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The ultimate strength and deformation of plastic hinges in reinforced concrete frameworks*

by W. W. L. Chan, B.Sc.(Eng.), Ph.D., D.I.C.

Contribution by Professor A. L. L. Baker, D.Sc., M.I.C.E., M.I.Struct.E.

Dr Chan's analysis shows that in columns, even when the stress due to bending is small in relation to the total stress, ultimate deformation closely approximates to the case in which plastic hinges are assumed to be concentrated at points. This is a valuable contribution.

His analysis is based on the assumption that the bending-moment diagram has a linear distribution and that the column is subject to bending action as in a beam. This is a close approximation to the actual behaviour. It is possible that the plastic concrete behaves almost like a confined fluid, so that there is no variation in strain over the plastic zone. If this is the case, the behaviour of the plastic hinge will, I think, approximate even more closely to one concentrated at a point.

Contribution by R. Gartner, D.Sc., M.I.Struct.E.

Dr Chan's paper contributes substantially to the theory of plastic design of indeterminate structures in reinforced concrete. As far as I know there are only two other methods for the calculation of the angles of plastic hinges. Professor A. L. L. Baker[1] derives a formula for this angle assuming a circular deflexion. This formula yields approximate values only which, however, are on the safe side. The present writer[2] advocated taking the plastic angle δ_r as a percentage of the angle δ_{or} of the determinate case ($\delta_r = \nu\delta_{or}$) and arbitrarily assumed $\nu = 30\%$. Dr Chan, in the above paper, is the first to investigate this method thoroughly; he gives Tables for the calculation of all the unknowns and draws attention to the fact that the angle is small for a moment with a short branch (fully fixed beam).

I wish to point out, firstly, that only concrete hinges are treated. Admittedly the plastic part of the stress-strain curve of steel is long; on the other hand, it must be kept in mind that the cracks under critical load must not be too wide, and that by a rise of the neutral axis the concrete will eventually be crushed. A calculation for steel hinges is therefore also required.

*Pages 121-132.

Secondly, Dr Chan explains that plasticity in columns, especially under a heavy direct load, can spread extensively, even over the whole column. Would it not be better to call this "plastic zone action" instead of "plastic hinge action"?

REFERENCES

(1) BAKER, A. L. L. Recent research in reinforced concrete, and its application to design. *Journal of the Institution of Civil Engineers.* Vol. 35, No. 4. February 1951. pp. 262–298.

(2) GARTNER, R. Design of indeterminate structures by the "plastic" method. *Concrete and Constructional Engineering.* Vol. 48. No. 1. January 1953. pp. 3–8. No. 2. February 1953. pp. 85–94.

Reply by the author

Professor Baker's observation that the plastic concrete could behave almost like a confined fluid and having no variation in strain over the plastic zone is consistent with his assumptions in deriving simplified safe limiting formulae for the available rotation of plastic hinges[1]. However, if the imperfect elasto-plastic behaviour depicted in Figure 1 of the paper is correct, then for a varying bending-moment distribution, it follows from statical considerations that between H and B, the moment of resistance varies from M_y to M_u, and therefore the values of ε and n_1 must also vary accordingly.

The procedure for calculating rotations in steel hinges (i.e. in under-reinforced members) mentioned by Dr Gartner is identical to that for concrete hinges. In deriving the depth of the neutral axis at critical loads, suitable F′ factors as proposed by Professor Baker[1] can be introduced to allow for the effect of wide cracks.

Since it has been shown that even extensive spreading of plasticity does not sensibly affect the assumption of point hinges, the term "plastic hinge action" would appear to be justified.

REFERENCE

(1) BAKER, A. L. L. Recent research in reinforced concrete and its application to design. *Journal of the Institution of Civil Engineers.* Vol. 35, No. 4. February 1951. pp. 262-298.