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A photoelastic investigation of anchorage bearing stresses*

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This paper is a useful addition not only to research on anchorage bearing stresses, but also to problems involving the application of concentrated loads, for example, the transfer of shear by dowel action in reinforced concrete components and by shear connectors in composite beams. As the Professor Rasheeduzzafar and his co-authors have shown in the paper, the theoretical solutions for the two-dimensional problem of symmetric strip loads on prisms are in reasonably close agreement, but the theoretical solutions for the three-dimensional problem of concentric patch loads give widely varying results. As a result of the additional complications caused by ducts and eccentric loadings, solutions to this type of problem have been sought experimentally by testing concrete prisms, but these tests only give the ultimate strengths and rarely show the methods by which the load is transferred or the modes of failure. However, the photoelastic tests described in the paper give a clear picture of what is actually happening.

When a concentric strip load (EFGH in Figure I) of width $2a_1$ is applied to a prism of width $2a$, the ratio between the splitting force, T, and the applied load, L, can be deduced from the equilibrium of forces on a slab subjected to a uniform shear flow[1]:

$$T/L = (1 - a_1/a)^2/\pi \qquad \text{(I)}$$

This ratio applies to specimens in which the concentrated load is applied uniformly over the strip and when $l > 2a$ (Figure IIb) and was deduced by ignoring shear lag and using the fact that, as $a_1 \to 0$, T/L approached the inverse of π[2]. This relationship is in good agreement with a finite element analysis which allowed for shear lag and in which the strip load was applied as a uniform displacement[3], with Iyengar's analysis[3], and with the two-dimensional data in Figure 14 which are included in Figure III. Equation 1 converges onto the point ($a_1/a = 1$, $T/L = 0$) because

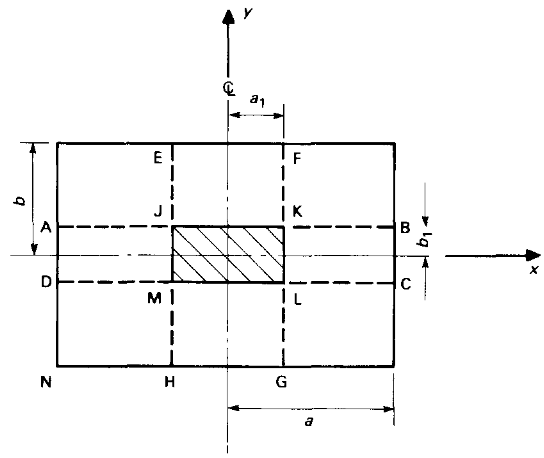
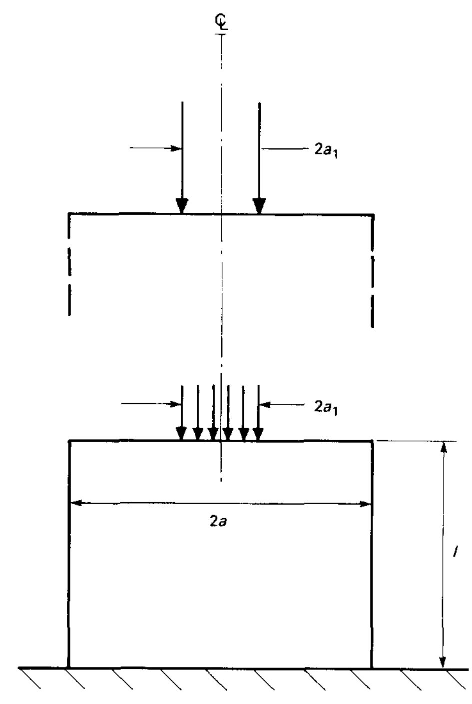


Figure I: Concentric patch load.



(a)

(b)

Figure II: Types of concentrated load.

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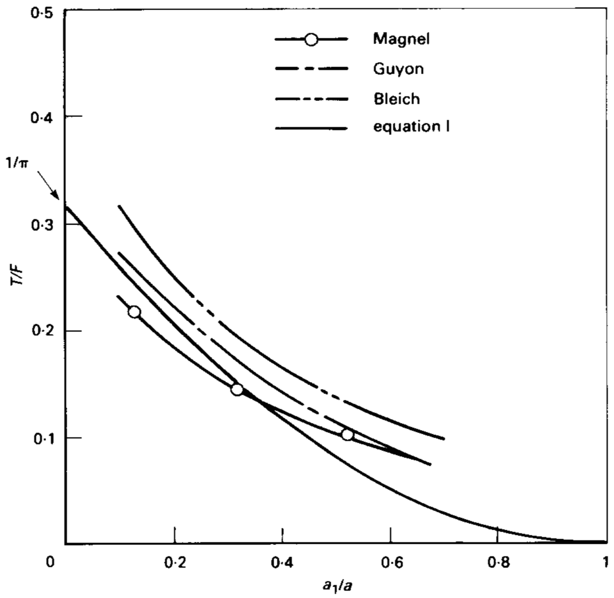

Figure III: Splitting force of concentric strip loads.

the concentrated load is applied uniformly and hence there is no lateral dispersal when $a_1 = a$; this condition is not true for all of the other theoretical analyses in Figure III. The large divergences with equation I at high values of a_1/a can therefore be attributed to the method of applying the strip load and to shear lag.

The splitting force from the two-dimensional analysis of the strip load (equation I) is compared in Figure IV with the splitting force from the three-dimensional analyses of concentric patch loads which are shown in Figure 12. The total splitting force

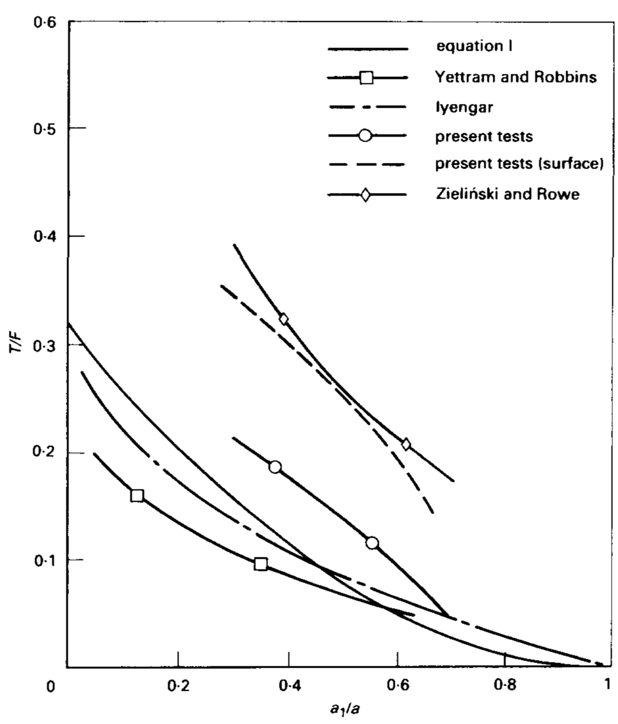

Figure IV: Splitting force of concentric patch loads.

appears to depend upon a_1/a and not upon whether the load is dispersed in two or three dimensions. It it is assumed that T is the sum of the splitting forces along the orthogonal axes in Figure I, as is indicated in Table 5, it is not clear what the distinction is between the 'present tests' and 'present tests (surface)' in Figure 12. Equation I is reasonably close to the 'present tests', but falls well below those of the 'present tests (surface)' and below Zieliński and Rowe's results (reference 4 of the paper). It would be of interest to know how the total splitting force was determined from Zieliński and Rowe's experiments when, as stated in the paper, only surface strains were measured on the concrete prisms.

The 'present tests (surface)' and Zieliński and Rowe's results in Figure IV suggest that the bursting force is approximately six times the value given by equation 1 when $a_1/a = 0{\cdot}7$; this factor appears to be too large for the following reason. Consider a concentric square patch load acting on a square prism be derived from equation I, is the same as if the patch load JKLM acts as a strip load on element ABCD of the prism, the splitting force in this prism, which can be derived from equation I, is the same as if the patch load acted on element EFGH of the prism. Therefore, it is conceivable that, if the patch load acted on the prism without the four corner elements, such as DMNH, then the splitting force could be double that of the strip load due to dispersal in three directions instead of two. It is improbable that the splitting force could be six times that of the strip load, since the corner elements only comprise 9% of the area of the prism when $a_1/a = 0{\cdot}7$. If it were six times, this would imply that the splitting strength of the patch JKLM on element ABCD actually reduced when $b > b_1$, which is contradictory to the available experimental data[3].

A theoretical relationship between the total concentrated load, F, and the maximum splitting stress, P, can be derived from equation 1 and from the distribution of the splitting stresses[3,4] for the case of a strip load EFGH acting on the whole prism in Figure I:

$$F = \frac{2{\cdot}4\,a\,b\,P}{(1 - a_1/a)^2/\pi} = 4\sigma ab \quad \ldots\ldots\ldots \text{(II)}$$

The conditions used in deriving equation I are applicable to this equation and the results have been validated experimentally[3]. This variation is compared with the theoretical variations from Figure 13 in Figure V and forms a lower bound and hence overestimates the load at which a prism splits. As the authors have stated in the text and shown in Figures 6a and 6b (I assume the sub-captions for Figures 6b and 6c are incorrect), the duct moves the position of the maximum transverse tensile stress from along the centre-line to the edge line. This may indicate that the applied load is concentrated along the edge as in

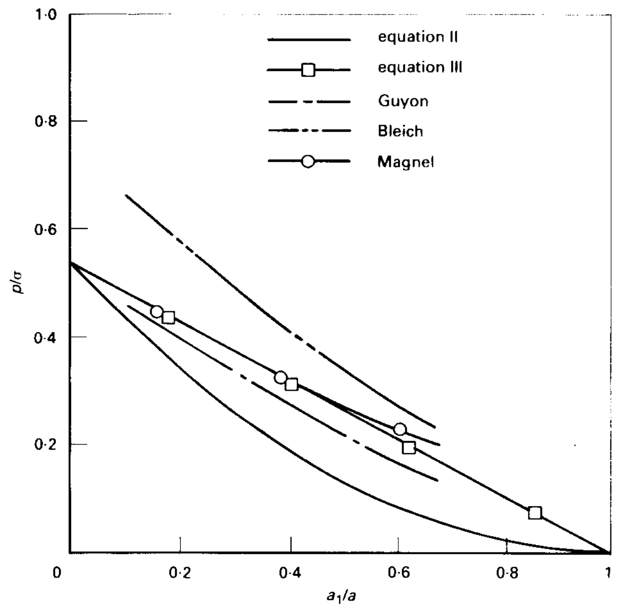

Figure V: Maximum splitting stress for concentric strip loads.

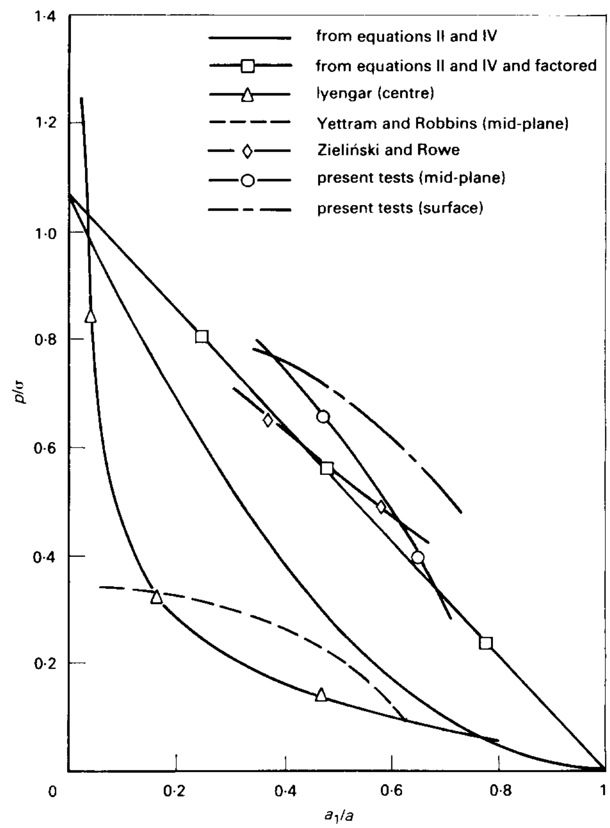

Figure VI: Maximum splitting stress for concentric patch loads.

Figure IIa; it would be of interest to know whether the interface stress distribution in the photoelastic tests did range between those shown in Figures IIa and IIb and whether it can be considered to be the same in a concrete structure. If the load is assumed to be concentrated at the edges of the patch as in Figure IIb, it can be deduced from the equilibrium of forces on a slab subjected to a uniform shear flow(1) that the total splitting force does not change, but that the length of the zone in which the splitting force occurs reduces, causing the maximum splitting stress to increase by a factor of $(1-a_1/a)^{-1}$ and hence cause the linear relationship between F and P in Figure V:

$$F = \frac{2{\cdot}4\, a\, b\, P}{(1-a_1/a)/\pi} \quad \text{.............(III)}$$

This relationship is close to the other theoretical variations.

A reciprocal relationship(3) using equation 2 can be used to derive the splitting strength of a prism subjected to a patch load:

$$\frac{1}{P} = \frac{1}{P_y} + \frac{1}{P_x} \quad \text{.............(IV)}$$

where P is the load to cause the prism to split when subjected to the patch load JKLM in Figure I, P_y is the splitting load when the strip ABCD is applied and P_x is the splitting load when the strip EFGH is applied. The only theoretical basis for this reciprocal relationship (equation IV) is that it is correct for extreme values since, as $a_1 \to a$, $P_x \to \infty$ and hence $P \to P_y$, but it has been shown to agree with experimental data(3). For the case of a concentric square patch load on a square prism, equation IV shows that the splitting stress is twice that derived from equation II when $b_1 = b$ and lies between the theoretical and experimental values of Figure 11 which are shown in Figure IV. When the load is concentrated near the edge of the patch, the length of the splitting zone is reduced as in the case of the strip load; if the same factor, $(1-a_1/a)^{-1}$, can be applied, the splitting stress has the linear variation shown in Figure VI. This is in reasonably close agreement with the 'present tests' and Zieliński and Rowe's results.

Professor Rasheeduzzafar and his co-authors propose that a distinction be made in design between external and internal anchorages by allowing part of the concentrated load to be transferred by shear across the steel–concrete interface in the internal anchorages. Could the shear strength and distribution in a concrete and steel component be different from the photoelastic models, and can this bond or friction be relied upon, in particular, when the anchorage zone is subjected to cyclic variations in load or overloads? Could the shear strength depend upon the method by which the insert is placed and upon such variables as the shrinkage strain and tensile strength of the concrete?

The authors have shown that there is only a slight overlap of splitting stresses in their specimens. I suggest that this is because the spacings of the anchorages were at least half the depth of the anchor block.

Should the spacings be much less than the depth of the anchor block, there would be a substantial overlap of the bursting stresses beneath each anchorage and furthermore the anchorages would act as one unit causing large bursting stresses between them[5]. The authors have based their design rules on tensile failure, but I feel they should limit its applicability, since compressive failure will control design when $a_1 \rightarrow 0$ and $a_1 \rightarrow a$. The range in which tensile failure controls the design of concrete prisms subjected to concentric patch loads has been found, experimentally, to be[3]

$$0{\cdot}2 < a_1/a < 0{\cdot}6 \quad \text{(V)}$$

However, this range may be affected by ducts.

The authors suggest that the transverse reinforcement is placed according to the elastic distribution of the splitting stresses. After splitting, will this distribution be the same in the reinforcement and can the concrete be relied upon to resist the compressive load, since the triaxial restraint to the concrete under severe compression may have been reduced by the split, particularly when a duct is present? It was found in tests on stud shear connections[1] that the transverse reinforcement did not increase the strength after splitting, whereas in reinforced prisms the strength increased by a factor of 1·7 to 2·2 times the splitting load; however, the prisms failed well before the lateral reinforcement yielded[3]. Could the strength of the prisms after splitting depend not only upon the strength of the lateral reinforcement but also upon the lateral restraint the reinforcement gives to the compression zone and hence upon its position with regard to this zone? However, it has been shown experimentally[1,3,4] that the transverse reinforcement does prevent sudden catastrophic failures by allowing a gradual reduction in load.

This is a most interesting paper; not only does it give recommendations for determining the strength of the end-block and the splitting force which, when compared with other research, verge on the conservative side and hence can be used for design, but it also shows how the duct influences the distribution of the lateral stresses by concentrating them at the edges of the anchorage. Since possible premature compressive failure of the concrete under the concentrated load after splitting is to be avoided, and in order to prevent splitting at serviceability loads, I would suggest it would be more appropriate to design the end-block against splitting by using Figure VI and also place transverse reinforcement according to Figure IV in order to ensure that there would be a gradual failure should the system be overloaded.

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