Discussion of articles published in the *Magazine of Concrete Research*

Experiments on the strength of reinforced and prestressed concrete beams and of concrete-encased steel joists in combined bending and torsion*

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Contribution by K. Kretsis

In the recommendations for the design of reinforced and prestressed concrete beams and concrete-encased steel joists in torsion, on page 18 of the article by Cowan and Armstrong, the two equations

$$M_T = \alpha b^2 d F_{ct} \quad \ldots\ldots(3)$$

$$M_T = \sqrt{2\lambda}\, b'd' \frac{A_v}{p} \sin(\psi + 45^\circ) F_v \quad \ldots\ldots(4)$$

are given for plain and reinforced concrete respectively.

The first equation agrees with another expression

$$F_{ct} = \frac{KM_T}{b^2 d} \text{ where } K = 3 + \frac{2{\cdot}6}{0{\cdot}45 + d/b}$$

used by Continental writers on reinforced concrete for the maximum shear stresses occuring at the points A (Figure 1). It is evident that at one of these points A, shear stresses due to vertical shearing forces will be added to those due to torsion and therefore

$$F_{ct} \text{ total} = F_{ct} \text{ shear} + F_{ct} \text{ torsion}$$

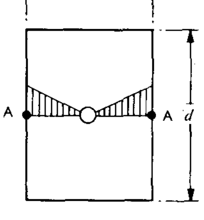

Figure 1

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This really means principal tension. Furthermore I should like to add here that practically all regulations specify two values, F'_{ct} and F''_{ct}, for shear stress (= principal tension) such that if $F_{ct} \leqq F'_{ct}$ no special reinforcement is required and if $F_{ct} \geqq F''_{ct}$ then the section is to be enlarged; therefore reinforcement will be used if $F'_{ct} < F_{ct} < F''_{ct}$. In the British Standard CP 114:1948 for 1:2:4 concrete $F'_{ct} = 100$ lb/sq.in. and $F''_{ct} = 4 \times 100$ lb/sq.in.

To the best of my knowledge, it is stated in Continental technical literature that the above provisions will apply also in the case of torsion or torsion + shear and therefore the maximum permissible twisting moment of a rectangular reinforced concrete section will be

$$M_{T\,max} = \alpha b^2 d\,(F''_{ct} - F_{ct} \text{ shear})$$

which determines the size of the section. In the article it is stated that the maximum permissible twisting moment of a rectangular reinforced concrete section will be given by the above-mentioned equation (4) (according to CP 114:1948), and no upper limit is given.

Do the authors suggest that the permissible twisting moment may be increased up to the extent determined only by the amount of reinforcement which can be accommodated in the section, regardless of the value of

$$F_{ct} = \frac{M_T}{\alpha b^2 d} + F_{ct} \text{ shear?}$$

In equation (4), and similarly in equations (5) and (6), the inclined reinforcement (or the stirrups in the case of split-up reinforcement) is given in terms of the sectional area of each bar and the pitch p for obvious reasons, but the longitudinal reinforcement in the second case is not determined in the same way; instead the total area only of such reinforcement is given.

Do the authors suggest that once the total sectional area of the longitudinal reinforcement is determined, then the choice of diameters need not depend on the spacing of the bars A-B (Figure 2), and that therefore the three arrangements in Figure 3 of the longitudinal reinforcement in a rectangular section are equivalent if the total sectional area of the steel is the same?

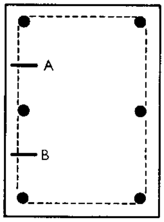

Figure 2

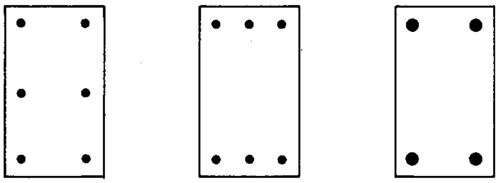

Figure 3

Reply by the authors

Mr Kretsis is quite correct in stating that the value of $\frac{1}{\alpha}$ is given as a close approximation by $3 + \frac{2{\cdot}6}{0{\cdot}45 + d/b}$. The expression was derived by Bach* as a simplification of St Venant's torsion theory, and it has been used by Rausch† in Germany as well as by Anderson(6)‡ in America.

Since Bach's formula is a very close approximation to St Venant's theory, it can certainly be used in place of the tabulated values of α, if the designer prefers to do so. Mr Kretsis is correct in stating that, in the presence of transverse shear, the net shear stress is given by the difference between the transverse and the torsional shear stresses on one face, and by their sum on the other, the latter condition being critical for the design of the section.

The authors are obliged to Mr Kretsis for pointing out that there is an upper limit to the shear resistance of the section irrespective of the amount of reinforcement. This point is discussed in the theoretical section of the investigation,(15) but no experiments were carried out since torsional shear stresses of this magnitude are very unlikely in practice. The authors consider that Mr Kretsis's suggestion of an upper limit to the permissible torsional shear stress of 400 lb/sq.in. for a 1:2:4 concrete is a reasonable one.

Mr Kretsis is correct in assuming that the longitudinal steel is more efficient if spaced around the whole of the circumference of the section, and the closer the spacing the better would be the resistance to torsion. This point has been considered by a number of previous investigators, notably Miyamato.(4) No specific experiments were carried out by the authors because it was felt that the amount and spacing of the longitudinal steel was unlikely to determine the resistance to torsion, provided there is at least one bar in each corner of a rectangular section. As in the case of shear reinforcement, the amount of stirrup (or spiral) reinforcement is critical for the design of the section. An expression for the minimum amount of longitudinal reinforcement required has been derived by one of the authors.(17)

*BACH, C. *Elastizität und Festigkeit.* Berlin, Verlag von Julius Springer. 1911. pp. 666.
†RAUSCH, E. *Drillung, Schub und Scheren im Stahlbetonbau.* 3rd edition. Düsseldorf, Deutscher Ingenieur Verlag. 1953. pp. 168.
‡The index numbers refer to the items in the list of references in the original paper.