

Discussion

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Shear transfer across cracks in reinforced concrete due to aggregate interlock and to dowel action*

Shear transfer in cracked reinforced concrete†

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Dr Millard and Professor Johnson are to be congratulated on their extensive and thorough experimental analysis on the roles of aggregate interlock and dowel action in cracked concrete shear transfer.

The roles of the two transfer mechanisms are analysed separately in the first paper (*MCR 126*), by means of different concrete specimens, whilst the combined effects are presented and discussed in the second paper (*MCR 130*), which refers to several tests performed on reinforced concrete specimens.

The main feature of the first set of tests (*MCR 126*) is that the direct stiffness restraining crack widening (i.e. the transverse normal stiffness across a crack) is constant throughout each test; as a consequence, both crack width and slip vary during each test, and generally increase with increasing shear values. In these tests, not one of the main parameters, characterizing aggregate interlock (namely the interface stresses and the crack relative displacements) remains constant: as a consequence, the response curves are certainly useful for understanding the many aspects of the problem, but hardly provide the basis for the development of new analytical models.

With reference to aggregate interlock, we would like to make a few comments, mostly on the comparison of the experimental results with those predicted by available analytical models.

Only very few consistent analytical models have been proposed so far in order to describe and to predict aggregate interlock behaviour: to our knowledge, only three models are suitable for general use, namely the model based on the frictional sliding of the two rigid crack faces⁽⁸⁾, the 'two-phase model'⁽⁹⁾, and the 'rough model', proposed first by Bažant and Gambarova⁽²⁰⁾ and then improved by Gambarova and Karakoç⁽²²⁾.

The rough crack model is based on a few general properties to be expected for the cracks, because of the random distribution of the contact points at the crack interface. The (total stress)-(total displacement) relations are deduced by considering the crack surface as a regular array of trapezoidal asperities (Figure I): as a consequence, the stresses are essentially a function of the slip width ratio (called $r = \delta_s / \delta_n$, where δ_s is the shear displacement, and δ_n is the crack width).

The mathematical formulation of the stress-displacement relations (equations I and II) is based on Paulay and Loeber's test results at constant crack width, and (to a lesser extent) on Daschner and

*Pages 9-21 of *MCR 126*.

†Pages 3-15 of *MCR 130*.

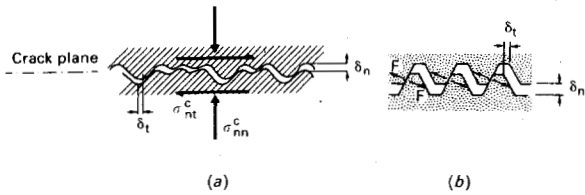


Figure 1: Crack morphology (a); simplified model with trapezoidal asperities (b).

Kupfer's test results at constant confinement stress⁽²²⁾:

$$\delta_{nt}^c = \tau_0(1 - \sqrt{2\delta_n/d_a})r(a_3 + a_4r^3)/(1 + a_4r^4) \dots (I)$$

$$\delta_{nn}^c = -a_1a_2\sqrt{\delta_n\sigma_{nt}^c}/(1+r^2)^{0.25} \dots (II)$$

where $\tau_0 = 0.25f'_c$; d_a = aggregate size; a_1, a_2, a_3, a_4 are constants, which depend on τ_0 .

In the paper $\delta_t, \delta_n, \sigma_{nt}^c, \sigma_{nn}^c$ are denoted by $\Delta_s, c_w, \tau_s, \sigma_c$ respectively.

In equations I and II the ratio r is introduced as a

handy factor, which can be replaced with δ_t/δ_n ; generally, r varies during each test (either experimental or numerical), according to the test modalities.

A satisfactory fitting of many experimental results (obtained by other authors) is shown in references 20 and 21. Very recently, the constitutive equations I and II were successfully introduced into a model for the description of the ultimate behaviour of reinforced concrete beams subjected to torsion (Iori and Dei Poli⁽²³⁾).

As can be seen in Figure II (heavy full lines), the rough crack model predicts quite closely the test results obtained with a small initial crack width and a large transverse stiffness (Tests 9S and 15L, full lines with solid circles); the fitting is still acceptable when the initial crack width is large (Test 19L), but the computed stresses seem to be too large in the cases with small transverse stiffness (Test 11L). Here, Dr Millard and Professor Johnson observe that Tests 15L and 11L have rather similar displacements ($\delta_n = 0.88-1.18$ mm for $\delta_t = 1.5$ mm), whilst the maximum shear stresses are in the ratio 2.5 to 1, and the

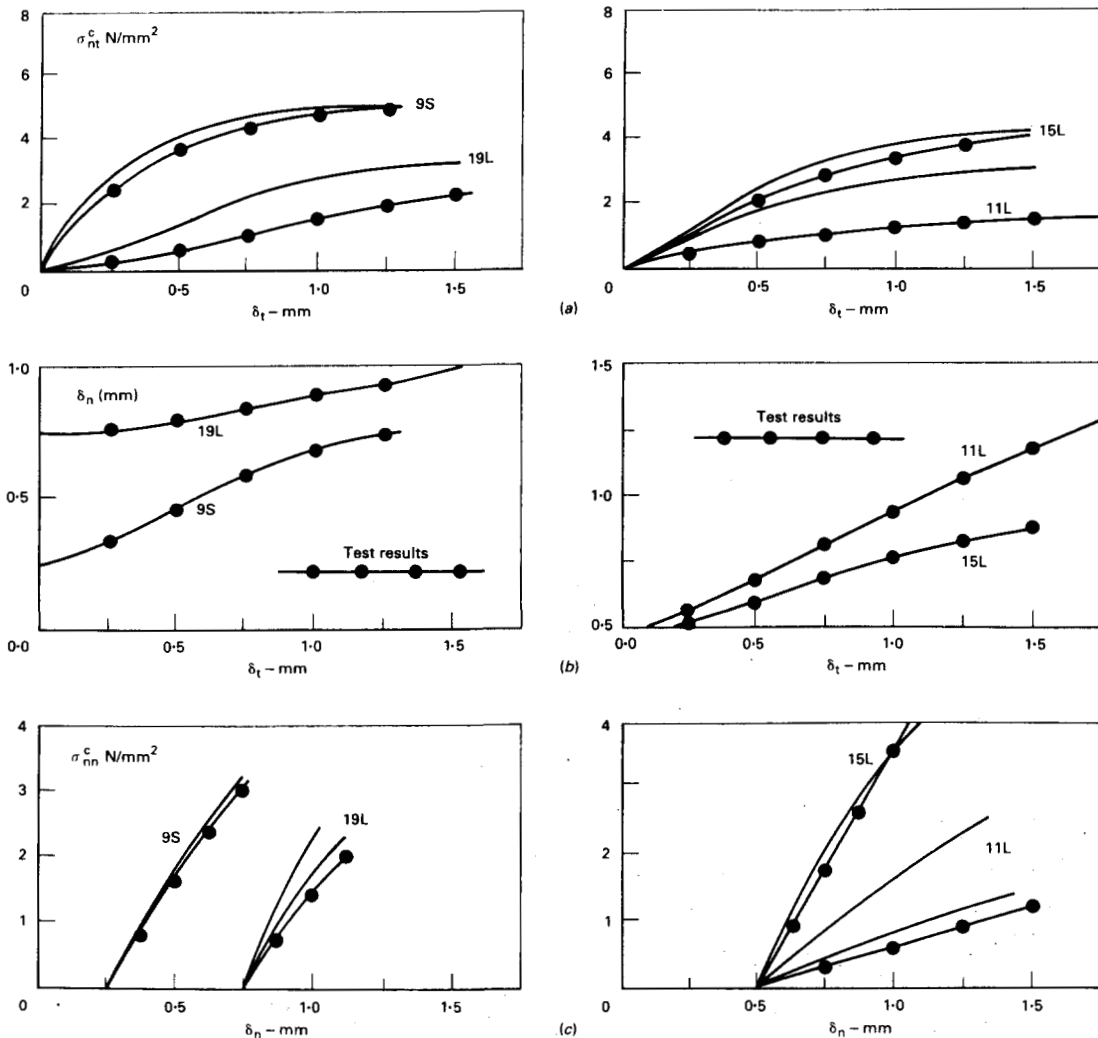


Figure 2: Comparison of rough crack model with the results of Tests 9S and 19L (different initial crack width), 15L and 11L (different confinement stiffness); $f'_{cc} = 35.4$ N/mm², $f'_c = 29.3$ N/mm²

confinement stresses 6 to 1: should the results of Test 11L be confirmed, they would suggest a remarkable path-dependency for aggregate interlock. Some of the results shown in Figure 7, with large variations of the stress values for similar displacements, seem to suggest even sliding-frictional behaviour when the crack width is large.

In Figure II, the theoretical curves (Figure II a,c) are based on the measured displacements (Figure II b) which are used as input data for the rough crack model. The thin curves shown in Figure II c refer to the confinement stress, and are based also on the experimental values of the shear stress; these curves are very close to the experimental ones, which means that equation II is reliable also in the case of small transverse stiffness.

With reference to the second set of tests (MCR 130), a really wide spectrum of different problems is considered by the authors. We agree that the most important points are: (a) the intersection of aggregate interlock and dowel action, which sometimes improves their combined shear behaviour, but sometimes makes it worse than might be expected from a simple superposition of the two separate mechanisms; (b) the role of steel-to-concrete bond, which increases the steel stiffness by reducing the effective bar length (the tensile strains in the bars tend to be localized close to the crack interface); (c) the 'pinching' effects of the bars on the crack width.

The tests show definitely that the cracks also tend to remain plane in reinforced concrete specimens (no pinching effects), and that the shear response is heavily dependent upon the coefficient stiffness, which in turn depends upon the initial crack width, upon the steel ratio, upon the bar diameter and (more gener-

ally) upon the parameters affecting bond behaviour. It seems really difficult to separate the various effects! For instance, in Figure 10 the role of the initial crack width seems very important, whilst in Figure 11 (and –to a lesser extent– in Figure 9) the same role is much more limited.

For the same steel ratio, bonded and unbonded bars seem to guarantee the same maximum shear strength (Figure 15), but the experimental evidence is still too limited. For the same confinement stiffness (i.e. axial stiffness), both the displacements and the shear responses practically coincide (Figure 21) for bonded and unbonded bars.

If we bear in mind other test results (for instance, those by Walraven and Reinhardt), it appears that the tests on reinforced concrete specimens performed by the authors behave rather softly: a possible explanation is that the crack faces are fairly smooth, owing to the relatively small aggregate size ($d_a = 10$ mm).

Certainly the role of the maximum aggregate size should be further investigated, as well as the effect of possible concrete deterioration close to the bars, at the crack interface (such deterioration may produce a little debris, which increases the roughness of the crack).

Also in the case of reinforced concrete specimens, the rough crack model fits closely the experimental curves (Figures III and IV): for fitting the Test 2 curves, the experimental displacements were used as input data (Figure IIIb), whilst for Test 5 the experimental values of the confinement stress and of the crack width (Figure IVc) were introduced as input. In both cases, the dowel action contribution was evaluated with the elastic-plastic model proposed by the authors. As emphasized in the paper, a mere

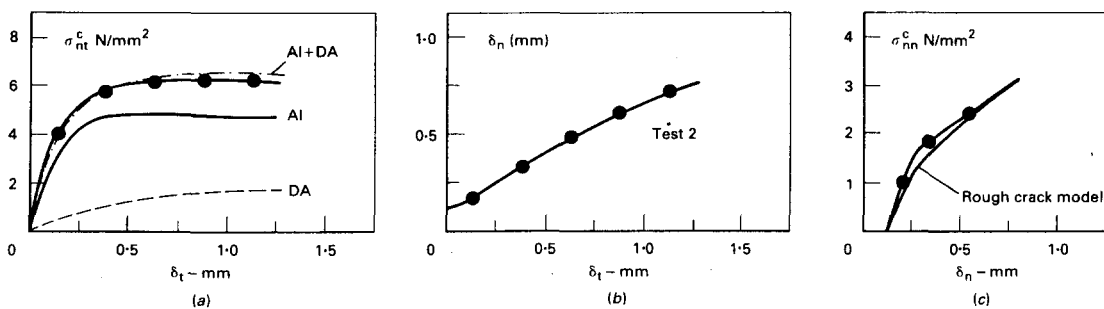


Figure III: Comparison of rough crack model with the results of Test 2 (reinforced concrete specimen); $f'_{cc} = 38.9$ N/mm², $f'_c = 32.3$; DA = Dowel Action, AI = Aggregate Interlock.

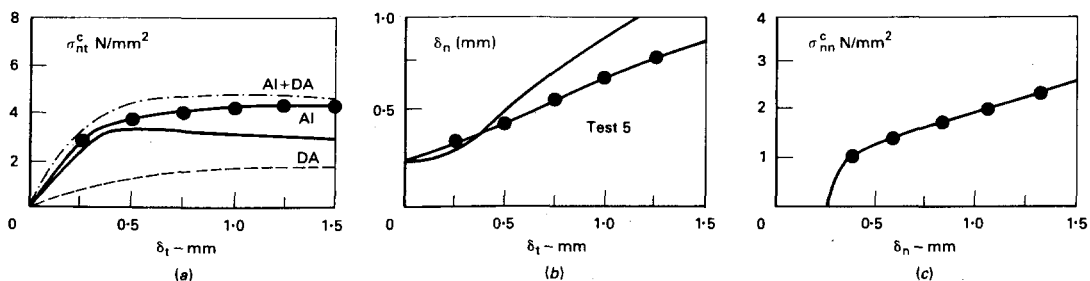


Figure IV: Comparison of rough crack model with the results of Test 5 (reinforced concrete specimen); $f'_{cc} = 33.2$ N/mm², $f'_c = 27.6$ N/mm².

superposition of the effects of the two transfer mechanisms leads to an over-evaluation of the shear response; for an initial crack width of more than 0.5 mm, the error tends to become unacceptable (the same is true for the two-phase model).

Last but not least, we agree that the role of concrete compressive strength should reasonably be greater in reinforced concrete specimens than in aggregate interlock specimens, because of the favourable effects of concrete strength upon bond (which is present only in reinforced concrete specimens). On the other hand, the better the bond is, the earlier the yield of the reinforcement, with greater deterioration in the concrete. Everything considered, the advantages to be expected from a stronger concrete would not increase in proportion with its added strength. In fact, Walraven and Reinhardt's tests show that increases in concrete strength of up to 100% (from 30 to 55 N/mm²) improve the crack shear strength by only about 50%. Further tests are needed to confirm the role of concrete strength.

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Reply by the authors

We thank Professor Dei Poli, Professor Gambarova and Dr Karakoç for their interesting discussion paper. There is still much to be learned about the mechanism of shear transfer in cracked reinforced concrete before an over-all theoretical model can be developed, based upon a clear understanding of the mobilization of stresses within and adjacent to cracks.

We chose to study the aggregate interlock effects while maintaining the restraining stiffness normal to the crack rather than maintaining the restraining force, because this is closer to the situation which occurs in cracked reinforced concrete. Shear slip across a crack causes crack widening due to the wedging action of the rough crack forces. This, in turn, causes an increase in the force normal to the crack, provided by the reinforcement. To conduct tests with constant normal restraining forces would result in an unrealistic amount of crack widening due to frictional

overriding of the aggregate protruding from each crack face. To conduct tests at constant crack width would bear little resemblance to the real situation, as the shear forces would be provided almost entirely by crushing of the cement matrix surrounding aggregate particles.

It may be possible to model the response from a particular crack shearing and widening path using results from either of these two methods. However, this can only be done if this path is already known and if it can be assumed that all aspects of aggregate interlock are independent of the path travelled to reach a particular shear displacement and crack width. Our test results have displayed a degree of path dependency. Professor Dei Poli et al. correctly observe that the shear stiffnesses of specimens 11L and 15L are quite different, in spite of a fairly similar crack opening path. This is, in fact, due to the very different restraint stiffnesses normal to the plane of cracking. In test 11L, there is a small normal restraint, which means that the crack faces are relatively free to override during shear slip. It would seem reasonable to assume that, in this case, much of the shear resistance is provided by friction. In test 15L, the larger normal restraint will restrict overriding and much of the shear resistance will be provided by crushing along the crack faces.

The other side of the coin is seen when two similar specimens have the same shear slip and crack width, but have reached them by different paths. This has occurred a number of times in our tests as can be seen in the first paper. A study of the resulting shear and normal forces at these points reveals that they are not always the same. Hence there is some measure of path dependency displayed by aggregate interlock. This is not altogether unexpected. If a cracked specimen undergoes shear displacement without crack widening occurring, there must be a great deal of irreversible crushing damage caused. If the crack is then widened, it might be expected that the shear and normal stresses would be quite different from those that would result if the crack was first widened and then caused to shear.

In spite of this, the two-phase model does predict the aggregate interlock behaviour reasonably well, even though it is a path-independent model. This is perhaps because such extreme differences in the path followed by the crack as described above do not usually occur, and this may explain why the model is less accurate when the initial crack width is 0.5 mm or more, a situation approaching that described.

We are surprised to see professor Dei Poli et al. modelling aggregate interlock as an interlocking of trapezoidal asperities. The model proposed requires a knowledge of the crack opening path before it can be carried out, i.e. a value for r must be input into equations I and II. In comparing this model with our own test results, this information was available—see, for

example, Figure IIb. How is a value for r , which is not in fact constant, to be obtained when predicting the behaviour of an untested specimen? More progress in this subject is to be made by work directed into improving understanding of the aggregate-interlock and dowel-action mechanisms, than by searches for more sophisticated curve fits to existing test data.

The two-phase model is based upon a combination of crushing and sliding of spherical particles, of a realistic range of sizes, embedded in a softer cement paste. This is an approximation to what we believe to be occurring in aggregate interlock. Furthermore, this model gives a quite close prediction of the shear-slip/crack-width path and hence the shear stiffness, which cannot easily be obtained from the alternative model proposed.

In cracked reinforced concrete, the interaction between aggregate interlock and dowel action is indeed complex. In Figure 11 (MCR 130) the role of the initial crack width is less marked than in Figure 10, partly because the range of initial crack widths is smaller and partly because the breakdown of bond in

test 4 causes a significant reduction in the normal stiffness and hence masks the effect of a smaller initial crack.

It would seem unlikely that the 'softness' of these test results is related to the maximum size of aggregate used. For crack widths of 0.1 to 1 mm a change of the maximum aggregate size from 10 mm to 20 mm is not going to make much difference. Previous tests (MCR 126, reference 16) have shown very little sensitivity to the maximum size of aggregate. Walraven's (MCR 126, reference 9) work has shown that the whole range of fine and coarse aggregates contribute towards aggregate interlock. It is more plausible that any differences may be due to localized damage to the concrete around the reinforcing bars. Walraven has reported that cones of crushed concrete were seen around each bar and that the crack-widening effect in reinforced concrete was greater than that expected from aggregate interlock tests. Neither effects were observed in our tests and this certainly warrants further study.

Permeability of normal and lightweight mortars*

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A mortar made with lightweight aggregate may have a different permeability from that of a sand mortar (made with a similar water/cement ratio and aggregate volume and grading) due to the influence of a number of factors:

- (1) *aggregate permeability*—lightweight aggregates are more porous and hence generally more permeable than normal aggregates;
- (2) *aggregate/cement paste bond*—the porous nature of lightweight aggregate generally enhances aggregate-cement-paste bond, reducing flow paths at the aggregate/cement-paste interface; and
- (3) *the availability of water to allow prolonged hydration*—lightweight-aggregate particles contain a greater volume of absorbed moisture and may act as internal reservoirs allowing prolonged hydration, thereby producing cement paste of reduced permeability.

Unfortunately Dr Nyame has introduced several other variables, owing to the way that he has designed his mixes, which do not relate to current practice and which may have masked the effects of the factors listed above.

In practice, lightweight aggregate is generally pre-soaked so that a sufficiently workable yet cohesive

mix is achieved. Dr Nyame used dry aggregates, "so that absorption effects would be brought into the composite action", and did not even adjust the water content to allow for aggregate absorption. As a result, Dr Nyame's supposedly equivalent normal and lightweight mortars have *different free water/cement ratios*. If Dr Nyame has data relating to the absorption and moisture content of the aggregates, an assessment could be made of the actual free water/cement ratios. For example, if the lightweight aggregate absorbed 5% (by weight) of water during the first 2 or 3 h after mixing, the free water/cement ratio of the 65% aggregate volume concentration mix would be reduced by around 25%, i.e. from 0.47 to 0.35. Such a reduction would be expected to cause a very substantial reduction in permeability. Furthermore, the absorption of mix water reduces the volume of cement paste and hence increases the aggregate volume concentration. In this context, it should be pointed out that equation 1, the expression apparently used by Dr Nyame to calculate the aggregate volume concentration, is incorrect.

Secondly, the use of sand and lightweight aggregate of different gradings has resulted in supposedly equivalent normal and lightweight mortars having *different effective path lengths* and also *different areas of aggregate/cement paste interface*, thereby distorting the effect of aggregate/cement paste bond (factor 2

*Pages 44–48 of MCR 130.