

# Discussion: Modified Bolomey equation for strength of lightweight concretes containing fly ash aggregates

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## Contribution by M. C. Nataraja and L. Das

The paper by Rajamane and Ambily (2012) presents experimental results from four series of lightweight aggregate (LWA) concretes (series A, B, X and Y) made using commercially available sintered fly ash aggregates. Different volume fractions ( $a$ ) of LWAs and water/cement ( $w/c$ ) ratios were considered. Based on the 28 d compressive strength of these mixes, Rajamane and Ambily developed a relationship between the  $w/c$  ratio, compressive strength and the volume fraction of LWA based on the Bolomey equation, which the authors call a modified Bolomey equation (MBE) generalised for  $w/c = 0.4$ . It is of the form

$$23. \quad f_c = f_{0.4}(0.5cw - 0.25)^{(1-a)}$$

where  $cw$  is the cement/water ratio.

To validate the proposed equation, Rajamane and Ambily (2012) designed two further mixes using different cements. From the 28 d test results, the authors plotted the relation between compressive strength and ( $cw - 0.5$ ). The graph shows that there is no significant difference in the compressive strength for the two volume fractions considered. Although there was a significant difference in the volume fraction of the LWAs ( $a = 0.53$  and  $0.37$ ), the compressive strength variations were within 5–10%, as shown in Figure 1 of the original paper. In any concrete mix design, variation in strength to the extent of 10% is always possible from the point of view of practical applications. From this, it is clear that the effect of volume fraction of LWA on compressive strength is rather small and is greatly influenced by the  $w/c$  ratio in line with normal-weight concrete. The authors modified the Bolomey equation, generalised for  $w/c = 0.4$ , and claim that this  $w/c$  ratio is useful to prepare concrete in practice without the need for superplasticiser. The discussers feel that this  $w/c$  ratio is rather very low and the concrete would be highly unworkable. Even for  $w/c = 0.5$  and with the use of LWAs (which are generally more porous), plasticiser is necessary to

obtain even a medium workability. Taking this into account, the discussers consider that the authors should have generalised their equation for a  $w/c$  ratio of 0.5. Many earlier investigators have normalised the  $w/c$  ratio equation for  $w/c = 0.5$ . The discussers have thus modified Equation 23 for  $w/c = 0.5$  to obtain

$$24. \quad f_c = f_{0.5}(0.67cw - 0.34)^{(1-a)}$$

There are many reports in the literature on concrete mix design using unconventional aggregates and cementitious materials (Girish *et al.*, 2006; Nataraja and Das, 2011). Many concrete mix design works carried out by the discussers are based on the generalised Abrams law using the law of composite mechanics (Nataraja *et al.*, 2005, 2007). This method of concrete mix design is well established and the feasibility of the method has been proven on different types of aggregates. The method is quite satisfactory for a mix design where the  $w/c$  ratio alone controls the design for all types of aggregates (normal to lightweight). In addition, the method is applicable for any type and combination of cementations materials.

With this in mind, the discussers feel that the generalised Abrams law approach is also best suited for LWA concretes. To prove this, the experimental data presented by Rajamane and Ambily (2012) and data from various other works presented in the original paper were used to check the applicability of the MBE for a  $w/c$  ratio of 0.5. The results presented in Table 5 show that the predicted strength based on Equation 24 matches the actual strength of the concrete determined by testing very well.

In order to further check the applicability of the methods, a few more concrete mixes using lightweight cinder aggregate were cast and tested. The experimental results are compared with values predicted by the generalised Abrams law and Equations 23 and 24 in Table 6. The three methods predict the strength of LWA concrete very well, within about 5% error. However, the discussers feel that

| Sample | w/c  | Density:<br>kg/m <sup>3</sup> | <i>a</i> | $f_c$<br>(Experiment):<br>MPa | $f_c$<br>(Equation 23,<br>w/c = 0.4): MPa | Difference: % | $f_c$<br>(Equation 24,<br>w/c = 0.5): MPa | Difference: % |
|--------|------|-------------------------------|----------|-------------------------------|-------------------------------------------|---------------|-------------------------------------------|---------------|
| 1      | 0.50 | 2170                          | 0.37     | 18.00                         | 19.20                                     | 6.10          | 18.00                                     | 0.00          |
|        | 0.60 | 2170                          | 0.37     | 16.00                         | 16.30                                     | 2.00          | 15.43                                     | -3.69         |
| 2      | 0.40 | 2173                          | 0.36     | 20.00                         | 20.00                                     | 0.00          | 21.69                                     | 7.78          |
|        | 0.50 | 2141                          | 0.36     | 17.00                         | 16.70                                     | -2.10         | 17.00                                     | 0.00          |
| 3      | 0.60 | 2123                          | 0.36     | 14.00                         | 14.10                                     | 0.90          | 14.57                                     | 3.90          |
|        | 0.30 | 2175                          | 0.18     | 41.70                         | 43.40                                     | 3.90          | 50.51                                     | 17.44         |
| 4      | 0.40 | 2117                          | 0.18     | 32.60                         | 32.60                                     | 0.00          | 38.00                                     | 14.00         |
|        | 0.50 | 2072                          | 0.18     | 29.80                         | 25.70                                     | -15.70        | 29.80                                     | 0.00          |
| 5      | 0.30 | 2117                          | 0.24     | 37.50                         | 38.40                                     | 2.50          | 41.91                                     | 10.50         |
|        | 0.40 | 2059                          | 0.24     | 29.50                         | 29.50                                     | 0.00          | 32.19                                     | 8.00          |
| 6      | 0.50 | 2014                          | 0.24     | 25.70                         | 23.70                                     | -8.40         | 25.70                                     | 0.00          |
|        | 0.30 | 2060                          | 0.30     | 35.00                         | 35.20                                     | 0.60          | 36.56                                     | 4.00          |
| 7      | 0.40 | 2002                          | 0.30     | 27.60                         | 27.60                                     | 0.00          | 28.67                                     | 3.00          |
|        | 0.50 | 1957                          | 0.30     | 23.30                         | 22.60                                     | -3.30         | 23.30                                     | 0.00          |
| 8      | 0.30 | 2002                          | 0.36     | 31.80                         | 28.80                                     | -10.60        | 31.15                                     | -2.00         |
|        | 0.40 | 1944                          | 0.36     | 23.00                         | 23.00                                     | 0.00          | 25.75                                     | 10.67         |
| 9      | 0.50 | 1899                          | 0.36     | 21.30                         | 19.10                                     | -11.40        | 21.30                                     | 0.00          |

Table 5. Applicability of Equation 24 for LWC to authors' data

| Volume<br>fraction,<br><i>a</i> | w/c  | $f_c$<br>(Experiment):<br>MPa | Target<br>strength $f_c$<br>from GLA <sup>a</sup> | Difference: % | $f_c$<br>(Equation 23):<br>MPa | Difference: % | $f_c$<br>(Equation 24):<br>MPa | Difference: % |
|---------------------------------|------|-------------------------------|---------------------------------------------------|---------------|--------------------------------|---------------|--------------------------------|---------------|
| 0.412                           | 0.75 | 24.34                         | 25.00                                             | 2.60          | 26.43                          | 7.91          | 25.42                          | 0.42          |
| 0.412                           | 0.60 | 30.64                         | 30.00                                             | -2.10         | 30.50                          | -0.45         | 29.35                          | -4.39         |
| 0.412                           | 0.51 | 34.54                         | 35.00                                             | 1.31          | 36.87                          | 6.32          | 35.44                          | 0.29          |
| 0.412                           | 0.50 | 36.00                         | 36.00                                             | 0.00          | 37.44                          | 3.85          | 36.00                          | 0.00          |
| 0.412                           | 0.40 | 44.23                         | 45.00                                             | 1.71          | 44.23                          | 0.00          | 42.67                          | -3.66         |
| 0.412                           | 0.35 | 49.06                         | 50.00                                             | 1.88          | 48.64                          | -0.86         | 47.00                          | -4.38         |

<sup>a</sup> Generalised Abrams law:  $S/S_{0.5} = -0.2 + 0.6(c/w)$ , for  $S_{0.5} > 30$  MPa. *S*: compressive strength at any water to cement ratio;  $S_{0.5}$ : compressive strength at water to cement ratio of 0.5. *c/w* cement to water ratio by weight

Table 6. Application of generalised Abrams law and MBEs to predict compressive strength

the generalised Abrams law is a better method as it is a function of w/c ratio alone, which is line with any other conventional concrete. The volume fraction of coarse aggregate has a minor effect and can be neglected based on the above observations.

### Authors' reply

■ The influence of volume fraction is not linear, as seen from the equation itself. Therefore, when comparisons are made with two other quite different volume fractions, rather than 0.53 and 0.37, the effect may be seen and the use of volume fraction *a* in the power portion of the equation becomes relevant. In normal-weight concrete (NWC), the volume

fraction of aggregate would not significantly affect the concrete strength, mainly because the intrinsic strength of normal-weight aggregate is far more than the cement mortar matrix surrounding it. Moreover, the cracks in NWC go around the aggregates, thereby providing more ductile failure to NWC in contrast to lightweight concrete (LWC), where LWAs are weaker than the cement matrix and failure occurs by actual fracturing of the LWAs and this is aided by the excellent inherent bond between the porous surface of the LWA and the cement matrix.

■ A w/c ratio of 0.5 is practically useful to prepare NWC without any need of superplasticiser. However, in the case of

LWC, especially with fly ash aggregates,  $w/c = 0.4$  is quite adequate to prepare workable concrete considering the good shape (nearly spherical) of LWAs and when LWAs in saturated surface-dry conditions are used. A  $w/c$  ratio of 0.5 for LWC is sometimes considered rather high since this  $w/c$  could produce a lower strength mortar and therefore a lower strength level of the LWC. In structural concrete, a lower  $w/c$  ratio in LWC is preferable from a durability point of view.

- For LWCs, along with  $w/c$  ratio, aggregate content also becomes important as aggregate strength also affects LWC strengths. This has been discussed in numerous reports in the literature on concretes containing various types of LWA. Hence, the nature of LWAs should be considered in designing LWC mixes.
- It is interesting to see that a  $w/c$  ratio of 0.5 could be used instead of  $w/c = 0.4$ . In fact, this was considered by the authors while developing Equation 23. It was found that the constants of the equation need to be different for a better data fit and, in order to keep the constants at reasonably simple values,  $w/c = 0.4$  was chosen. Moreover, LWCs have to be made with lower  $w/c$  ratios in order to achieve structural concretes of higher grades. Therefore, 0.4 was selected for the equation.
- Table 6 indicates that the quantity of aggregate plays a minor

role in determining the strength of the concrete matrices prepared. However, the authors feel that their suggested equation is general in nature and applicable to most types of LWAs. In fact, the authors used much published data on LWAs before arriving at the MBE.

#### REFERENCES

- Girish RU, Nataraja MC and Nagaraj TS (2006) Re-proportioning concrete mixtures for rapid production of pre-cast units. *Indian Concrete Journal* **80(4)**: 40–44.
- Nataraja MC and Das L (2011) A simplified mix proportioning for cement based composites with crushed tile waste aggregate. *Journal of Scientific and Industrial Research* **70(5)**: 385–390.
- Nataraja MC, Nagaraj TS and Basavaraj SB (2005) Reproportioning of steel fiber reinforced concrete and its impact resistance. *Cement and Concrete Research* **35(12)**: 2350–2359.
- Nataraja MC, Nagaraj TS, Bavanishankar S and Ramalinga Reddy BM (2007) Proportioning cement based composites with burnt coal. *Materials and Structures* **40(6)**: 543–552.
- Rajamane NP and Ambily PS (2012) Modified Bolomey equation for strength of lightweight concretes containing fly ash aggregates. *Magazine of Concrete Research* **64(4)**: 285–293.