The influence of size of concrete test cubes on mean strength and standard deviation*

by A. M. Neville, M.Sc.(Eng.), A.M.I.C.E., A.M.N.Z.I.E.

Contribution by A. B. Harman, B.Sc., A.C.G.I., D.I.C., A.M.I.C.E.
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Without attempting to deal in detail with Mr Neville's article, may I offer a few observations on the matter of comparative cube sizes, and a few test results to supplement Mr Neville's.

In mass concrete work, particularly in concrete dams, maximum sizes of aggregate commonly range from 3 to 6 in., and occasionally up to 10 in. With very large aggregate sizes, cubes of practical dimensions can only be made after the concrete has been screened to eliminate the large particles. With 3 in. maximum size, however, cubes or cylinders larger than usual may be called for, without removal of the larger aggregate particles. As Mr Neville's tests were limited to cubes no larger than 6 in. and aggregate of ¾ in. maximum size, the following results from a large number of site mixes and a few laboratory mixes may be of interest.

(1) When gravels of 3 in. maximum size and of mixed rock types were used in vibrated mixes of low workability (from 1:8½ to 1:12 by weight, water/cement ratio 0·52–0·58), 10 in. cubes gave compressive strengths similar to those of 6 in. cubes made with the "screened" mix (i.e. with 3–1½ in. aggregate removed), but 12–20% lower than 6 in. cubes made with the "unscreened" mix (i.e. with 3–1½ in. aggregate retained). They were also lower by a similar percentage than the results that would be expected from the normal water/cement ratio curves based on 6 in. cubes.

(2) When flint river gravel of ¾ in. maximum size was used in laboratory mixes (1:6 by weight, water/cement ratio 0·50), 8 in. cubes gave 7-day compressive strengths 5% lower, and 10 in. cubes compressive strengths 8% lower than comparable 6 in. cubes.

Firstly, these results confirm the conclusion reached by Mr Neville and others that the larger the cube, the lower the compressive strength.

Secondly, these results suggest that, with some aggregates, the larger the aggregate size, the greater is the reduction in compressive strength with increasing cube size. From visual observation, one reason for this appears to be that, with the larger cubes, fracture of the larger aggregate particles is more likely to accompany failure of the mortar.

Thirdly, these results suggest that 6 in. cubes can be used for the quality control of mixes containing 3 in. aggregate, even if the larger aggregate particles are not screened out. (In the latter case, however, the results may have a wider scatter than usual, so that it would be advisable to consider the average of at least two results.)

This brings me to a point made by Mr Neville that "the recommended ratio of the minimum test specimen size to the aggregate size varies between 5 and 4". I think he will find that a ratio of 3 is sometimes accepted and, in accordance with the above remarks, I would suggest that a figure as low as 2 might be acceptable in certain circumstances, if cube results were not considered individually.

Finally, Mr Neville's article raises in one's mind the old question, "what exactly are we measuring when we test a concrete cube?" It would take up too much space to give all the possible answers to this question, but in these days when strength specifications are being more widely adopted (a welcome trend), we sometimes lose sight of the object of making and testing concrete cubes. For instance, in much mass concrete work, the criterion is durability rather than compressive strength. Other things being equal, both are related to the water/cement ratio and, if we really knew what the water/cement ratio was all the time, only occasional concrete cubes might be necessary. This is not yet the case, but I suggest that, in specifications for mass concrete work of any importance, particularly if it involves large aggregate sizes, there should be more emphasis on a limiting value for the water/cement ratio, and that, as the cube size affects the compressive strength, any compressive strengths specified should take this factor into account.

Contribution by T. N. W. Akroyd, M.Sc.(Tech.), A.M.I.C.E., A.M.I.Struct.E.
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Mr Neville's paper on one aspect of concrete cube testing is of interest in that, by drawing attention to the effect of size on the results of the compression test, it helps to stress the complex nature of this so-called simple test.

I feel that it would have rendered his paper even more easy to read if the actual results of the compression tests on the three sizes of cube had been shown graphically. In Figure I, prepared from the results given in Table 3, one method of doing this is shown. An inspection of this Figure raises the question why the *a priori* assumption was made that it would not be possible to determine a significant difference between the mean strength of the 6 in. and of the 5 in. cubes. From Table 3 it can be seen that $\bar{x}_5 < \bar{x}_2$ and $\bar{x}_6$ for all the tests except two (values of

*Pages 101–110.

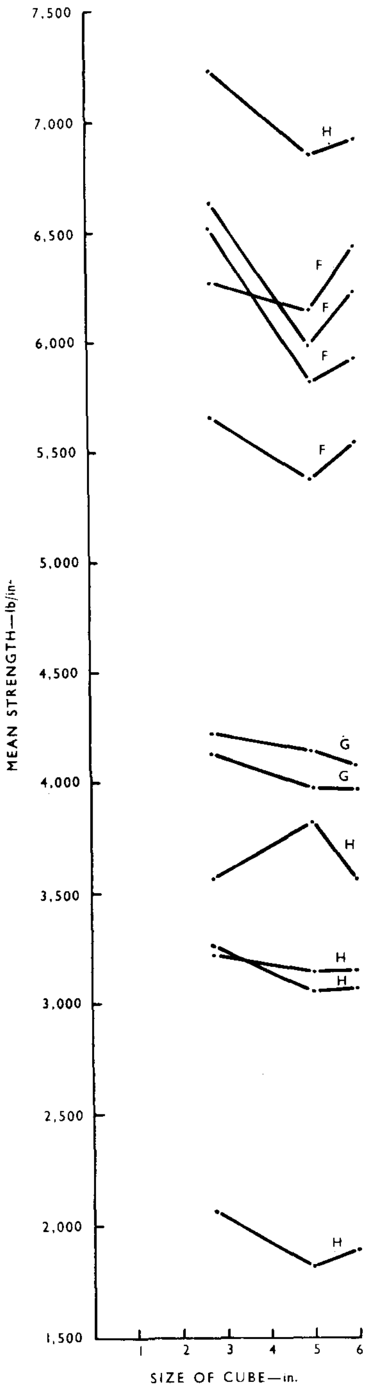

Figure I: Mr Neville's results (from Table 3). The letters indicate the batch of cement.

3,982 and 3,984 for Test No. 6 are considered to be equal). This compares with the fact that $\bar{x}_2 > \bar{x}_5$ and $\bar{x}_6$ for all tests except one for each case, not, as stated on page 102, $\bar{x}_2 > \bar{x}_5$ for all tests.

I suggest that in fact the results of the tests on the 5 in. cubes, when compared with those on the 6 in. cubes, are highly inconvenient in that, instead of the mean strengths for the 5 in. cubes being somewhere between the values for the 2·78 in. and the 6 in. cubes, generally they were below the strengths of both. Can Mr Neville make any suggestions to explain this, for I feel that it spoils his hypothesis, which I support? It has generally been my experience that the smaller the cube, the higher is the mean strength, and as an example I give below some recent results of site cubes made from the same mix, cured and tested in accordance with B.S.1881.

	6 in. cubes	*4 in. cubes*
Number of cubes	26	26
Mean strength	6,325 lb/in²	6,725 lb/in²
Range of variation	33·8%	40·3%
Standard deviation	539 lb/in²	792 lb/in²
Coefficient of variation	8·5%	11·7%

From this single example it will be seen that the standard deviation is higher for the smaller (4 in.) cube, which agrees with the results given in the paper for the difference between the 2·78 in. and 5 or 6 in. cubes, but is distinct from the standard deviations for the 5 and 6 in. cubes in Mr Neville's tests, which appear to be of the same order.

In describing the tests, Mr Neville implies that, since the cubes were made under excellent laboratory conditions and all tested in the same manner, variations in the cube results may be directly due to size effects. I suggest, however, that the actual leanness or richness of the mix may have an effect upon the variation in mean strengths between cubes of different sizes, the difference in mean strengths of the cubes being greater in the 5,500 to 7,500 lb/in² range than in the 3,000 to 4,500 lb/in² range. This effect of richness was discovered while the effect of maturity on the strength of cube specimens was being considered.

Contribution by H. W. W. Pollitt

(Associated Portland Cement Manufacturers Ltd)

Mr Neville concludes that there are significant differences between (a) the means and (b) the standard deviations of 2·78 in. and 6 in. concrete cube strengths. Two questions arise: first, are the differences real, and second, if they are real, are they due to difference in size alone?

On the first question, some doubt must arise, because it is possible by means of an analysis of variance to detect differences between nominally identical batches (e.g. the 6 in. cubes in Tests 2, 3 and 4 and all cube sizes in Tests 7, 8 and 9) that are as highly significant as the effects discussed by Mr Neville. The methods of making the cubes, therefore, could well have contributed to those effects.

On the second question, there are other factors, apart from the testing techniques already referred to, that could influence the results. The most obvious one is the fact that small and large cubes of the same concrete require two different loads on the same testing machine; and it is possible for a grade A machine (B.S. 1610:1950) to be $\pm 0{\cdot}5\%$ in error at one load and $\mp 0{\cdot}5\%$ in error at the other. Thus a grade A machine could give a 1% difference between the mean strengths of two sizes of cube. It is also often said that the dynamic behaviour of a testing machine is not fully tested by a static calibration and, for this reason, comparisons of testing machines are often made by means of identical concrete cubes. Testing machine error could therefore account for differences greater than 1%.

Mr Neville refers to Williams's results in support of his conclusions on standard deviations, but has not perhaps appreciated the importance of the fact that Williams's small cubes were of mortar. Experience of a number of mortar tests suggests that it can be more difficult to obtain uniformity with mortar than with concrete in strictly controlled laboratory tests, although the B.S. 12:1947 vibrated mortar cube test is the most satisfactory of mortar tests at present. It is, therefore, quite possible that Williams's interpretation is correct and that the larger standard deviation of his mortar cubes is due to their being of mortar rather than to their size.

A relatively minor issue is whether the standard deviation of such test results as these is independent of the mean as Mr Neville suggests, following Himsworth and others.

If Mr Neville reads the discussion of Himsworth's paper [Mr. Neville's reference (15)] he will see that this view is disputed, for laboratory tests at least, and if, using the present data, he plots the pooled standard deviations for identical batches against their pooled means he will see why.

In conclusion it should be mentioned that investigations of this type made in the A.P.C.M. Research Laboratories have failed to reveal any systematic difference between the strengths of 4 in. and 6 in. cubes. Differences have sometimes been observed, but they varied in sign and magnitude with the age of the specimens and with the type of testing machine used. A comprehensive and properly planned experiment would be necessary to show whether or not Mr Neville's conclusions are correct.

Reply by the author

I am most grateful to Mr Harman and to Mr Akroyd for supplying further information on the effect of cube size on the compression test results. I have always believed that the best confirmation of the limited range of results obtained in a University laboratory can be provided only by the contractor who utilizes the breadth and variety of his experience to obtain comprehensive and carefully collected test data.

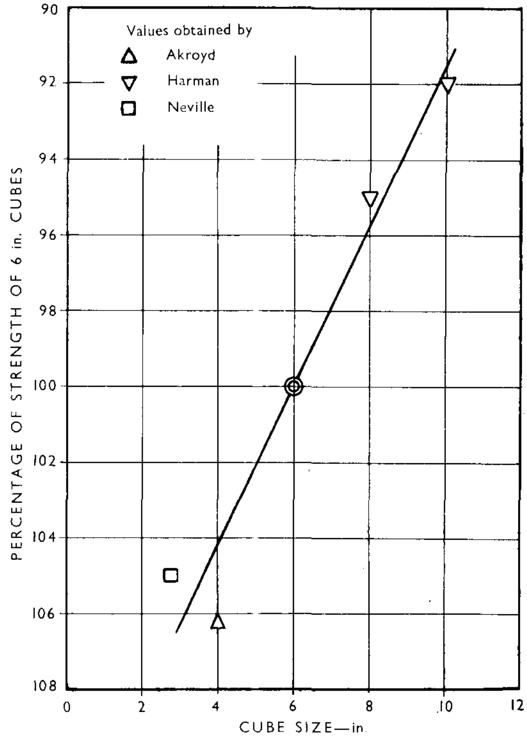

Figure II: Strength of cubes of various sizes as percentage of strength of 6 inch cubes.

In the conclusion of the paper, I indicated that the results obtained were believed to apply only to the range of mixes and cube sizes tested. I find it gratifying to see that Mr Harman's and Mr Akroyd's data, together with those listed in the original paper, present an integrated picture of size effects, as shown in Figure II. It would seem, therefore, that the effect of cube size on the mean strength is approximately uniform over a wide range of cube sizes and holds good for a variety of conditions of preparation of samples.

A comparison of strengths of 5 in. and 6 in. cubes for group C of the samples would show that the 5 in. cubes are, on the average, 28 lb/in^2 stronger than the 6 in. cubes. The standard errors of means of the two sizes are given in Table 7 as $S_{A5} = 24$ and $S_{A6} = 15$ lb/in^2. Thus, the significance of the observed average mean can be measured by the t test, using equation (12):

$$t = \frac{28}{\sqrt{24^2+15^2}} = 1$$

This represents odds of the order of 70% against the null hypothesis, and that is why no conclusion could be reached about the difference between the strengths of the cubes of these two sizes unless the standard errors of the means were reduced by increasing the size of the samples.

The results of group A would appear to show that $\bar{x}_5 < \bar{x}_6$ but, since the samples of this pilot part of the project were small, the standard deviations were high, and little can be inferred from the data for this group. The sensitivity of the value of the mean to the variation in the size of the sample was shown by Prôt(1), who found that about 400 specimens were required for the mean not to deviate by more than 1% on increase of the size of the sample. With 150 specimens, deviations up to 3% were possible.

If the 5 in. cubes conform to the pattern of behaviour shown in Figure II, then their mean strength would be expected to be 2% higher than that of the 6 in. cubes. This value (assuming all samples to consist of 12 cubes) is 3,806 lb/in^2 for group C, and thus the mean strength of 5 in. cubes in that group should be (3,806+76) lb/in^2. The actual difference, therefore, falls short of the theoretical value by 48 lb/in^2. The probability of the difference, measured at random, being 48 lb/in^2 below the population mean (28 lb/in^2), is given by statistical tables as about 10% ($t = \frac{48}{28} = 1{\cdot}71$). This would mean that there is a 10% probability of the actual results being obtained by random choice while the population mean strength of 5 in. cubes is such as predicted by the curve of Figure II.

I agree with Mr Akroyd about the possibility of richness affecting the magnitude of the size effect: this would mean that the size effect is a function of mean strength, but would not invalidate the basic argument. The data obtained by the author to date(2) do not show clearly whether or not there is such an effect.

Figure II should help to dispel Mr Pollitt's doubts about the reality of size effects.

Recent rests conducted by the author at Manchester University confirm Mr Pollitt's assertion that the standard deviation is not independent of the mean. This does not, however, appreciably affect the calculations in the paper, since the pooling of the standard deviations was done for similar strengths for each cube size: the average difference in strength within tests was of the order of 5%. If the corresponding difference in standard deviations is assumed to be proportional to this difference of strengths, then the error induced represents considerably less than one-tenth of the observed difference in standard deviations.

Mr Pollitt is right in stressing the errors of testing machines. If, however, the strength of concrete cubes belonging to one batch is to be the basis of measurement of accuracy of machines, and not merely a check on their performance, then all statistical analysis of variation within a batch is futile: Mr Akroyd quotes, in this discussion, a range of variation of cubes from the same mix as high as 40·3% of the mean strength. A scatter of strength results is inherent in the tests of brittle materials, and indeed, the quality of concrete can be described properly only if both the mean strength and the standard deviation are quoted.

Prôt(1) compared the scatter of strength of mortar and concrete prisms of three sizes, and found no appreciable inherent difference in the scatter of results for the two materials. Table I gives the summary of his data.

Finally, I would like to express my wholehearted agreement with Mr Pollitt about the desirability of further tests on the effects of size in concrete testing. That such tests are required was emphasized in the paper, and it is hoped that data obtained on construction sites, such as those given by Mr Harman and Mr Akroyd, will in time provide a comprehensive picture.

TABLE I

Prism size (cm)	Number of prisms tested	Material	Mean deviation (kg/cm^2)
7·08 × 7·08 × 22·3	160	concrete	40
	160	mortar	43
7·08 × 7·08 × 45	80	concrete	37
	80	mortar	41
15·8 × 15·8 × 50	80	concrete	22
	80	mortar	25

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2. NEVILLE, A. M. The use of 4 in. concrete compression test cubes. *Civil Engineering and Public Works Review.* Vol. 51, No. 605. November 1956. pp. 1251–1252.