Discussion on papers published in the

Magazine of Concrete Research

Volume 36, Number 126 : March 1984

Shear transfer across cracks in reinforced concrete due to aggregate interlock and to dowel action*

S. G. Millard and R. P. Johnson

Contribution by D. J. Oehlers

University College, Cork, Department of Civil Engineering

It may be of interest to compare this research by Dr Millard and Professor Johnson on shear transfer across cracks in reinforced concrete with research on stud shear connections in composite beam construction; the two systems are similar, as a reinforcing bar can be considered to be a very long stud without a weld collar. In composite beams the studs are normally welded vertically to the top flange; however, in some beams, they are welded horizontally to the webs of inverted T sections(1), where the analogy between the two systems is more obvious. In a composite beam the shear force is transferred across the steel–concrete interface by friction and by the dowel action of the stud, whereas in reinforced concrete beams it is transferred across the crack by aggregate interlock and by the dowel action of the reinforcing bar. In composite beams failure of the shear connection can occur by fracture or embedment failure of the stud and by splitting or shear failure of the slab(2), but in reinforced concrete beams failure is unlikely to occur by embedment or shear failure unless the length of the reinforcing bar on one side of the crack is small. Splitting of the slab and fracture of the dowel occur therefore in both systems.

The shear force transferred across the crack by the dowel action is applied to the reinforcing bar as a normal force which is concentrated near the crack. If the distribution of this normal force is similar to that on a stud(2), the shear load on a single reinforcing bar which causes the slab to split is given by(3,4)

$$P_{sp} = \frac{1{\cdot}2\pi\varphi d_s f_{ct}}{(1-\varphi/d_s)^2}$$

when the reinforcing bar is not subjected to significant axial loads and where φ is the diameter of bar, d_s is the depth of concrete slab and f_{ct} is the indirect tensile strength of the concrete. This equation is in close agreement with the results of specimens 21L to 24L and explains the small variation in strength between specimens 21L and 24L, since the large variation in the cube strength of the concrete, f_{cu}, only causes a small change in f_{ct} according to the linear relationship between these two variables. It begs the question of how important is splitting in the transfer of shear in cracked reinforced concrete beams, since it has been shown that it can affect the strength of composite L beams(5) where the lateral cover to the dowel is small, as may occur in cracked reinforced concrete slabs. There appears to be a contradiction in the cube strength of specimen 21L; it is quoted as 27·6 N/mm² in the text and 37·6 N/mm² in Table 2.

The dowel strength of stud shear connections in push tests was determined experimentally by Ollgaard, Slutter and Fisher(6) as

$$F_{du} = 1{\cdot}74 A_s f_{cu}^{0{\cdot}3} E_c^{0{\cdot}44} \qquad \text{(I)}$$

*Pages 9–21 of *MCR* 126.

where the units are in N and mm and where A_s is the area of the shank of the stud, f_{cu} is the cube strength of the concrete and E_c is the elastic modulus of the concrete. The ultimate tensile strength of the stud material, f_u, was 490 N/mm² and the mean concrete cube strength was 35 N/mm². This relationship was simplified to

$$F_{du} = 0{\cdot}46 A_s\,(f_{cu}E_c)^{0\cdot5} \quad \ldots\ldots\ldots\ (\text{II})$$

which has a similar form to the theoretical dowel model of equation 3 in this paper. Semi-empirical research at Warwick University[2] showed that the strength of long stud shear connectors without weld collars and in push tests can be written in the form

$$F_{du} = 0{\cdot}64 A_s K_e K_f f_u \quad \ldots\ldots\ldots\ (\text{III})$$

where K_e and K_f are factors which allow for variations in E_c and f_{cu}/f_u. These methods of predicting F_{du} are compared in Figures I to III for the case of a 16 mm high-yield reinforcing bar; the relationship between f_{cu} and E_c given by CP110[7] for normal-density concrete is used where necessary. Ollgaard's results are always an upper bound (Figures I and III), since his studs had weld-collars, but the variations are similar. It would appear from Figure II that equation 3 is applicable to stud shear connectors if the 'yield' strength, f_y, of the high-yield reinforcement can be assumed to be close to the ultimate tensile strength and therefore this equation is useful in the analysis of stud shear connections without weld-collars in which f_u was varied, since there is a scarcity of research in this area. Ollgaard tested specimens with lightweight and normal-density concrete and found that the dowel strength depended on E_c as well as f_{cu}; it would appear that equation 3 is applicable to normal density concrete slabs and its scope could be enlarged if the strength was defined as a function of E_c as well as f_{cu} and f_y. An analysis of the specimens without initial axial load using equation III showed that only specimen 25L would fracture before the slab split and this would occur at more than twice the experimental load when f_u is assumed to be 500 N/mm². Is the value of E_c in the linear regression analysis correct, as it appears to be twice the value given in CP110[7]?

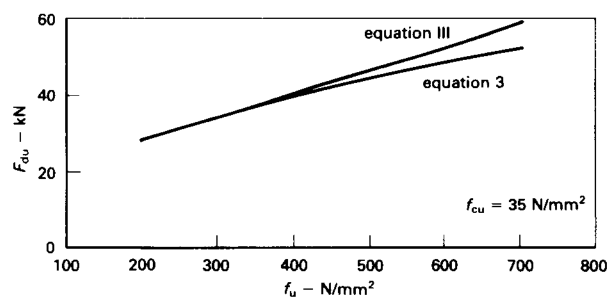

Figure II: Variation of the dowel strength with the ultimate tensile strength of the steel.

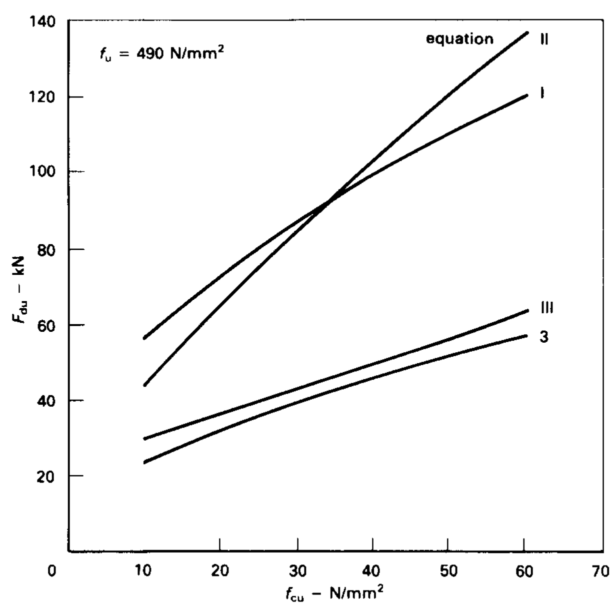

Figure I: Variation of dowel strength with cube strength of the concrete.

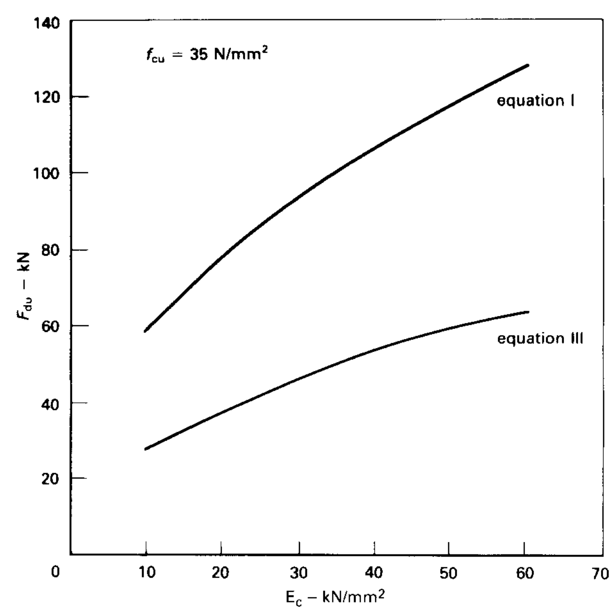

Figure III. Variation of the dowel strength with the stiffness of the concrete.

Reply by the authors

We thank Dr Oehlers for his interesting comments upon the comparison between test results on dowel-bars and shear stud connectors. There are, indeed, many similarities, but one must take care also to observe the differences and their consequent effects upon the behaviour of the test specimens.

The larger-diameter weld collar at the base of a normal shear stud undoubtedly causes a reduction in the bearing stresses in the concrete at the point where they are highest and explains why the empirical and dimensionally inconsistent equation I gives an overestimate of the ultimate dowel force in a reinforcing bar. That equation III, relating to shear studs without weld collars, more closely follows equation 3 confirms this explanation.

However, another important difference between the behaviour of shear studs and that of reinforcement dowel bars embedded in cracked concrete is that tensile forces are induced in the dowel-bars due to over-riding of the rough crack faces when a shear displacement occurs across the cracks. These tensile forces have a significant effect upon the mode of ultimate failure and they will be less pronounced in shear studs. It is seen in Figure 15 that, when no tensile forces are present in dowel-bar tests, ultimate failure is caused by splitting of the concrete. However, when a tensile force is applied and maintained during shear loading, the mode of failure changes to a crushing of the concrete adjacent to the crack and flexural yielding of the reinforcement further away from the crack caused by the redistribution of bearing forces. This change in behaviour is presumably due to micro-cracking and softening of the concrete adjacent to the crack, resulting from high localized bond stresses. Hence equation 3 is modified to equation 5 to allow for the effect of tensile stresses upon the ultimate bending moment of the dowel-bar. The effect of tensile stresses induced in reinforcing bars embedded in cracked concrete specimens due to overriding of the crack faces is discussed further in a second paper on pages 3 to 15 of this issue.

We thank Dr Oehlers for pointing out two misprints which appeared in the first paper. The strength of specimen 21L is the value 37·6 N/mm² quoted in Table 2 and not 27·6 quoted in the text. The linear regression equation for E_c should read:

$$E_c = (20{\cdot}88 + 0{\cdot}175 f_{cu}) \times 10^3 \quad \text{N/mm}^2$$

This gives values consistent with Table 64 of CP110.

REFERENCES

1. TOPRAC, A. A. and EYRE, D. G. Composite beams with a hybrid Tee steel section. *Proceedings of the American Society of Civil Engineers*. Vol. 93, No. ST5. October 1967. pp. 309–322.
2. JOHNSON, R. P. and OEHLERS, D. J. Analysis and design for longitudinal shear in composite T-beams. *Proceedings of the Institution of Civil Engineers. Part 2 : Research and Theory*. Vol. 71. December 1981. pp. 989–1021.
3. OEHLERS, D. J. and JOHNSON, R. P. The splitting strength of concrete prisms subjected to surface strip or patch loads. *Magazine of Concrete Research*. Vol. 33, No. 116. September 1981. pp. 171–179.
4. OEHLERS, D. J. *Analysis of 180 push specimens subjected to fatigue loadings*. May 1984. University College, Cork, Department of Civil Engineering. pp. 46.
5. JOHNSON, R. P. and OEHLERS, D. J. Design for longitudinal shear in composite L beams. *Proceedings of the Institution of Civil Engineering. Part 2 : Research and Theory*. Vol. 73. March 1982. pp. 147–170.
6. OLLGAARD, J. G., SLUTTER, R. G. and FISHER, J. W. Shear strength of stud connectors in lightweight and normal-density concrete. *Engineering Journal of the American Institute of Steel Construction*. Vol. 8. April 1971. pp. 55–64.
7. BRITISH STANDARDS INSTITUTION. *The structural use of concrete. Part 1 : Design, materials and workmanship*. London. pp. 154. CP110 : Part 1 : 1982.