to be somewhat inconsistent and I should therefore like to offer the following comments.

(1) I am sure there were valid reasons for restricting the concentrations of aggregate to 80%, although these have not been mentioned in the paper at all. Ordinary mixes for structural concrete contain approximately 70 to 75% of aggregate by volume and, since Figures 1 and 2 of the paper suggest a steady rise in strength over the interval 60–80% of aggregate and the economies possible, an extension of the experimental programme beyond the 80% limit is very desirable (Figure I).

(2) Since the tests for strength were limited to aggregate volumes up to 80%, it was surprising to find a diagram for modulus of elasticity in tension in which the concentration of the aggregate had been extended to a full 100%. In the context of the paper where the properties of concrete specimens in which the volume of the aggregate was represented by *a number of particles* are dealt with, the diagram mentioned (Figure 8) does not fit and could be misleading. This Figure appears to be based on an assumption that the gradual elimination of cement paste would ultimately lead to a 'fusion' of the particles of aggregate into one solid mass of aggregate (Figure II). Since the modulus and strength in tension of an assembly of unbonded aggregate particles will tend towards zero, it is difficult to see why the investigators did not stop at 80% here also, to make all the diagrams comparable. For an aggregate which remains in the form of discrete elements, conclusion (4) of the paper clearly does not apply.

(3) The authors are rightly concerned about the uniformity of the distribution of the aggregate particles within the concrete specimens. It would be inter-

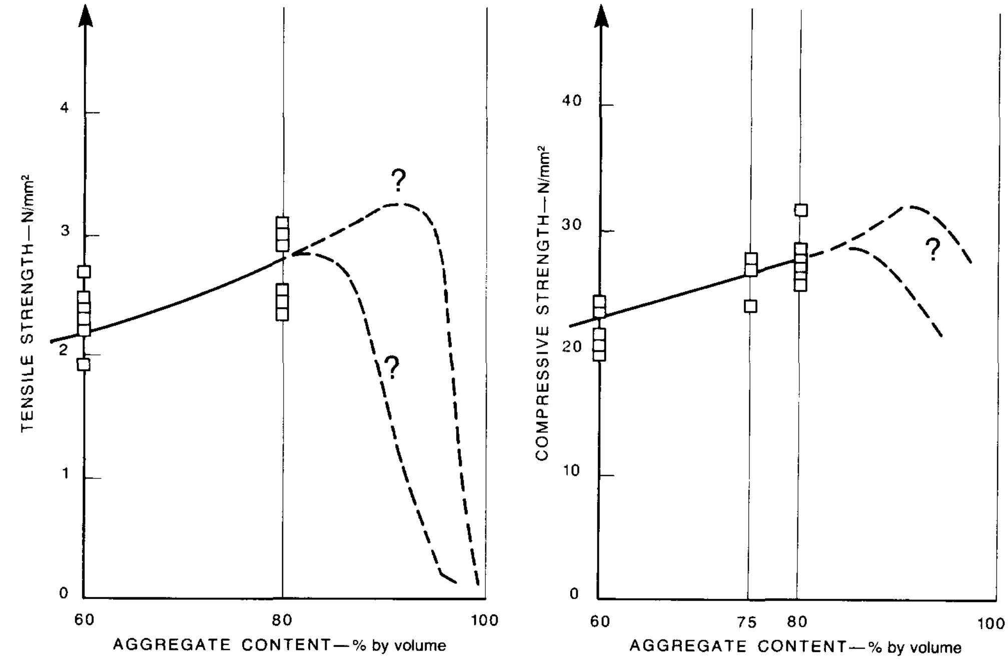

Figure I: Relationships between strength of concrete and concentration of aggregate should be extended above the 80% limit in order to find the optimum concentration.

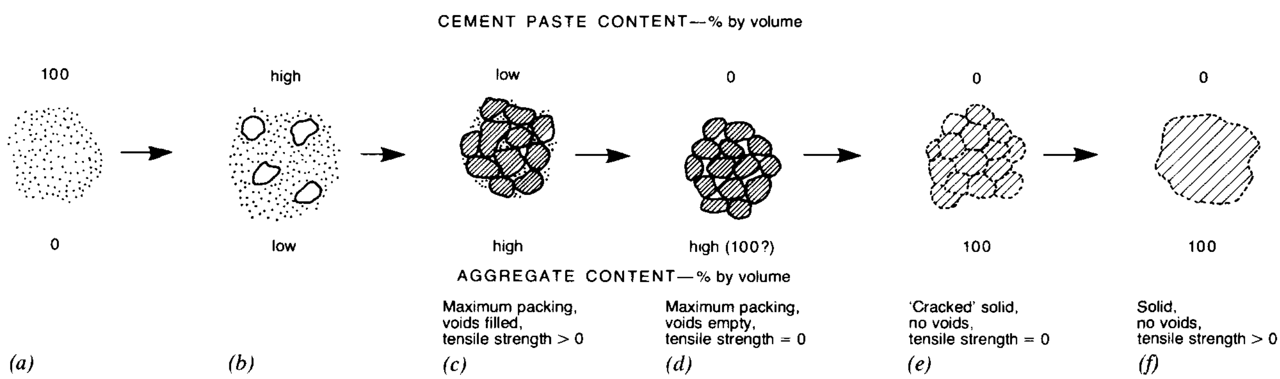

Figure II: Interpretations of volume concentrations of aggregate in concrete. Cases a and d represent practical limits; cases e and f apply only to simplified models of 'composite elements'.

esting to know how they assessed the 'good distribution' which was eventually achieved or judged the specimens to be 'acceptably uniform'.

(4) The authors themselves considered it inappropriate to attempt any predictions of properties of a composite from properties of only one constituent. It is therefore difficult to see a justification for the best-fit equations presented.

It is possible that some, if not all, the answers to this discussion can be found in the reference 17 of the paper, which is an MSc thesis. Unlike other publications, theses for higher degrees are generally difficult to get hold of even in the country of their origin; for readers abroad, such references are virtually impossible to obtain. In cases with this background, the scope of a paper may have to be restricted to permit all the basic data to be included and thus to minimize its dependence upon reference to the original work.

Reply by the authors

We would like to express our appreciation to Dr Hobbs, Professor Akman and Dr Bartos for their discussions on our paper. We find it most gratifying that our paper aroused sufficient interest to merit comment. For convenience we will reply to the discussions individually.

TO DR HOBBS. We are pleased to see such close agreement between Dr Hobbs' idealized model and our results. Despite the limitations of predictive equations, it is clear that they provide a valuable basis for a qualitative, if not quantitative understanding of some aspects of the behaviour of concrete.

Our comment on Dr Hobbs' 1972 paper related to the observed values rather than to his theoretical predictions. For example, in Table 6 of his 1980 paper[7], mixes with a free water/cement ratio of 0·59 and volume fractions of 64% to 80% have observed cube strengths within the range 32·4 to 33·7 N/mm^2.

TO PROFESSOR AKMAN. Professor Akman's comment regarding the interlocking of the coarse aggregate particles, with the attendent mobilization of some sort of skeletal structure within the concrete as it fails, could be relevant to the behaviour of concrete in compression. However, we believe that this type of behaviour is less likely in uniaxial tension, since our specimens had only one failure plane.

The aggregate used for our investigation was an irregular smooth flint gravel, in which some of the flint particles had a weathered surface layer. We made no attempt to measure any surface characteristics of the aggregate, since the experimental programme only included one type of aggregate, a limitation which we regret.

TO DR BARTOŠ.

(1) To provide meaningful comparisons between strength at different aggregate volumes, it is necessary to attempt to ensure that all specimens have the same air content. Preliminary studies indicated that the graded aggregate could not be compacted to less than 20% void content by techniques normally used for concrete. As Dr Bartoš indicates, ordinary mixes for structural concrete contain approximately 70–75% by volume of aggregate and our literature search did not reveal any mixes with significantly more than 80% aggregate volume (Figures 1 and 2). In our experience, even for dry lean concrete which is designed to maximize the aggregate content, the maximum volumetric content of the aggregate is approximately 84%. Thus an extension of the programme beyond 80% is unnecessary.

(2) As indicated in our paper, the plot in Figure 8 follows the practice of Dougill[8] in considering phase relationships in composite materials. Further information can be obtained from the chapter by Newman in *Composite materials*[9]. In this type of graphical plot, it is common practice to include an end point to indicate the modulus of the aggregate alone.

(3) The wash-out tests mentioned in our paper involved dividing stiffened but not hardened specimens into four sections and recovering the aggregate from each section. Since the quantity of aggregate in each section, expressed as a percentage of the total material in that section, did not vary by more than 3% from section to section, the specimens were judged to be uniform.

(4) We feel that the best-fit equations and the limitations thereof are adequately discussed in the paper. Also the MPhil thesis referred to as reference 17 may be obtained on request from the University of Surrey Library.

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