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The statistical distribution of the drying shrinkage of concrete using Monte Carlo simulation

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The paper by Professors Al-Sugair and Almudaiheem presents techniques to account for inherent variability of shrinkage modelling. Variability of shrinkage data is, as yet, poorly understood and techniques need to be further developed to account for this variability. These techniques provide information required in critical state design procedures.

The method presented in the statistical study was based largely upon use of the Modified Ross Equation (MRE) and the choice of appropriate statistical distribution for the coefficients N_s and ε_u. Al-Sugair and Almudaiheem assume that values chosen for the two quantities N_s and ε_u, are taken from log-normally distributed populations. From these assumptions they show how Monte Carlo methods may be used to determine the distribution of shrinkage data.

That N_s is drawn from a log-normal population is crucial to the remainder of the paper. Two sentences before equation (5) the authors state that this assumption is valid because the predictions of ln N_s have been derived by linear regression methods. This is unjustifiable, for there is no formal link between regression analysis and normality. Least squares provides best linear unbiased estimators under a set of assumptions which do not include normality. The normality assumption in regression is only added for inference purposes, i.e. to validate *t*- and F-tests in small samples. Further investigation is therefore needed before the log-normal assumption for N_s can be supported.

The authors assume that ε_u too is taken from a log-normal population. This assumption is based entirely upon a reference cited in the text. In work by McDonald[1] it was shown that this conclusion was incorrect.

Log-normal populations have skewness values greater than 0. In the cited reference it was assumed that sample distributions taken from log-normal

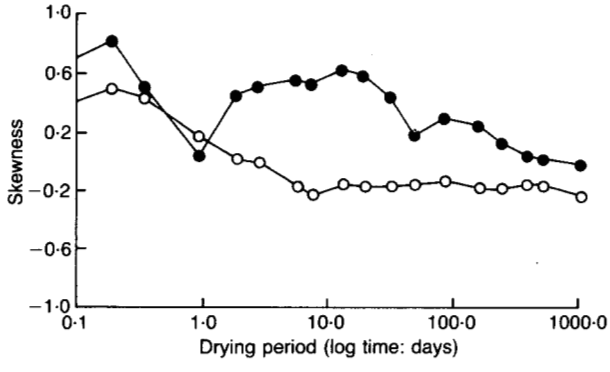

Fig. 1. Skewness of shrinkage data versus drying period; size of specimen: ○ 83 mm, ● 160 mm

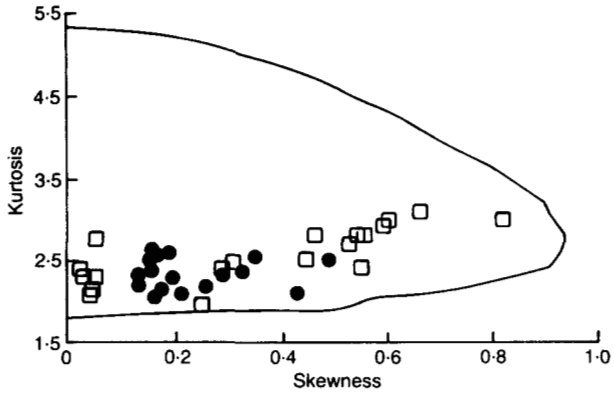

Fig. 2. Skewness and kurtosis of shrinkage data; 35 specimens, 95% confidence limit: □ 160 mm, ● 83 mm

populations would also have a skewness coefficient greater than 0. When the skewnesses of the shrinkage values are plotted, negative values are found (Fig. 1). As the drying period increases, the skewness tends toward negative values. The values of skewness do not support the contention that the shrinkage data are taken from log-normal populations.

When the method proposed by Bowman and Shenton[2] for jointly analysing skewness and kurtosis is used on the cited data, both the normal and log-normal distributions can be seen to fit the data, which lie within the 95% confidence bands (Figs 2 and 3).

Because the paper by Al-Sugair and Almudaiheem is largely dependent upon the density functions from which N_s and ε_s are taken, we seriously question the validity of the conclusions presented by these authors. This does not imply that the methodology chosen by these authors is inaccurate, but that extensive experimental investigations of the statistical distribution of shrinkage data are required before methods such as the Monte Carlo simulation may be used with certainty.

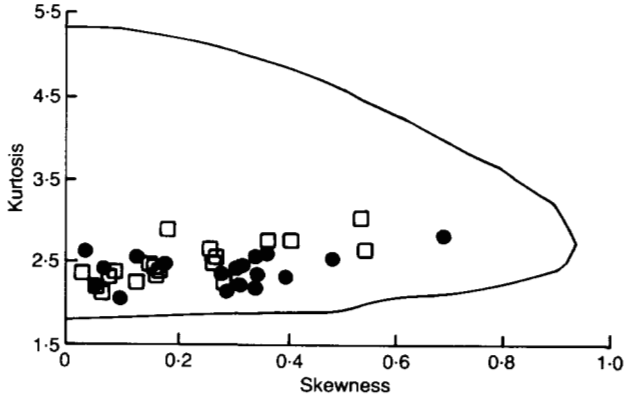

Fig. 3. Skewness and kurtosis for the logarithm of the shrinkage data; 35 specimens, 95% confidence limit: □ 160 mm, ● 83 mm

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Reply by the authors

Professors McDonald *et al.* have focussed their discussion on the Authors' assumptions for the statistical distributions of the two main random variables in the shrinkage equation, the shrinkage half-time N_s and the ultimate drying shrinkage ε_u. Before replying to these specific questions, we would like to elaborate on the approach presented in the paper. The formulation of the model and the equations presented (equations (8–15)) provide a clear methodology for computing the statistics of the drying shrinkage at any time. The model can be easily modified if future research indicates that different distributions better describe variables effecting the drying shrinkage process. Nevertheless, there is strong support for the statistical distributions assumed in the paper.

The equation for calculating N_s is

$$\ln N_s = C_1 + C_2(V/S) \qquad (1)$$

where C_1 and C_2 are constants found via least-squares regression, and V/S is the volume-to-surface ratio of the member. If the distribution of V/S is normal, then it can be shown that $\ln N_s$ is also normally distributed,[1] and thus N_s is log-normally distributed.

V/S is a measured quantity in which the variability is mainly due to human variability and error. This type of variability can be assumed normally distributed in the absence of contrary information.[2] For a more elaborate discussion of human errors see Reference 2.

As for the assumption of ε_u being lognormally distributed, the work of Bazant *et al.*[3,4] featured a large number of replicate specimens cast and shrinkage measurements systematically made at various times. The distribution of the drying shrinkage at any time, including the ultimate drying shrinkage, was found to be well-described by a log-normal density function. This finding is further supported by the work in the Paper, where assumptions of the distributions of the

main random variables coupled with Monte Carlo simulation produce similar results.

Finally, we agree with the discussers that more data is required in order to improve our understanding of the drying shrinkage process.

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Dr A. W. Beeby's affiliation was omitted. At the time of writing his editorial comment he was at the British Cement Association; he is now at Leeds University, UK.