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A fractal model for simulating the formation of microcracks in the fracture process zone and a theoretical explanation of the size effect of the fracture energy of concrete

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Contribution by P. S. Addison

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I read with interest the paper by Ji *et al.* There has been for some time now much interest in the application of fractal geometric techniques to cracking phenomena in a variety of brittle and quasi-brittle materials. Over the past 5 or so years some of this attention has focused on concrete cracking.

I found some difficulties in intepreting the theoretical framework of the paper, and I wonder whether the authors would like to comment on these. I detail them as follows:

(1) May I take Fig. 1(c) to be a schematic diagram of a 'snapshot' of the choices of movement that the crack has available to it as it moves through the FPZ (in a random sense). The fractal curve of Fig. 1(c) is not a strictly self-similar fractal, that is, it is not self-similar all along its length, but only as we progress towards the tips of the branching tree. Cracks, on the other hand, are strictly self-similar, that is, we can zoom into a part of the crack and find more, self-similar detail anywhere along it.

(2) Equation (3) relates to the divider dimension, whereby a fractal curve is covered by successively smaller step lengths of size r and the number of steps required N_r scales with r to some exponent $-D$. This is true; however, it is a well known fact[15,16] that the divider dimension is really only suitable for self-similar fractal curves (e.g. the Koch curve, Brownian motion trajectories) and that it may produce erroneous results for self-affine fractals when large step lengths are used. I put it to the authors that their cracks are not self-similar processes but rather self-affine, and perhaps another, more robust, method of dimension estimate would be more appropriate, for example the box counting dimension.

(3) Why should r_0 (the 'very large unit length') not be taken to be L_0. Or, to put it another way, why do the authors presume that $r_0 < L_0$ and hence that there is a size effect? I personally would interpret size effects in terms of a limited spatial memory in the fractal process.[17]

(4) Equation (4) does not follow from equation (3). The length L obtained for a fractal curve using a

step length r is simply the number of steps N_r multiplied by r, that is,

$$L = rN_r$$

which according to equation (3) is

$$L = Cr^{1-D}$$

Hence, although the authors are correct in stating that the larger the value of step length used the smaller the total length obtained, the ratios do not follow the simple inverse law of equation (4), but rather follow the (fractal) scaling law:

$$\frac{r^{D-1}}{r_0^{D-1}} = \frac{L_0}{L}$$

(5) Concerning equation (8), it is true to state that below a certain step length r we should find the true length of the curve. This is because 'natural fractal' objects are not true fractals, that is, they do not possess self-similarity over all scales—only sufficient to allow a fractal geometric interpretation to be useful.[16] Thus, below a certain resolution the natural fractal curve becomes smooth and possesses a finite length (something that a true fractal curve does not have!). What the authors appear to have done is use some arbitrary small step length d—'the diameter of an atom'—to find the length using a fractal scaling law. However, below the cut-off scaling size, whatever that might be, the scaling law no longer holds, and the curve length becomes constant—independent of step size. It is for this reason that we cannot use equation (8) to find a true length.

(6) I cannot understand why the area of the fracture surface A_T is simply the breadth B multiplied by H_T (as given in equation (9)). If the crack profile is a fractal, then we can be reasonably certain that the internal crack surface is a fractal, hence the effective breadth should have a similar formulation to H_T in equation (8).

Reply by the authors

We are grateful to Addison for his interest in our paper, and will try to answer his questions, in order, to the best of our knowledge:

- *Point (1)*. Fig. 2 presents only an example of the fractal-tree model for crack branching and propagation: many other variations of the fractal-tree model can also be formulated. We admit that Fig. 2(c) is self-similar only as we progress towards the tips of the branching tree, but we could modify this model so that it is self-similar all along its length, for example in Fig. 2(b) we could reformulate the generator as follows: after dividing the line segment of unit length in Fig. 2(a) equally into three parts, we do not discard the lower part in Fig. 2(b), instead we directly attach the bending line segment (which has a length of $1/3$ as shown in Fig. 2(b) in our paper) to the very end of the original line segment in Fig. 2(a); after obtaining this new generator, we can redraw Fig. 2(c), which would now be self-similar all along its length.
- *Point (2)*. The theoretical deduction from equation (3) does not rely on Fig. 2, although Fig. 2 provides some help. During our deduction, we assume that the cracks are self-similar.
- *Points (3) and (4)*. The development of equation (4) is based on the work by Pfeifer[18] (p. 3561, second column, section V: D and monolayers on adsorbent particles of different size): given equation (3) in our paper, now a change of step length from a fixed value r_0 (step length) to some value r is the same as measuring everything in units of, say, (r/r_0) angstroms instead of just angstroms. In units of (r/r_0) angstroms then, the curve has a length of $L = (r_0/r)L_0$, so the change in N_r from Cr_0^{-D} to Cr^{-D} can be attributed by expressing r in terms of L to a change in curve length from L_0 to L. That is,

$$N_r = C(r_0L_0)^{-D}L^D = C'L^D$$

which is equation (6) in our paper.
- *Point (5)*. We believe that the fracture surface and microcrack network of concrete are 'natural fractal' objects, which should have a definite area. Within certain resolutions the fracture surface is still self-similar. In order to find the accurate fracture surface area of concrete, we need to find a very small step length within the resolution where the fracture surface is still self-similar, because the smaller the step length the more accurate the fracture surface area. That is why we use an extremely small step length—'the diameter of an atom'—to find the fracture surface area, although we admit that the fracture surface area may not be still self-similar with such a small step length (which requires extremely high magnification of the fracture surface)
- *Point (6)*. It is possible for breadth B to have a similar formulation to H_T in equation (8). The breadth B should have its own B_T, but the purpose of our paper is to examine the effect of the height of ligament, H, on the fracture energy of concrete, and, to simplify the analysis, we do not incorporate B_T in equation (8).

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