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Determination of specimen-size independent fracture toughness of plain concrete*

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The fracture of concrete is a complex process which is characterized both by localization in the strain field and by strain-softening. Several researchers$^{(1-4)}$ have observed these phenomena, but few quantitative test data are yet available, particularly with regard to localization. The zone of localization is usually referred to as the fracture process zone. The size of this zone may not be insignificant compared with the sizes of typical laboratory specimens and this has resulted in so-called 'size effects' in fracture mechanics test results. Various processes take place sequentially within the zone. Firstly, microcracking occurs at the aggregate/paste interfaces, followed by crack bridging between the aggregate particles, and then the formation of a macroscopic crack. Stress can still be transferred across the fracture process zone by aggregate interlock within the crack, although the stress that can be transferred (the stress-transferring capacity) reduces with opening of the zone. The development of fracture process zones in stable direct-tensile tests has recently been observed in research undertaken at Imperial College$^{(5)}$. The preliminary findings of this research indicate that, for the concrete tested, which had a maximum aggregate size of 10 mm, the opening of the zone at which the stress-transferring capacity reduces to zero may be greater than 0·2 mm.

In stable notched-beam tests of the type reported by Dr Nallathambi and Professor Karihaloo in their paper, the fracture process zone is initiated at the tip of the notch during the ascending portion of the load/deflection plot. The zone develops and propagates away from the notch throughout the remainder of the test. As suggested in the paper, the length of the zone is a function of the testing geometry as well as the material characteristics; however, it will also vary during the test. The straight line approximation to region 2 leads to a simplified view of the development of the zone. The zone is modelled by an increase in the effective notch depth. This increase is assumed to be constant, i.e. a stress-free crack of fixed length ahead of which the material is not damaged. The justification for this model is that it appears to give a reasonably consistent size-independent measure of fracture toughness. However, the model does not represent the true state of the material, and it is not yet clear how useful the results will be.

The fracture toughnesses determined in the paper are dependent on the slope of the straight line approximation to region 2. This in turn depends on the limit of the elastic response. It would be of interest to know how accurately this limit could be determined and the corresponding sensitivity of the calculated fracture toughness.

An alternative to the method adopted in the paper is the use of non-linear fracture process zone models$^{(6,7)}$. These models represent more accurately the relationship between the stress transferred by the zone and its opening. The models developed so far depend on the

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*Pages 67–76 of MCR 135.

assumption that the fracture energy, G_F, is a material constant. The results given in the paper show G_F varying with specimen dimensions. However, it appears that the effect of the self-weight of the beam has not been included in the calculations. Petersson(1,8) showed a significant dependence of the calculated value of G_F on the energy supplied by the weight of the beam. This energy increased the value of G_F by 50–60% for 50 mm-deep beams and by 150–250% for 200 mm-deep beams. RILEM Committee 50-FMC(9) gives an approximate method for calculation of this additional energy based on Petersson's work. The additional energy depends on both a/d and L/d and increases with both parameters. For example, a beam of the size given in Figure 1(b) would have an additional energy of approximately 15 J/m^2, whereas if the value of a/d was increased from 0·3 to 0·6 then this energy would increase to approximately 30 J/m^2. The dimensions of the beams are not given in Figures 12 and 13, but if they were similar to the beam used in the example above, it can be seen that with the additional energy due to self-weight the dependence of G_F on a/d in Figure 12 is significantly reduced.

The authors also put forward three arguments against the use of G_F. The first suggests that the area under the load/deflection plot is dependent on the stiffness of the testing machine. There does not appear to be any evidence to support this suggestion. Provided that the test is stable and no energy is lost in dynamic effects then the observed response is independent of machine stiffness. The authors state correctly that if an insufficiently stiff machine were used then the softening response would not be apparent although it would still exist. The problem is one of appropriate test design rather than variation in the material property. The second argument concerns the energy expended in the specimen away from the primary fracture zone. This was recognised by Petersson(8) as a potential source of inaccuracy. This problem can be limited by avoiding low values of a/d and RILEM(9) recommends an a/d ratio of 0·5. The third argument discusses the difference between the true fracture area and the projected area of the fracture surface. It would seem that a value of G_F based on the cross-sectional area of the member fractured would be the most useful measure of toughness. To this end, RILEM recommends that the total measured energy be divided by the area of the ligament of the beam.

Although some of the comments made above would improve the results obtained, it seems that it is not yet possible to obtain values for G_F from notched-beam tests that are totally independent of specimen size(10). Gopalaratnam and Shah(2) suggested that direct-tensile tests might be more appropriate. However, insufficient data on size effects in this type of test are available for an adequate analysis to be made.

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Reply by the authors

We agree completely with Dr Raiss on his discussion of the reasons for the observed size effect in the fracture test results. We agree also with his observation regarding the residual stress-transferring capacity across the fracture process zone. Indeed, this observation forms the basis of the two-parameter fracture model proposed by Jenq and Shah (Reference 5 in our paper).

We had stressed in our paper that the straight line approximation to region 2 is an oversimplification of the fracture process zone. In fact, we had noted that the real stiffness in this region reduces from E at the elastic limit to nothing at the peak load. In other words, we had recognized that the increase in the pre-notch depth cannot remain constant during the test. We are currently investigating the effect of relaxing this rather strict restriction.

On the point of the sensitivity of the fracture toughness to the accuracy in the determination of the elastic limit it need only be said that it is of the same order as is normally accepted in a compression test.

We are quite familiar with the non-linear fracture process zone models cited by Dr Raiss. There is however no agreement yet on the model that best describes the fracture process in concrete. It should be noted that the influence of self-weight is included in our results for G_F.

It may interest the readers of *MCR* to know that RILEM Committee 89-FMT (in succession to Committee 50-FMC mentioned by Dr Raiss) is currently investigating the test method and physical model best suited to concrete. One of the two sub-committees of RILEM 89-FMT is charged with the task of formulating recommendations on the method of determination of mode I fracture toughness from notched-beam tests. This sub-committee is chaired by one of the authors (BLK). Preliminary analysis of notched-beam test results from several laboratories around the world indicates that Bažant's size effect law and Shah's two-parameter model give fairly consistent and reproducible results for G_F. What is rather surprising is that G_F determined on the basis of these non-linear fracture process zone models is almost equal to the G_c reported in our paper! A draft report from this sub-committee will be presented to 89-FMT at the joint RILEM/SEM International Conference on Fracture of Concrete and Rock, Houston, June 17–19, 1987.

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Relationship between strength and volumetric composition of moist-cured cellular concrete*

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The Editor very much regrets that two points on Figure 2 on page 15 of the above paper were shown incorrectly. The correct figure is shown here. The corrected points occur at strengths, air/cement ratios of 9·5 N/mm^2, 1·75 (correct water/cement ratio 0·80) and 5·0 N/mm^2, 2·8 (correct water/cement ratio 0·80).

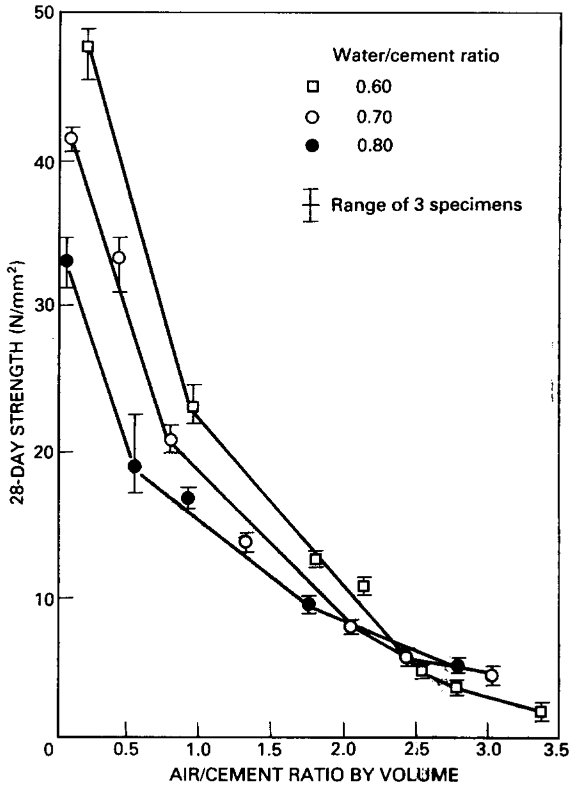

Figure 2: Effect of air/cement ratio on strength.

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