Magazine of Concrete Research, 2001, **53**, No. 6, December, 429–431

Discussion on paper published in *Magazine of Concrete Research* 2000, **52**, No. 4, 275–285

Effect of compression reinforcement on the shear strength of reinforced concrete bridge beams

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Gale and Ibell are correct in asserting that plastic analysis is a powerful tool for the assessment of existing structural concrete bridges, in that all steel can be adequately accounted for. However, I find their case for a significant contribution from compression reinforcement to the shear capacity less than convincing.

First, it is puzzling that the authors choose to analyse a rotational mechanism with a straight yield line. It is commonly acknowledged that if the concrete is modelled by the modified Coulomb criterion under plane stress then a hyperbola is the optimal shape of a yield line separating two parts rotating relative to each other, compare for example the June 1982 discussion of the article by Kemp and Al-Safi (reference 4 of the paper). Only if the centre of rotation is at infinity does the discontinuity degenerate to a straight line.

Moreover, the relative rotation of the beam end at failure is outwards, the centre of rotation being on the extension of the beam. At first glance this may seem counter-intuitive, but upon reflection it makes sense, and is indeed what is observed during testing. Consider for example the photograph of the failure of specimen 1 (Fig. 9. of the article). The width of the horizontal splitting crack is decreasing towards the beam end, and the thin piece of concrete at the top of the beam is breaking off in a way that is consistent with a clockwise rotation of the beam end. The authors assume the centre of rotation to be at the support of the loaded beam part (Figs 2 and 3 of the article), which is by no means necessary (note that we are considering relative rotations, the individual beam parts will of course have to rotate about their supports).

To calculate the energy dissipation in the yield line there is no need to perform an approximate, piecewise summation. The concrete contribution to the rate of internal work is simply given as

$$ED_c = \tfrac{1}{2}bk(1 - \sin\alpha)f_c \qquad (1)$$

Here k is the chord length and α is the angle to the chord of the relative displacement rate, measured at the chord mid-point, cf. Fig. 1. The figure is reproduced from reference 1, which derives coinciding upper and lower bound solutions for beams without web reinforcement, expanding on earlier work by J. F. Jensen, compare discussion of reference 4 of the article.

The authors evaluate the effective concrete strength as $f_c = \nu f_{cu}$, where the effectiveness factor ν depends upon a number of parameters, of which the most im-

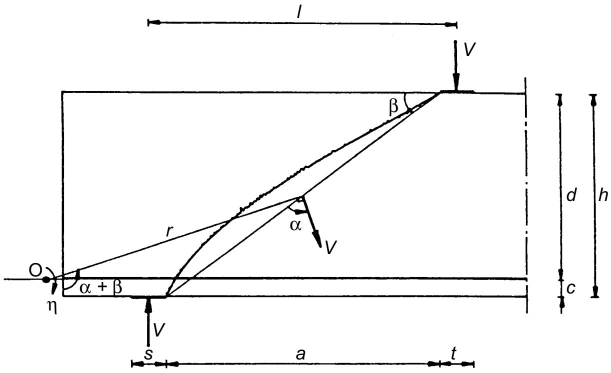

Fig. 1. Hyperbolic yield line with rotation about extension of bottom reinforcement

portant is the concrete grade. Insertion of the numbers for specimen 1 (with longitudinal reinforcement ratio 2·3% and no stirrups) into equations (3)–(9) results in the effective strength $f_c = 0{\cdot}24 f_{cu}$ (for $f_{cu} \geqslant 40$ MPa).

Reference 1 uses a different empirical formula, but suggests the conservative estimate

$$\nu_{cyl} = 2{\cdot}0/\sqrt{f_{cyl}}$$

to be applied on the cylinder strength. Assuming the cylinder strength f_{cyl} to be 70% of the cube strength f_{cu} we find the effective strength

$$f_c = 2(0{\cdot}7 f_{cu})^{1/2} \quad (2)$$

For $f_{cu} = 50$ MPa this gives $f_c = 0{\cdot}24 f_{cu}$, which is the same as to the value assumed by the authors.

According to reference 1 the plasticity solution is

$$P = \tfrac{1}{2} b f_c [(a^2 + 4(h - y_0) y_0)^{1/2} - a] \quad (3)$$

where

$$y_0 = h\Phi/\nu \leqslant h/2$$

Here Φ is the mechanical ratio of longitudianl reinforcement, calculated on the basis of the full cross-section $b \times h$, i.e. $\Phi = A_s f_y / b h f_{cyl}$. For the test beams with two bars T16 we find $\Phi = 0{\cdot}23$, and since $\Phi > \nu/2$ this means that even specimen 1 is over-reinforced ($y_0 = h/2$), in the sense that the longitudinal steel does not yield at shear failure. Thus the addition of an equal amount of top steel in specimen 2 would not be expected to increase the shear capacity.

The failure mechanism of specimen 1 would be a rotation about a point on the extension of the bottom reinforcement, as described above. For specimen 2 the centre of rotation should move to infinity to preclude any dissipation in neither top nor bottom steel. This agrees with Fig. 11 of the article, which shows no sign of rotation at failure. Consequently, there can be no buckling of the top steel, invoked by the authors to explain the lack of additional shear capacity.

Whatever the failure mechanism, the plane stress distribution corresponding to the coinciding lower bound is a single compressive strut running from the load to the support (compare Fig. 2). However, as explained in reference 1 a certain size s of the support platen is required for equilibrium to be satisfied. Indeed, we must have

$$s \geqslant s_1 = \frac{(h - 2c)[(a^2 + 4(h - y_0) y_0)^{1/2} + a]}{2(h - y_0)} - a \quad (4)$$

Here $c = h - d < y_0/2$ is the distance from the beam soffit to the centroid of the reinforcement. Inserting the numbers for specimen 1 we get $s_1 = 290$ mm, which is not satisfied by the 100 mm platen. Consequently, the compression zone depth y_0 at ultimate is less than $h/2$, and the capacity is given by[1]

$$P = \frac{(h - 2c)(2ac + hs) b f_c}{(a + s)^2 + (h - 2c)^2} \quad (5)$$

Equation (5) corresponds to the failure mechanism of Fig. 1, with rotation about a point on the extension of the bottom reinforcement, so there is no contribution from the (un-yielding) reinforcement.

For specimen 2, however, the centroid of the reinforcement is at mid-depth, which means that the platen need only be large enough to resist the bearing pressure, and the capacity is given by equation (3) above (with $y_0 = h/2$). Equation (3) corresponds to a purely translational failure mechanism with a straight yield line. For $y_0 = h/2$ the relative displacement rate is perpendicular to the beam, i.e. there is no yielding of the reinforcement (over-reinforced case), for $y_0 < h/2$ the relative displacement rate would be oblique to the beam axis, corresponding to reinforcement yielding.

With the topical concrete strength, and the effectiveness factor given by equation (2) above, the predicted shear capacities for the two beams without stirrups are

Specimen 1: $P_1 = 19{\cdot}6$ kN
Specimen 2: $P_2 = 21{\cdot}4$ kN

This is less than half of the experimental capacities reported in the paper, a fact I find hard to explain, unless the peak failure loads given in Table 1 are in fact the total loads, contrary to the statements on page

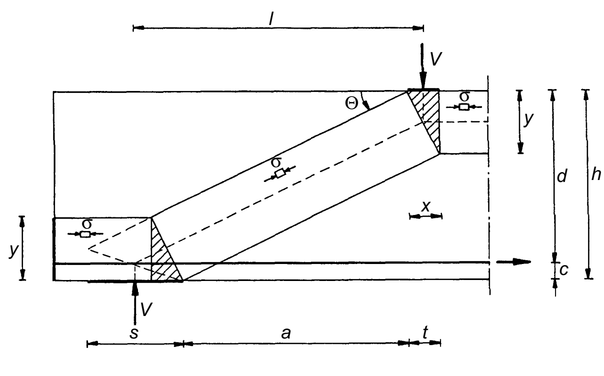

Fig. 2. Stress distribution with compression strut from load platen to support platen

279, but more in line with the narrative on page 280. Then the corresponding observed loads are 26·3 kN, respectively 25 kN.

The value of P_1 and particularly P_2 are lower than the predictions of the authors, which is to be expected although the adopted effectiveness factor is the same, as equations (3) and (5) are optimal upper bound solutions, indeed coinciding lower bounds.

The beams with stirrups are quite another matter. Although the stirrup reinforcement is minimal, with a shear span ratio of $a/h = 2{\cdot}6$ it is just sufficient to ensure that the optimal yield line does not extend all the way to the support. The shear strength predicted by plasticity theory is then given by the Web Crushing Criterion, compare reference 6 of the article

$$P = bz[\rho_{sv} f_{yv}(\nu_{cyl} - \rho_{sv} f_{yv}]^{1/2} \quad (6)$$

Here z is the internal moment lever arm, estimated at $z = 0{\cdot}9d$, and the effectiveness factor may be assessed at

$$\nu_{cyl} = 0{\cdot}8 - f_{cyl}/200 \quad (7)$$

Equation (7) was proposed[2] as a central estimate, and for code applications the more prudent value $\nu_{cyl} = 0{\cdot}7 - f_{cyl}/200$ was suggested, with the cylinder strength represented by the characteristic value, compare also reference 5 of the article. This has been adopted by the Danish code DS411 and by the Eurocode 2.

For the topical test beams we find from equations (6) and (7)

Specimen 3: $P_3 = 47{\cdot}5$ kN

Specimen 4: $P_4 = 42{\cdot}6$ kN

Again, this is approximately 50% of the experimental loads reported by the authors, which would seem to reinforce the suspicion that these should be halved, compare above. Thus the corresponding observed loads are 47·5 kN, respectively 48 kN.

The values P_3 and P_4 cannot readily be compared with the authors' predictions, as the effectiveness factors are different.

No yielding of longitudinal reinforcement is predicted for either specimen, which is corroborated by the failure pictures (Figs 12 and 14 of the article). Consequently no increase in shear capacity would be expected from the compression reinforcement. The fact that specimen 4 exhibits a higher load, in spite of the weaker concrete, may be explained by the added confinement provided by the heavy top bars, which increases the concrete effectiveness factor.

In summary, the conclusion that I derive from the application of plasticity is exactly the opposite to that of the authors. For beams without stirrups the addition of compression reinforcement may give a marginal increase in shear capacity, even for overreinforced beams, by allowing a more favourable stress distribution, whereas for beams with stirrups any shear capacity enhancement from top bars can only be due to the increased confinement of the web concrete provided by the reinforcement cage.

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