The Author. tests of the theory which he had given. In all three cases the calculations were based on a previously observed mean depth of the old section, from which the maximum depth under the tightened section had been calculated by means of equations (4) and (9). Equation (9) was only advanced tentatively, and in applying it Colonel Anderson had presumably assumed a value for K. Further, the mean scour of the old section was probably based on a higher flood than had occurred after the remodelling, because figures for the great flood of 1924 were not given. By the courtesy of Mr. L. H. Grant of the North-Western Railway, the Author had been able to examine the register of scour of the Jumna bridge between Kalanour and Sarsawa (cited by Colonel Anderson as his second example) for the big flood of September, 1924. During the highest period of the flood, the 29th and 30th September, 1924, the depth could not be observed, owing to the velocity being too high for the sinker. The maximum flood occurred on the 30th September and reached R.L. 881·9. The depth was observed to be 40 feet on the 1st October when the flood-level had fallen by 7 feet to R.L. 874·9. If, therefore, the bed had been at the same level, the day before, under the highest flood-level, the depth of scour would have been 47 feet; it was probably rather greater. The maximum anticipated depth of scour had been calculated from formulas (4) and (9) by Colonel Anderson as 46 feet, which agreed well, since this flood exceeded all past records.

Correspondence.

Mr. Ballester. Mr. RODOLFO E. BALLESTER, Chief Engineer of the Rio Negro Irrigation Works, observed that in his judgment the Author was not justified in establishing his formulas on the assumption that the Kennedy law (which had been based on the observation of irrigation-canals of small slope) was applicable to all watercourses. Irrigation-canals were dimensioned in such a way that the silt which came from the source of supply could traverse the canal and distributaries to the fields without being deposited in the canal and without being added to by erosion of the bottom or sides. In a river the eternal levelling work of Nature had to be taken into account, and eventually material scoured from the high reaches of the river would always try to find its way to the sea. In consequence, there would not be "equilibrium" or "balance" between silting and scouring, which was the real meaning of the Kennedy formula. With reference to the value of the coefficient in the formula $V_o = cd^m$, he was able to give the results of his own observations in the investigation of the application of the Kennedy law to the irrigation-canals of the Rio Negro Superior in Argentina.[1] From the plotting of fifty-one observations of depths and velocities he had obtained the following formula, which represented the application of the Kennedy law to those canals: $V_o = 0 \cdot 52 d^{0 \cdot 44}$, in which V_o denoted the velocity in metres per second, and d the depth in metres. This formula, expressed in English units, was $V_o = 1 \cdot 01 d^{0 \cdot 44}$. Mr. Ballester.

He had made a determination of the fineness of the silt of the Rio Negro canals, as deposited, with the results shown in the following Table:—

FINENESS OF SILT DEPOSITED IN THE MAIN CANAL OF THE RIO NEGRO IRRIGATION-WORKS.

Location.	Sample No.	Quantity of Dry Silt Weighed.	Coarse Sand.[2] Mesh No. 100.		Fine Sand.[2] Mesh No. 200.		Fine Silt and Clay (by Difference).[2]
		Grams.	Grams.	Per Cent.	Grams.	Per Cent.	Per Cent.
Main Canal.							
From Km. 8 to Km. 18 .	1	300	1·50	0·5	6·43	2·1	97·4
,, ,,	1	300	1·93	0·6	7·37	2·4	97·0
,, ,,	2	300	3·35	1·1	9·75	3·2	95·7
,, ,,	2	300	4·04	1·3	9·90	3·3	95·4
,, ,,	3	300	12·45	4·1	68·45	22·8	73·1
,, ,,	3	300	7·20	2·4	96·4	32·2	65·4
,, ,,	3	250	5·30	2·1	50·7	20·2	77·7
Main Canal.							
Km. 72	4	250	1·55	0·6	19·3	7·7	91·7
,, ...	4	300	3·31	1·1	25·48	8·5	90·4

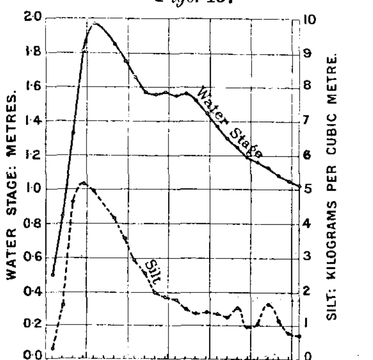
There was a wide difference between the value of (0·64) for m in the Kennedy formula for the Punjab and the 0·44 found by Mr. Ballester for the Rio Negro canals. This difference affected in a large measure the results arrived at by the Author. For instance

[1] "*Velocidades y Coeficientes de Aspereza para el Cálculo de Canales en el Rio Negro,*" *Contribución al Estudio de las Ciencias Físicas y Matemáticas, Serie Técnica*, vol. III, part 5, p. 435. Buenes Aires, 1927.

[2] The silt-classification is that detailed in Minutes of Proceedings Inst. C.E., vol. ccxvi (1923), p. 204. The mesh number denotes the number of meshes per linear inch.

Mr. Ballester. (p. 245), the maximum depth, with sufficient bed-slope to give the velocity required to prevent bed-silting, assuming the same values of $c=1$, and $V_o=5$ feet, would be $5^{1/0 \cdot 44}=38 \cdot 5$ feet, instead of 12 feet given by the Author with $m=0 \cdot 64$. Mr. Ballester had made determinations of silt-content in floods on the Neuquen river, which fed the main canal of the irrigation-works at the Neuquen barrage, in 1915, and at Punto Unido, 7 kilometres upstream of the Neuquen barrage, in 1917 and 1918. Samples of a litre of water were taken in the same place at 0·50 metre below the surface-level, and the silt-content was determined by evaporating the sample, and drying and weighing the silt, the results being given in kilograms of dry silt per cubic metre of water. *Fig. 15* gave results at Punto Unido for the flood of the 17th and 18th May, 1918, *Fig. 16* for the same place for the 8th to the 13th May, 1919, and *Fig. 17* for the same river at the Neuquen barrage for the flood of the 23rd to the 29th May, 1915. He had tried to establish a relation (which subsequent observation had not confirmed) between velocity and silt-content, in a Paper written in 1917.[1] *Figs. 15* to *17* showed clearly that the relation between depth and silt-content in the Neuquen floods was far from simple. For the same depth (water stage) the quantity of silt varied with different floods, with height of flood wave, and with river stage, being greater with a rising river; and the maximum content of silt carried was always reached before the maximum river stage. These observations showed that the formulas developed by the Author for the calculation of contractions, river-training, weir-widths, and bridge-foundations, were not reliable, because the Kennedy formula on which they were based was an "equilibrium formula," not appliable to the non-uniform movement of a river in flood.

Figs. 15.



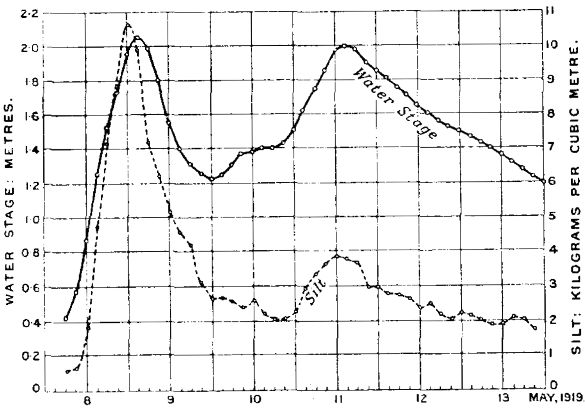
NEUQUEN RIVER AT PUNTO UNIDO.
FLOOD OF 17–20 MAY, 1918.

Mr. Bellasis. Mr. E. S. BELLASIS remarked that thanks were due to the Author for his presentation of interesting facts and river sections and for

[1] "*Contribución al Estudio del Régimen de Rio Negro: Crecidas del Rio Neuquen.*" *La Ingeniería* (Buenos Aires), 1919, pt. 1, p. 83.

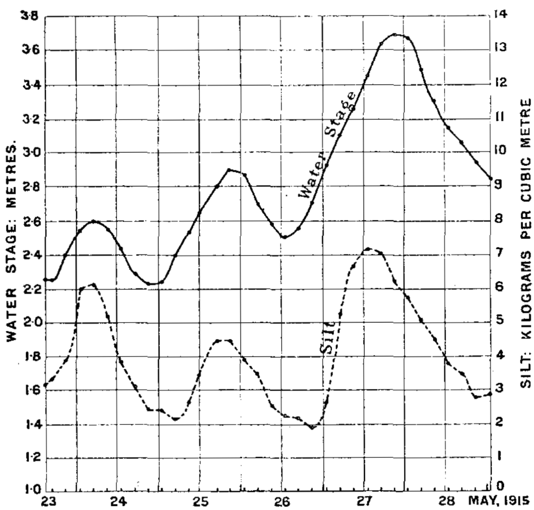
his suggestions for calculating the effects of obstructions in streams. Mr. Bellasis.
The Author stated (p. 247) that, as the flood rose, V increased only

Fig. 16.



NEUQUEN RIVER AT PUNTO UNIDO. FLOOD OF 7–13 MAY, 1919.

Fig. 17.



NEUQUEN RIVER AT NEUQUEN BARRAGE. FLOOD OF 23–29 MAY, 1915.

as $d^{0.5}$. Mr. Bellasis had shown elsewhere[1] that it increased approximately as $d^{2/3}$; also that in Mr. Kennedy's formula the

[1] "Hydraulics, with Working Tables," p. 173, London, 1924.

Mr. Bellasis. index of d should be $0 \cdot 57$ rather than $0 \cdot 64$. Mr. Kennedy's formula applied when the bed was silt and sand. The contraction of width from 4,700 feet to 1,700 feet (p. 255) might cause d to increase to D, such that, roughly, $1{,}700\,D^{5/3} = 4{,}700\,d^{5/3}$, but D would probably be less than this—at least at first—because the slope would tend to be greater in the narrow channel. For greater exactness it would be necessary to look up Kutter's coefficients for the particular sections concerned, to see the exact rate of their variation, and to take Mr. Kennedy's index as $0 \cdot 57$. No formula could suit all channels. Even calculations made as above would be unreliable if the material of the channel was not uniform all over it. Every river needed careful study. For a bed of boulders it would be necessary to consider the rolling power of the stream. In any case, any depth actually attained by the stream would be no more permanent than was the depth of the unrestricted section. The Author mentioned (p. 252) the section "determined by" the flood. A flood might leave a section silted or scoured.[1] Above the Bhimgoda weir the deep channel appeared to have shifted from left to right, but there was no general rise in the bed-level. In what year had the weir first come into use? Were all the cross sections taken looking down stream?

Mr. Claxton. Mr. P. Claxton remarked that the Author's use of the empirical formula for flow could not be altogether justified. In Papers[2] read for 2 years in succession at the Punjab Congress, Mr. Claxton had tried to show that the laws of flow were governed by two factors: one was restraint of boundary, and the other was what he termed "potential." Restraint had been fully dealt with by Professor Osborne Reynolds, who had shown that the presence of the boundary broke down steady flow in straight lines and made it sinuous or eddying. The law of resistance to flow thenceforward changed and was a function of velocity with an exponent in the vicinity of 2. That gave a general formula with V^2 as one function. As the eddies burst throughout the mass at one time, and their resistance was also constant throughout the mass, this law, based on boundary restraint alone, might be used in the way it had been by the Author, but it could not be so used when potential also was taken into account. Restraint, in fact, only depended on the presence of the boundary, but did not take into account the condition of its surface.

[1] E. S. Bellasis. "River and Canal Engineering," pp. 52 and 53, London, and New York 1924.

[2] Proceedings Punjab Eng. Congress, Lahore, vol. xiii (1925), pp. 47 to 67*j*, and vol. xiv (1926), pp. 33 to 51*l*.

Mr. Claxton.

In order to study it separately it was necessary to consider an absolutely smooth boundary. The meaning of potential could be explained by consideration of a clear stream with its bed rolled in dunes. The water flowed over these dunes, with a corresponding wave-form at its surface, though hardly, if at all, noticeable. Particles of silt could be observed to settle on the upstream faces of the dunes and to roll along till, at the crest, violent eddying took place. Such eddying at the crest was the result of potential which had been acquired up the face of the dune and stored till it was released at the crest and reacted in eddies. The fact that particles were deposited on the upstream face of the dune indicated loss of kinetic energy, which was converted into potential energy. If this were so, it followed that potential was an added force to restraint, and both gave rise to eddies. A combined formula, therefore, was irrational, and could not be applied in the way attempted by the Author. Each grade of silt had its own ruling angle of repose, and so raised dunes which had the general capacity for potential peculiar to that grade. When another grade entered the stream, it laid down another ruling angle, and silt was either picked up or deposited till conditions were adapted to this ruling angle. Thus each grade of silt gave a distinct formula. Mr. Kennedy's formula was only applicable to the Punjab. It was, of course, empirical and not rational.

Mr. Ghaleb.

Mr. K. O. Ghaleb observed that he had adapted Kennedy's law to Egyptian canals. Expressed logarithmically, the formula became—

$$\log V_o = n \log d + \log C,$$

Plotting the logarithms of the values given for the water-depths and the velocities in five canals alleged to be non-silting (the first five in the following Table), and drawing what seemed to be the "best" straight line through the points thus obtained, C was found to be 0·283, and n was found to be 0·727. Thus Mr. Kennedy's law, when applied to Egyptian canals, could be written $V_o = 0{\cdot}283\, d^{0{\cdot}727}$. The Table on page 286 gave the velocities as measured and as computed from this formula, for non-silting canals.

The close agreement between the results observed and those computed was remarkable for several reasons: the canals given in the Table were distributed all over the country, and their full supply level was not stated in every case; while the measurements of the velocities had been made by different observers, and neither at the same period of the year nor, probably, in the same year.

Mr. Ghaleb.

Canal.	Water-Depth.	Velocity. Measured.	Velocity. Computed.	Authority.
	Metres.	Metre per Second.	Metre per Second.	
Sahel Markas	2·60	0·580	0·567	Th. Yenidunia.[1]
Rayah Belkas	3·60	0·640	0·718	
Mansouria	5·10	0·930	0·925	
Abdalla Wahby Km. 17 1/2	2·65	0·570	0·575	
,, ,, Km. 30	1·66	0·410	0·409	
Bagouria	2·80	0·600	0·598	
Rayyah Menoufia	3·80	0·730	0·747	
Khadrawia	1·05	0·310	0·293	
Khandak Sharki Km 19·2	2·74	0·592	0·589	Chosen from amongst carefully taken cross sections at the end of 1916.
,, ,, Km. 30	2·07	0·473	0·480	
,, ,, Km. 34	2·14	0·490	0·492	
Ibrahimia (Deirout)	4·22	0·792	0·808	Sir William Willcocks.[2]
,, (Assiout)	9·00	1·400	1·398	Edmond Béchara.[3]
Small distributaries in Behera Province	1·40	0·370	0·361	G. R. Kinder.[1]
	1·20	0·330	0·323	
	0·90	0·270	0·262	
	0·80	0·250	0·241	

The following Table had been computed in order to test the formula further:—

Data Relating to Discharges of the Kasra Canal (Girga Province) During the 1916 Flood.

Date of Discharge.	Water-Levels on Head Regulator. Up-stream.	Water-Levels on Head Regulator. Down-stream.	Up-stream Water-Depth.	Upstream Velocity. As Measured.	Upstream Velocity. As Computed.	Height of Silt Deposited Upstream.
	Metres.	Metres.	Metres.	Metre Per Second.	Metre Per Second.	Metre.
28th August	65·95	65·50	4·755	0·870	0·879	0·000
4th September	65·78	65·60	4·950	0·928	0·905	−0·380
11th ,,	66·05	65·08	4·950	0·562	0·905	−0·110
18th ,,	66·29	64·93	4·610	0·476	0·860	0·470
25th ,,	66·38	65·08	4·650	0·398	0·865	0·520

[1] Private communication.

[2] "Egyptian Irrigation," vol. 1, p. 438, London, 1913.

[3] "Irrigation pérenne des Bassins de la Moyenne Egypte," Lausanne, 1905.

The Kasra canal had a well-placed head reach which did not get obstructed by the entrance of river sand. Though the discharges tabulated above had been taken during the high flood of 1916, the deposit had not been formed, as would be expected, during that first period of the flood, when the water was most heavily charged with silt. At the beginning, when the measured velocity about equalled the computed one, the bed-level of the canal remained unchanged; then, when the measured velocity exceeded the computed, scouring took place. The discharges of the 11th September and after indicated that settlement began as soon as the headworks were heavily regulated for water-control purposes, and the velocity was thus reduced below that shown by the formula to be necessary for keeping the silt in suspension. A series of articles on "Factorized Hydraulic Formulæ" had recently been published[1] which showed that:— Mr. Ghaleb.

(i) The exponent of d in Kennedy's formula, and in all critical velocity formulas, should be about 0·7 apparently; as he had stated, it was found to be 0·727 for Egyptian canals.

(ii) The numerical coefficient in the formula, which was probably an index of the silt-density, varied with the conditions of the feeder canal, but would be constant for each system. For the Upper-Punjab canals, Kennedy's formula was: $v = 0{\cdot}84\ d^{0{\cdot}64}$, and the equivalent Barnes's formula was $v = 0{\cdot}78\ d^{0{\cdot}694}$. The formula for Egyptian canals was $v = 0{\cdot}283\ d^{0{\cdot}727}$, and the equivalent Barnes's formula was $v = 0{\cdot}295\ d^{0{\cdot}694}$.

(iii) When the canal was non-silting, the ratio of the wetted perimeter to the depth of water was a maximum. Taking as an example the following data: Discharge, $Q = 106$ cubic metres per second; side-slopes, 3 to 2; slope, $i = 0{\cdot}00006$; depth, $d = 3{\cdot}40$ metres; bed-width, $b = 40$ metres; and $v = 0{\cdot}283 \times 3{\cdot}40^{0{\cdot}727} = 0{\cdot}689$ metre per second; in this case $b/d = 12$. From a graph in the articles referred to,[2] the maximum ratio of wetted perimeter to depth of water which corresponded to a value of b/d 12 was 0·885; therefore, by Barnes's formula in metric units,

$$v = 40{\cdot}6 \times 0{\cdot}885^{0{\cdot}694} \times 3{\cdot}40^{0{\cdot}694} \times 0{\cdot}00006^{0{\cdot}496} = 0{\cdot}686 \text{ metre}$$

per second; that was, the same result.

Colonel Sir Gordon Hearn, R.E., welcomed the Author's contribution to a subject which was still obscure, namely, the behaviour of rivers in a country of gentle slopes and easily erodible soils. In this connection he agreed that the eroding velocities given by Colonel Hearn.

[1] "Indian Engineering," vol. 77 (1925), pp. 95, 109, 123.

[2] *Loc. cit.*, p. 109.

Colonel Hearn. Mr. Parker were very low, and considered that 1·3 or 1·5 foot per second was probably correct for sand. The technique of bridge-construction was well understood in India and was efficient, provided the higher authorities could bring themselves to sanction the use of efficient plant, without which regrettable delays always occurred. But the art of river-training was a branch hardly at all studied, or, if studied by the constructing engineer, it was lost sight of by the maintaining staff. The consequences of neglect were costly, partly through destruction of expensive training-works, and partly through panic, which brought in its train hasty decisions and hasty restoration work. Therefore the Author's contribution to the study of scour was very valuable. He had read recently in the Press an account of what had happened to a certain bridge, and had some slight knowledge of the matter. The training-works having failed, a pier had been scoured out and communication cut off. In due course a committee recommended certain steps, but, before they could be carried into effect, the river cut through the approach and left the bridge high and dry. He had had similar threats to face in Bengal, but had been able to meet them, and recent inquiry had given him the satisfaction of knowing that in every case success had been attained. The cost of restoration was heavy, however, and much of it could have been avoided by better knowledge on the part of either the constructing or the maintaining engineers. It was extraordinarily difficult to arrive at a true section of such rivers, for the section varied with every stage of the flood. As a rule the section could be taken only at low water, when islands had been formed and silt had been dropped over the sectional area covered in ordinary and high floods. The course, also, was very different, the rule being that the smaller the volume the greater the sinuosity. He had obtained confirmation of this rule from Sir Robert Gales, M. Inst. C.E., and it was an interesting fact that the sinuosity of the Ganges, in the vicinity of the Hardinge bridge, was now much less than formerly, this being attributed by Sir Robert Gales to the smaller volumes now supplied to the Bhagirathi (Hooghly) river and other deltaic offshoots on the right bank. If the river should swing over and reopen the mouths of those rivers, an increased sinuosity could be expected. The investigation into the probable high-flood volume of the Ganges at Sara had been long and careful, but the bridge had never been filled and was too long by two to four spans. Still, the surface velocity had risen as high as 13·2 feet per second—a velocity not contemplated[1]

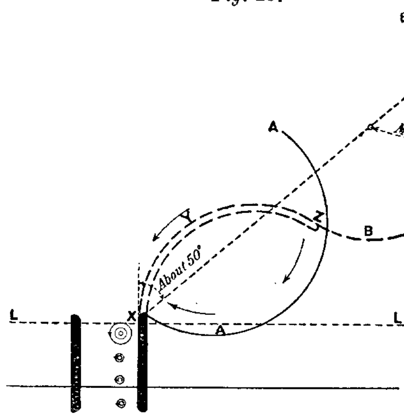
[1] "River Training and Control," Plate xlix. Simla, 1903.

by Sir Francis Spring, M. Inst. C.E., even with a depth of 130 feet, using the Mississippi formula. There was no afflux at the bridge; the stream was not restricted—or at all events had not been hitherto—owing to the excessive length of the bridge; and the guide-banks at the bridge had not been called on to resist attack. The outer works at Raita and Sara had both been subjected to a loop attack, but the steps taken had been effective, having been taken in good time. Colonel Hearn

The Author referred (p.255) to discrepancies between actual and calculated mean depths in the Jumna at Kalsi and Faizabad, but it appeared that the mean depth was calculated from a low-water cross section with a high flood-level plotted on it, and it was quite possible that the river had scoured its bed 5 feet in flood. This seemed hardly likely in a bed composed of shingle and boulders; but on the Khyber Railway he had allowed, when fixing the levels of the concrete floors of certain bridges, for a discharge, up to 3 to 4 feet below apparent bed, of half water and half shingle during floods. After the flood the river-bed was the same as before. The bed-slope was about 1 in 50. Having examined many cross sections of rivers taken at low water, he had nearly always found that a considerable width was covered by a few feet of water only when the high flood-level was plotted. The mean depth arrived at by dividing wetted area by wetted perimeter became much less in consequence, and his practice was to leave such shallow areas out of account in calculating velocity. For, when the flood was "bank high," the mean depth (and consequent calculated velocity) was high, and the discharge also high. When the flood rose still higher, owing to inability of the channel to carry an increased discharge, the calculation showed a smaller discharge, owing to reduction of velocity in conformity with a reduced mean depth. The Author was right, therefore, in his decision to leave out portions of the cross section of the Ganges at Gona (*Fig. 9*), and he might have gone even farther. The calculations for a railway-bridge in Bengal were often hampered by the absence of a free outfall, which had been the cause of great floods over large areas in 1918 and 1922. It appeared that water was practically stagnant with a slope of 1 foot in 12 miles. Subject to this, the contracted section should be such that free scour should be allowed in so much of the bed as was not prevented from scouring by the protection of the artificial banks. In his opinion the restriction should be as great as possible, so as to avoid local scour due to the meandering of the river through the canalized section. The practice of pitching around the piers was wrong, because it prevented free scour, and induced the throwing overboard of quantities of stone in times of panic. This stone itself induced

Colonel Hearn. scour and seldom reached an effective position. It should be recognized that a bed must scour; a caution point must be fixed, with some margin of safety and the pier must be sunk so far that a certain depth of it would still be firm in the ground when the scour exceeded the caution-point depth by that margin. It was not necessary for all the wells to be sunk to the same depth if local scour was prevented by constriction and adequate training-works. He had previously given, in an (unpublished) Paper read before the Bengal Association of Engineers, instances of the scour set up by loop attack acting on the relatively straight guide-bank, and of that scour (which developed off the armoured head) developing right through the bridge. He advocated a curved guide-bund, approximating in radius to the ascertained minimum radius of

Fig. 18.



loop attack of the particular river. Thus the water would take a reversed curve, instead of setting up a big whirl about X in *Fig. 18*, in which the line LL showed the centre-line with curved guide-banks. Although the curved bank might be considerably longer than a straight bank, designed on the arbitrary rule of length equal to contracted width, still the absence of toe scour should enable a relatively smaller quantity of armour to be used per linear foot. The Author was no doubt aware of the practice of providing aprons to drop on the slope formed by scouring and so prevent the sliding of the face armour, but Colonel Hearn was generally in agreement with him on the proposals shown in *Figs. 11*. Such a wall as the Author showed should be built in sections.

The unpublished Paper referred to proposed a theory of "nodes," from which the river did not depart appreciably in its vagaries; and this theory appeared to be supported by surveys of the Ganges between

1764 and 1924, one node being at Raita. It was possible that, if a curved bank were constructed at each of these nodes, the vagaries of the river might be restricted as desired by the Author. There was little doubt that a large bridge should be built normal to the general course and not to the local reach, but the Hardinge bridge was not so sited, though the outer training-works at the node had their effect. It was, however, urged by some engineers that a single guide-bank was not effective. Returning to the question of silt-transportation and scour, it seemed to him that the capacity to transport silt was undoubtedly a function of the velocity. That the river would require a higher velocity to scour seemed correct, since the quantity of silt in transportation was greater, while the canal water could pick up the silt and scour at a lower velocity. It was possible, by the way, in some cases that the flood-level was higher in the centre of the current than at the sides. He would like the Author to examine the case of the Ganges at the Hardinge bridge, with a surface velocity attaining 9·18 miles per hour in 1924 in a bed of soft silt, and to give an opinion as to what depth should be the maximum for scour by his theory. Records showed a depth of 95 feet between piers. The pier-foundations were still 80 feet below this level. Over a period of 5 months in 1925 there was not always a rise of velocity with a rising, or fall of velocity with a falling, flood-level. Also the bed did not necessarily silt up as the flood-level fell. The effect of the horizontal filaments was probably not so great as that of the swirls caused by impingement of the current on slow-flowing water or backwaters. The formation of pear-shaped islands, the point being and growing downstream, was interesting to watch, the silt being dropped as the velocity died out. Thus the position of these islands marked the edges of the main current, but they did not exist (even under water) at a high stage of flood. Colonel Hearn.

Mr. GERALD LACEY considered that the Paper should provide a useful stimulus to re-examination of the theory of silt-transportation and its bearing on the type of cross section that a controlled river should tend to assume. The Author based his conclusions on an acceptation of the Kennedy non-silting formula $V_0 = C d^{0 \cdot 64}$. The rivers with which the Author dealt contained "silt" ranging from boulders to fine sand, and did not reproduce the conditions governing the experiments made by Mr. Kennedy. Before any final conclusions as to the nature of ultimate channel sections could be reached, the Kennedy data required re-investigation. For this purpose it was desirable to read Mr. Kennedy's original Paper presented to The Institution, supplemented by the remarks in the preface to his "Hydraulic Mr. Gerald Lacey.

Mr. Gerald Lacey. Diagrams," the Paper[1] on "Flowing-Water Problems" by Mr. E. C. Thrupp, Assoc. M. Inst. C.E., and the comments made thereon by Mr. Parker in his treatise, "The Control of Water." The view taken by Mr. Kennedy of the silt-transportation problem had led to an orientation of subsequent investigation that had involved a consideration of the vertical depth d rather than the mean hydraulic radius r. Mr. Kennedy found that, so far as could be judged from his experiments, the bed-width of a channel produced no visible effect on results. When confronted with the problem of designing non-silting distributary channels on the Sarda canals Mr. Lacey had discovered a curious anomaly arising from rigid adherence to the Kennedy formula. That formula was a very useful means of calculating the non-silting slope for a given channel of given discharge; when, however, the slope was fixed by topographical considerations, the formula gave anomalous results. According to the Kennedy theory, if the discharge and slope were fixed, the wider and shallower the channel was made, the better able it was to support fine silt. This was not borne out in practice. Comparatively narrow and deep channels in fine silt carried their silt effectively, and that was due to the fact that the sides of the channels played a very important part in creating the turbulence on which the carriage of fine silt depended. Further, channels carrying fine silt and flowing in fine silt, so far from adopting the horizontal bed and steep side-slopes indicated by Mr. Kennedy, adopted a natural semi-elliptical cross section admirably designed to secure that the wetted perimeter played at every part its share in supporting silt by the creation of eddies normal to the channel-boundaries. Mr. Kennedy's theory had obvious limitations. At one point on the Sarda canal the whole discharge of 9,000 cusecs was siphoned under a torrent by means of a battery of pipes 6 feet in diameter. The Kennedy theory and the vertical-eddy theory broke down at once when such a section had to be considered. What was wanted was a formula in terms of r, the mean hydraulic radius. He had evolved in 1926 the formula $V_0 = 1{\cdot}16\ r^{0{\cdot}5}$, which would be found to fit the Kennedy data accurately throughout their range; it had also the advantage of being general instead of special. He was aware that from an examination of Mr. Kennedy's observations it had been found[2] that, if allowance was made for the vertical eddies created by the sides, Mr. Kennedy's formula became $V_o = C\,d^{0{\cdot}59}$. The Author had not stated,

[1] Minutes of Proceedings Inst. C.E., vol. clxxi (1908), p. 346.

[2] Cf. footnote to p. 245.

Mr. Gerald Lacey.

however, by whom that formula had been derived. Such a statement was a contradiction in terms; for, as soon as allowance was made for the sides (vertical eddies in such a connection were difficult to conceive), *r*, not *d*, was dealt with. The notion that *r* and *d* were interchangeable was incorrect and led to faulty conclusions. So long as the Kennedy formula was retained with *d* to any power, the error must persist. With the new formula in terms of *r* it would be found that the silt-carrying capacity of a channel in fine silt was a maximum when *r* was a maximum, which occurred when the section was semi-circular; and it was to such a section that rivers in very fine silt and mud tended to approximate. Mr. Parker had expressed the opinion that the deeper a channel was, the less likely it was to scour, and the coarser the material, the less marked was the effect of depth;[1] and he pointed out that Mr. Thrupp's figures[2] agreed very well with:—

Mud and silt not moved if	$V < 0 \cdot 4\, d^{0 \cdot 5}$
Fine silt moved	$V > 0 \cdot 4\, d^{0 \cdot 5}$
Fine sand moved	$V > 1 \cdot 5\, d^{0 \cdot 35}$
Coarse sand moved	$V > 1 \cdot 5\, d^{0 \cdot 30}$
Pea-size pebbles moved	$V > 2 \cdot 2\, d^{0 \cdot 30}$
Egg-size pebbles moved	$V > 5 \cdot 0\, d^{0 \cdot 25}$
Large stones moved	$V > 17 \cdot 0\, d^{0 \cdot 15}$

These deductions from Mr. Thrupp's figures were of interest. It was important to notice that Mr. Thrupp was dealing, not with *d*, but with *r*. It was clear that, the coarser the material, the more accurate was the theory of vertical eddies on which Mr. Kennedy based his formula, and the greater was the departure from the index of 0·64 that he adopted. The index vanished when the material was very coarse, and the depth vanished also.

In the cases of the rivers with which the Author dealt there must be a time in every flood when the coarse material would begin to roll. When this occurred, the material would roll towards the centre of the river, and there would be lateral translation, tending to a flattening of the bed, as well as forward transportation. The Kennedy formula, however, was derived from channels in fine silt constantly carrying the same discharge. Its application to a river in flood (carrying materials of every character according to the nature of the flood, and subject to a wide range of discharges), with a view to determine its ultimate section, would appear to be dependent on so many variables as to render a mathematical solution impossible. The finer the silt, the more feasible was a solution. In view of the

[1] *Loc. cit.* p. 492.

[2] Minutes of Proceedings Inst. C.E., vol. clxxi (1908), p. 353.

Mr. Gerald Lacey. foregoing remarks he doubted the accuracy of the Author's conclusions. It appeared to be incorrect to assume that, because the mean velocity was a function of $r^{0.5}\ s^{0.5}$ the velocities at different points in the cross section would vary as $r^{0.5}$ or, rather, $d^{0.5}$, since the existence of channel-boundaries would impose different velocities at the same depth—a phenomenon which was apparent in any channel with a horizontal bed and with a maximum velocity in the centre. Neglect of channel-boundaries was disregard of the mean hydraulic depth. Further, in view of the subsequent utilization of the Kutter formula, a formula of extreme and unnecessary accuracy, the elementary Chezy formula could hardly be employed. Manning's formula, a close approximation to Kutter's, might be written $V = K\ r^{0.67}\ s^{0.5}$. This formula might just as well be employed as the Chezy formula; if it was, the index was altered, and the Author's conclusions were vitiated. So far as fine silt was concerned the problem of silt-transportation required re-examination from a consideration of the mean hydraulic depth. Anyone who had attempted to deduce Mr. Kennedy's critical velocity from observations made on channels in natural regime soon discovered that the original slopes at the sides and the horizontal beds had vanished, and that the Kennedy value of d was guess-work. The value of r could, however, be readily calculated with great accuracy, and when this had been done, the problem presented little difficulty.

Mr. J. M. Lacey. Mr. J. M. LACEY observed that the Author stated that the silt-transporting power of a stream varied with the difference in velocities between the filaments of the current at unit distance apart in a vertical plane, and that this change in velocities was clearly a function of both the mean or the maximum velocity and the depth d. According to the above postulate, the transporting power should depend more on the ratio of maximum velocity to bed velocity, and not on the magnitude of the velocity alone. The Author, further, accepted the equation $V_o = Cd^n$ as representing the relationship existing between the velocity and depth of a stream with respect to its capacity to transport silt. That did not strictly agree with his opening postulate. Table I (p. 244) should be read in connection with the relevant part of Mr. Parker's treatise.[1] Mr. Parker remarked that the velocities necessary to move soft earth, fine clay, and soft clay were variable, and depended on the adhesion of the clay particles. In Dr. Deacon's experiments, cited by the Author, the velocity of 1·3 foot per second was observed surface velocity.

[1] "The Control of Water," pp. 495–6. London, 1913.

Mr. J. M. Lacey.

Dr. J. S. Owens, Assoc. M. Inst. C.E., had stated[1] that ordinary seashore sand composed of quartzite grains about $\frac{1}{50}$ to $\frac{1}{100}$ inch in diameter would begin to move along the bottom when the current attained a velocity of 0·85 foot per second. The Author observed that rivers emerging from the hills on to the plains of India had in their upper reaches a much steeper bed-slope and consequently higher velocity than in their lower reaches. This statement did not agree with the observations[2] of Mr. G. E. Lillie, M. Inst. C.E., that the velocity in a river was more or less constant, but that the depth and the slope varied from its source to its mouth. The upper reaches of a large river in India were comparatively broad and shallow, with steep gradients, and the material transported was larger than in the lower reaches, where the section became comparatively narrow and deep, with easy bed gradients. Mr. Lillie's observations thus in a manner confirmed the general equation $V_0 = Cd^n$. When stones and boulders were moved along the bed of rivers during floods, it was possible that the water was heavily charged with silt and its density was thereby increased, rendering the stones and boulders more buoyant than they would be in clear water. The general equation $V_0 = Cd^n$ appeared to indicate the velocity necessary to carry forward a certain quantity of silt for a given depth of stream d, the coefficient C and the power n depending on the nature of the silt transported. If the stream was fully charged with silt, the spiral eddies might be damped, or completely obliterated, and the flow would tend to become stream-line flow. In such a case the formula did not hold good; and a rising river might have a higher mean velocity and smaller sectional area of waterway than a falling river for the same gauge-level. In such cases there would be a tendency for the bed to rise. In that connection he would draw attention to a Paper[3] by Mr. A. B. Buckley, jun., M. Inst. C.E., and to the discussion upon it. The Author stated that the cold-weather supplies were more or less confined to the deep channel which wound within the broad bed of the river, and therefore the cold-weather channel was thus greatly lengthened and the slope proportionally reduced, as compared with the river in flood. Observations showed that the *live* stream of a large river in India during floods followed the cold-weather deep channel.[4] Progress up such a river in high flood, in a river-steamer, was only possible by hugging the bank

[1] Minutes of Proceedings Inst. C.E., vol. clxxxiv (1911), p. 199.

[2] *Ibid.*, vol. ccxvii (1924), p. 303.

[3] "The Influence of Silt on the Velocity of Water Flowing in Open Channels," Minutes of Proceedings Inst. C.E., vol. ccxvi (1923), p. 183.

[4] *Ibid.*, p. 150.

Mr J. M. Lacey. away from the live channel and crossing over to the opposite bank where the sinuous course of the live channel crossed over to the nearer one. In the case of a weir, the velocity of approach enabled a river or stream to scoop the silt up in front of the weir, and this velocity depended on the depth and slope of the stream, and not on the ratio between the width of the river and the width of the weir. In the case of the Godavari Anicut at Dowlaishwaram, the width of the river at Rajahmundry was about 10,000 feet when in flood; the river widened thence towards Dowlaishwaram (about 4 miles downstream) to about 18,000 feet, including the islands.[1] The actual lengths of the anicuts were :—

	Feet.
Dowlaishwaram branch	4,809
Ralli	2,859
Maddur	1,548
Vigeswaram	2,602
	11,818

The chart of the river taken in January and February, 1920, after the floods were over, showed the following amounts of scour in front of the various branches, anicut-crest being taken as 0·00.

	Feet
From left flank to right flank of the Dowlaishwaram branch	−1·00 to −25·00
Ralli branch, average scour in front	−10·00
Maddur ,, ,, ,, ,, ...	−7·00
Vigeswaram branch ,, ,, ...	−9·00

The right flank of the Dowlaishwaram branch was always a source of anxiety. The crest of the anicut was 8·75 feet above the sills of the head sluices of the three main canals, which were built at the approximate mean bed-level of the river.

Mr. Lindley. Mr. E. S. Lindley observed that the argument contained in the first seven pages of the Paper rested on a combination of Mr. Kennedy's relation for critical velocity and depth with a formula for the velocity of flow and depth; for the latter velocity was taken to vary with the square root of the depth. In every exponential formula that had been suggested the power of the depth was not the square root, but approximately the cube root of the square, and in non-exponential formulas the variation was similar. This modification of the premises entailed a modification of the conclusion to "the true value of n must be greater than $\frac{2}{3}$, but not very much greater." He had summarized[2] work on the subject, and was not aware

[1] Minutes of Proceedings Inst. C.E., vol. ccxvi (1923), p. 288.

[2] Proceedings Punjab Engineering Congress, pp. 74g, *et seq.* Lahore, 1919.

of any suggestion that Mr. Kennedy's value of 0·64 was too low; Mr. Lindley. while Mr. Kennedy himself had anticipated lower values with greater charges of bed silt, such as occurred on rivers in flood, with which the Author principally dealt. The cross sections of the channels of any of the canal-systems of northern India were the acme of regularity when compared with the rivers of the same country. The Author dealt with them at their emergence from the foot-hills, where they were even more irregular than in the plains. The data plotted on Plate IA of the reference given above showed that the dimensions at successive points of a canal did not merely vary slightly about a mean, but showed unexpected variations in ratios up to 1·5 or even 2·0. Those who realized how capricious were the conditions even on so regular a channel, how Kutter's N and V_0 as well as the concrete dimensions varied from point to point, how the silt carried affected the velocity, and how that responded by affecting the silt carried, would be chary of such extensions of theory and hypothesis to rivers. The confirmation found in the few observations cited by the Author, which had almost certainly not been made during the passage of floods, was unconvincing. The observation on the Jumna (p.225) gave in the unrestricted section maximum and mean depths of 20·5 and 9·85 feet, and in the restricted section 25·5 and 16·1 feet. Argument, based on the unrestricted section, that the maximum depth should be 10·65 feet more than the mean would find reasonable confirmation in the actual difference of 9·4 feet at the restricted section; the agreement was closer than that of calculated and observed mean depths, which might well be expected to agree better. It was as permissible to argue that, if the flood had lasted longer, the agreement might have been closer. Measurements taken within a short distance of the sections actually taken might well have given figures agreeing with both the Author's and the above theory. The Author was attempting to calculate the incalculable. The only possible aid to the prediction of the effects of such river works, apart from rule of thumb based on experienced judgment, was the testing of a model. The Author made the statement that "if the waterway is not materially reduced, the maximum increase of velocity due to an obstruction will not exceed that due to twice the velocity head." This, in its context, had the air of being based on observation. If so, would the Author give the observed facts?

Dr. J. S. OWENS observed that in several places the Author Dr. Owens. referred to the inclination of repose of saturated material; he would like to have some further information about the exact meaning which the Author attached to this phrase. He had found experimentally

Dr. Owens. that the inclination of repose of clean dry sand was the same as that of the same sand when completely saturated and submerged in water. He was inclined to think, since the movement of such sand was a rolling rather than a sliding one over the sloping surface, that the presence of water between the particles when the interstices were entirely filled should make no difference to the angle of repose. He was aware that with intermediate conditions, when the sand was not saturated, but contained air spaces as well as water, the inclination of repose was profoundly affected by the surface tension of the films separating the water from the air in the interstices, but that was evidently not the point in the Author's mind. The Author quoted figures for the velocity of current required to move particles of sand or stone. Dr. Owens had found that the behaviour of such particles depended largely upon whether it was a case of a single particle moving over a smooth bed or of the detaching and carrying away of particles of sand or gravel from a patch of the same material. For example, he had found that a current of 0·9 foot per second was capable of rolling a small stone $\frac{1}{2}$ inch in diameter over a smooth bottom, whereas to move the same stone from a patch of similar particles required a current of more than 3·3 feet per second. It was necessary, therefore, to be careful in applying tables of velocities required to transport material unless the conditions were clearly specified. Figures quoted from Dr. Deacon's experiments gave, for example, the first movement of sand occurring at a velocity of 1·3 foot per second. Dr. Owens had found by careful measurement that siliceous sand such as was found on the seashore—about 50 mesh—would commence to move in ripples when the current-velocity was between 0·6 and 0·8 foot per second, and that the rate of travel of the sand varied as the sixth power of the mean current-velocity. Also, denoting the latter by v, the ripple moved at a velocity of $0{\cdot}2v^6$. When, however, the current-velocity reached $2\frac{1}{2}$ feet per second, the method of movement changed from ripples to movement in a continuous sheet, so that for such sand there was what might be called a critical velocity of current. Two statements were made on p. 250 with regard to the rolling of stones over the bottom: first, that pure rolling was only possible for a series of true cylinders or spheres; second, that movement of large material was partly by rolling and partly by vertical eddies. If he understood the Author aright, he was afraid he could not agree with him, since his observation of the method of transportation of coarse material over the bottom showed that it was practically always effected by rolling, and that was independent of the shape of the particle. However flat a stone might be, if the current was capable of moving it at all, it would do so by rolling, and if the stone was at all disk-shaped and the bottom reasonably hard, it would roll along on its edge like a wheel. With regard to the general theory of silt and scour advanced by the Author, he would merely say that it appeared to be based entirely upon the validity of Mr. Kennedy's formula, from observation in artificial channels, and the conclusions arrived could not be more valid than the formula upon which they were based. Dr. Owens.

Mr. R. L. PARSHALL considered that possibly the Paper would be of greater interest if the Author had given more of the original data secured by Mr. Kennedy in his research on this problem, particularly the method of arriving at the value of $n = 0{\cdot}64$. As later discussed by the Author, it appeared that from theoretical considerations the value of n should be greater than 0·5. This, then, made it difficult to see why the value 0·64 was chosen. This might have been due to observations on conditions as existing in natural streams. Mr. Parshall.

Mr. R. E. PURVES remarked that years ago, when railway-engineers in India first, and after them irrigation-engineers, adopted the Bell-bund system of training-works, the difficulty of determining the most economical extent of necessary protection was at once recognized. The only data then obtainable were the depths of scour-holes in the unrestricted natural channel of the stream. There was nothing to connect those depths with depths likely to occur in a restricted channel under conditions of both bend and swirl scour. The matter was, at first, one of intelligent guess-work, modified later by experience gained on protective work that had either succeeded or failed. He was in charge of the canal head works at Khanki during a flood in the Chenab river in 1903, which was the maximum on record, and he was instructed to take soundings of the bed scour in the vicinity of the weir up and down stream during the currency of the flood. Unfortunately, owing to the high velocities prevailing, it was not possible to take any soundings until the flood had subsided. However, by probing with a long steel rod through the wet sand down-stream, he was able to locate sunken concrete blocks. That indicated approximately the full scour depth under the action of that big flood. Scour had occurred down to a depth of 40 feet below downstream flood-water surface. Checked by Mr. Kennedy's silt equation that scour-depth was 50 per cent. greater than was necessary for silt-equilibrium. The flood passing down the weir-glacis with accelerated velocity and impinging on soft sand would account for the depth found. These data connected facts with a formula which Mr. Purves.

Mr. Purves. was probably applicable generally. During the period 1908–12 he was engaged in the preparation of designs of works on the Upper Jhelum canal. There were a number of torrent works, and protective and training works in connection with them were required. The information obtained at Khanki, 5 or 6 years earlier, was applied to these works. It was now 12 years since he retired from service, and he was not able to say how severely the works had been tried or to what extent his designs had in this respect proved satisfactory. He referred to them, however, to show how, step by step, certain advances had been made. The Author's theory shed a new light on the subject, by helping to explain many phenomena not well understood hitherto. There was a distinct advantage in connecting Kutter's formula of flow, taking into account the channel rugosity, with Kennedy's formula of silt-equilibrium. Later information might lead to some modification of the theory, but, in the meantime, it was a great advance on what had gone before. *Figs. 7* and *8* (pp. 256 and 257) showed that the deep stream passed from the centre of the channel at Kalsi to the right bank at Hatnikund. In the course of that change, the more or less level water-surface at the upper site must have given place to a super-elevated surface on the extreme right at the lower site. That would probably make some difference in the average and maximum depths. There were some other circumstances, river-floods, which appeared to have been overlooked by the Author in his consideration of the matter. As a further contribution to the subject Mr. Purves would add the following facts connected with the great flood in the Chenab river at Khanki on the 24th July, 1903:—

(1) The weir at the head of the Chenab canal was 4,500 feet long, including sluices, piers, and a groyne.

(2) The river was 1½ mile to 2 miles wide.

(3) The bed material was coarse sand dotted with grass-covered islands of moderate height above low supply level, but covered in flood time.

(4) During the flood the main current followed the cold-weather course, passing down at a point 2 or 2½ miles above the weir from right to left diagonally at the canal-regulator and sluices, and then back to right at a point 2 or 2½ miles below the weir. Thus the stream, while flowing over the entire width of the channel, nevertheless had a strong current in the form of an arc with a chord about 4 or 5 miles long.

(5) There was a cross slope of 2 feet, or 1 in 2,250, from the left to the right flank of the river. That was quite a considerable super-elevation of the convex side.

(6) The longitudinal surface slope was 1 to 2,500. Mr. Purves.

(7) The flood was the maximum recorded, being about 1½ foot above the large flood of 1893. It rose in about 4 or 5 hours, remained at the maximum for 20 hours, and then subsided in 1½ day.

(8) There was a regular periodic pulsating flow which was measured as a succession of waves each ½ mile long and 2 feet from trough to crest. The duration of passage was 4 minutes. Each wave, as it came, rose slowly against the regulator face through a height of 2 feet in 2 minutes, and then went down in 2 minutes. Thus at every cross section the depths were constantly changing.

(9) The surface velocity, between 150 and 50 feet upstream of the weir, was 11 feet per second, the mean velocity being 10 feet per second. At the sluice-face the velocity was 12 feet per second, and under the standing wave 20 feet per second. From dip to crest the standing wave was 9 feet high.

(10) The mean velocity of approach at the left flank of the weir-crest was 16 feet per second, and at the foot of the standing wave on the weir-glacis 40 feet per second. The standing wave at this part of the weir was 7 feet from trough to crest.

(11) There was an average clear fall of 2 feet between the upper and the lower surface across the weir. The clear fall was 3 feet on the left and 1 foot on the far right flank at the chord of the arc of flow mentioned before.

(12) The flood scoured down to 40 feet below the water-surface downstream of the weir, and on the flood subsiding the scoured bed had silted up to normal level.

(13) There existed before the flood a large grass-covered island about 1,000 feet by 700 feet, and about 3 or 4 feet above weir crest-level, which had been growing annually under the silting action of low floods. It masked one and partly masked two other bays of the weir. During the course of this flood it was almost wholly removed by lateral erosion. Not only so, but, over the whole upstream face of the weir, erosion took place, providing a temporary enlargement of the cross section for the flood. This was ascertained by the subsidence of blocks placed there for protection. On the flood subsiding, the scour holes were found once more filled to weir crest-level.

In view of the foregoing facts it was very doubtful to what extent the data obtained from cross sections of a large river, taken after the flood had passed, could be accepted as representing the conditions of the channel during flow. Where the surface was not level across, where that surface rose and fell in long steady equal waves, and where scouring in the rising flood and silting in the falling flood took place, the depths of supply could not be used

Mr. Purves. without modifications, and the degree of accuracy required by mathematics was not obtainable.

Mr. Spreckley. Mr. J. A. SPRECKLEY remarked that it would seem possible to extend the principle of Mr. Kennedy's formula and the corollaries deduced by the Author to a condition that occurred in flat streams which formed the outlets of certain lakes in Western Canada, where the conveyed silt-content was almost negligible, after passing through a lake or series of lakes. He referred to the growth of vegetation, which was greatest under favourable conditions of temperature and stream-stage, and continued to obstruct the stream until a scouring velocity was able to remove the accumulation of decayed and other vegetation of several years' growth. There was a marked variation in the extent of the obstruction from winter to summer conditions, but in one case which he had had occasion to study the mean velocity had only reached 2 feet per second once in 14 years. The mean depth was then 8·75 feet with a slope of 6 inches per mile, and the observed mean velocity was 2·11 feet per second. These figures suggested a low value of V_o as compared with ordinary silt conditions. If it were assumed that $m = 0{\cdot}64$, the value of c in Mr. Kennedy's formula would not exceed 0·526, and would probably be lower, thus, $2{\cdot}11 = V = 0{\cdot}526 \times (8{\cdot}75)^{0{\cdot}64}$. Data for subsequent scouring and non-scouring floods suggested that the value of c lay between 0·50 and 0·41. This corresponded very closely with the minimum velocity of 0·45 given by Mr. Thrupp for fine silt for a hydraulic radius of 1 foot[1]. If the general theory applied to the lifting and removal of decayed vegetation, it should be possible to extend the Table of coefficients in Mr. Kennedy's formula to include these conditions, and he believed that such an application would be of value in cases similar to that described. The interesting feature in this case was that Nature tended to keep the level of the lake above its normal elevation during continued dry periods, and the lake-surface was liable to fall appreciably a short time after the flushing effect of a heavy spring flood, which might occur once in 6 or 7 years and was usually associated with a wet year.

Mr. Sykes. Mr. E. F. SYKES remarked that the Paper seemed to be based on a complete misunderstanding of Mr. Kennedy's theorem. Mr. Kennedy's observations were made on a number of moderate-sized artificial channels, and the relation between their characteristics was intended to express the condition that, taking the whole year through, they would neither silt nor scour. But that was not inconsistent with considerable alternations of silting and scouring; the only essential condition was that on the year's working they

[1] Minutes of Proceedings Inst. C.E., vol. clxxi (1908), Plate 5, Fig. 2.

should return to their original state. Hence the consideration of the coefficients in Mr. Kennedy's equation at particular periods, for example, at high flood (pp. 252 and 253) was meaningless. Further, the assumption was made that the equation was universally true (cf. p. 252, "if both Kutter's and Kennedy's formulas are correct in form"). Now Mr. Kennedy was a practical man, and his formula was directed to a practical end—the design of channels similar to those observed. It was obvious that the few data available could be expressed by an equation in any form; he selected a convenient form. But though the form covered all the cases observed, it was obvious that outside those limits it might be wholly incorrect; and it did not seem probable that Mr. Kennedy would have accepted without further inquiry any extrapolation even to channels of the same type—much less to the extraordinarily complex cases presented by rivers in flood. Mr. Sykes.

Mr. Edgar C. Thrupp remarked that the Kennedy formula had been widely quoted and very much misunderstood. It was based upon observations in canals ranging from 2 feet to 8 feet 6 inches in depth, and it had been stated that the deeper channels had to carry a higher silt-density than the smaller distributaries, and had to tolerate some deposit at flood time and scour it out later in the season, while the small channels received a more uniform silt-ratio all the time. In fact, the formula was a kind of give-and-take compromise which suited the conditions of the system in a fairly practical way, but was not applicable to channels of different depths having to carry the same silt-ratio or to operate just on the verge of moving sand. For it to apply to this latter purpose it would be necessary to reduce the value of n from 0·64 to less than 0·30 and to adopt a different value of C. Any general formula for movement of silt should be able to represent the required velocities for different depths and a constant silt-ratio. To do this it must also embody the correct index of r in the formula for the flow. The Author assumed that 0·5 was suitable for the purpose of his theory, but the most comprehensive study of the subject showed that it should not be less than 0·61. On the assumption that $d^{0 \cdot 64}$ and $r^{0 \cdot 50}$ were correct values, he got the expression $d^{0 \cdot 64} - d^{0 \cdot 50}$ as a factor in the supporting power of vertical eddies, but this should really be $d^{0 \cdot 30} - d^{0 \cdot 61}$, which was a negative quantity, and therefore upset the theory. Again, the Author asserted that, if Kennedy's and Kutter's formulas were correct, a river fully charged with silt should tend to adopt a mean depth depending on the slope of the bed and the coefficient of roughness of the bed, and the *depth* should be independent of the volume of flood-discharge. Surely Mr. Thrupp

Mr. Thrupp. this conclusion was sufficiently anomalous to indicate that this Kennedy-Kutter combination was wrong. It was true that rivers raised portions of their beds in flood time. On the Mississippi it had been necessary to dredge sand-bars regularly after floods in order to restore navigable channels rapidly. There was nothing unusual in the case of the Jumna acting in the same way on shingle and boulder beds, with slopes of 1 in 140 and 1 in 300. There were similar cases in British Columbia. The Author's analysis of such cases, in which sections of the river were treated on the assumption that the mean velocities were proportional to $d^{0 \cdot 50}$, took no account of the changing mode of motion of those high-speed streams as compared with irrigation-canals. In July, 1926, the following notes were made on the Fraser river between Lytton and Lillooet at a place where the width was about 1,000 feet, and the channel was not broken by rocks. The slope was about 1 in 700, and the mid-July discharge was about 170,000 cusecs. The surface velocity in mid-stream must have been about 20 feet per second. The striking features were that (1) the level of the water in the centre was obviously several inches higher than at the sides, as anyone could see by lying down on the beach and looking across from a level about 1 foot above the side water line ; (2) the motion of the water near the side was in irregular cycloidal eddies, sometimes actually turning a little upstream, although on a slightly concave bank; and (3) the water near the bank was carrying a large quantity of air in very minute bubbles which were audibly fizzing out at the surface. The fizzing of air was not a purely local condition, because it could be observed for several hundred miles and gave the water a slightly milky appearance. Evidently, therefore, the air must be entrapped in the rippling and choppy places in the middle of the stream and conveyed to the sides by the lateral cycloidal eddies. That action might account for the remarkable absence of scour on the banks of that river, which he had already reported.[1] The Author's calculations of depth of scour caused by an obstruction being independent of the nature of the bed, the slope, or the velocity, were untenable. All scour and silt-movement depended upon eddying motion and not on direct velocity alone. The whole problem of the relationship of all the hydraulic factors to the production of eddies, and then to transporting power, was mysterious, and would remain so until much more was known about the mechanism of gravitation. In the meantime it was far better to record hydraulic phenomena on charts which avoided confusion

[1] Minutes of Proceedings Inst. C.E., vol. ccxvi (1923), p. 284.

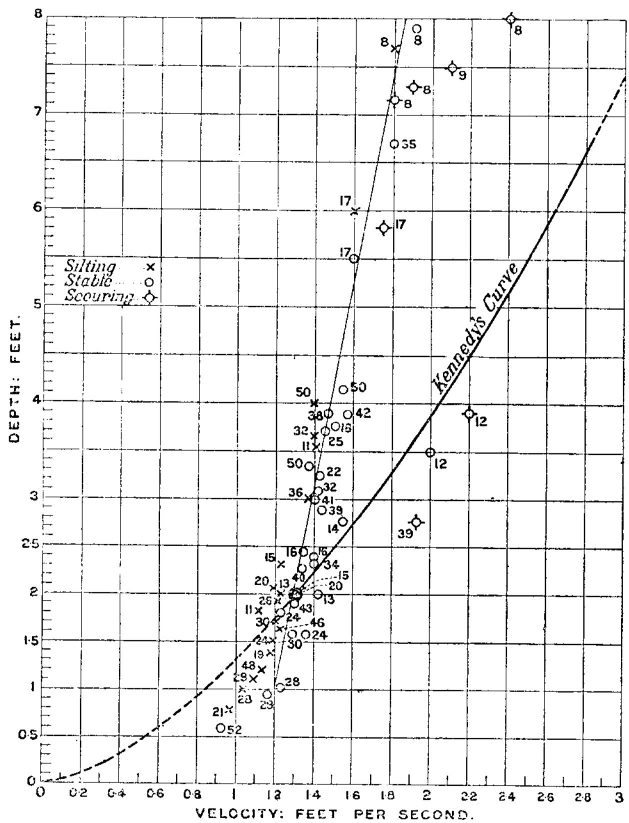
of data belonging to different zones of hydraulic conditions, involving different modes of motion. Mr. Thrupp.

Mr. W. L. C. TRENCH observed that the Paper was purely a mathematical argument based on a very limited amount of experimental data published many years ago, and apparently neither added to nor verified in subsequent years. The Author, in fact, sought to make deductions from the mathematical expression which Mr. Kennedy had derived to fit the data available to him, which had been obtained within a limited range of depths. The great value of Mr. Kennedy's work could not be overstated. It was he who showed that scientific principles could be applied to silt-laden streams, and, within the limits of his data and for the quality of the silt he had to deal with, the values obtained from his curves undoubtedly provided fairly satisfactory sections; but his expression was empirical, its form was almost certainly incorrect, and any attempt to justify it theoretically was unconvincing. It was in consequence an uncertain basis from which to draw mathematical deductions. In Table I the Author quoted velocities required to produce motion in materials of various grades, and he suggested that these values were very low. A similar Table[1] gave the following bottom velocities in feet per second: 0·25 for soft earth, 0·5 for soft clay, 1 for sand, and 2 for gravel, which seemed to be rather higher. The Author did not recognize, apparently, that these values, whatever they were, must affect the form of any formula dealing with silting. In fact, for the smallest value of depth any formula adopted must give a critical velocity equal to the minimum value required to move the material in question. *Fig. 19* gave the silting and scouring conditions for the Jamrao canal in Sind, extracted from a very large number of discharge observations taken over a number of years. The observations selected were taken between June and September, when the silt-content of the canal was a maximum, and when the gauge-reading was steady or nearly steady; and only those which gave an unmistakable indication of silting, scouring, or stability had been plotted. The following points emerged from a consideration of *Fig. 19*. Down to a depth of about 1·3 foot the location and direction of the line was clearly defined. This line produced cut the horizontal axis at about 1·1 foot per second, thus confirming the value given in the Table quoted above for the minimum velocity required to move sand. Below 1·3 foot the curve was somewhat uncertain, because it was difficult to find small channels flowing steadily for weeks Mr. Trench.

[1] Mansfield Merriman, American Civil Engineers' Handbook, p. 1109. New York and London, 1920.

Mr. Trench. at a time with a depth of less than 1·5 foot which did not silt. There was some indication, however, that a slope with a different inclination originating at a velocity of about 0·6 foot per second fitted the conditions of such small channels. The probable reason for this was that very small channels took off at a high level compared with the parent canal, and so carried silt rather than sand, which

Fig. 19.



gave a different minimum velocity. The Jamrao curve cut the Kennedy curve at $d = 2$ feet, and where $d = 1$ foot the value obtained from the Jamrao curve was as much as 1·3 times Mr. Kennedy's V_o. This result was due to the facts that Mr. Kennedy's data ended at a depth of 2·2 feet and his formula neglected the necessity of a minimum velocity in order to move the sand at all, however shallow the water might be. In *Fig. 19*, where several points bore the same number they referred to the same channel. An examination of these duplicate points showed that silting and scouring might occur within narrow limits of velocity. An increase or decrease of velocity of 10 or 12 per cent. might make all the difference (cf. Nos. 8, 17, 50, 28 and 29). The Author stated that to detach particles of such material as earth and clay from their natural bed " in any quantity " would probably require a velocity of $2\frac{1}{2}$ to 5 feet per second. That might be so, but the irrigation-engineer was equally interested in a slow, steady scour. Channel No. 15 (*Fig. 19*) scoured at a depth of 2 feet with a velocity of 1·3 foot per second, and a slow scour of even an inch a month might have serious results in a small channel. The Author's investigation of the behaviour of an irrigation-channel on the assumption that velocities of $2\frac{1}{2}$ to $3\frac{1}{2}$ feet per second were required to detach particles was quite unjustifiable. What actually happened was that the superficial $\frac{1}{8}$ inch or so of the silt sides was quite soft and ready to move with a very slight increase of velocity, revealing a fresh surface capable of being softened and carried away in turn. The whole paragraph of the Paper dealing with this point seemed to have no basis in ascertained fact. Mr. Trench,

The Author stated (p. 246) that the value of C in the upper and lower reaches of a river could only alter by the picking-up of material within its bed. The Author did not allow for the fact that the material transported by the river, especially the rolling silt, continually became finer by attrition as it proceeded towards the sea. This could be seen from the sieve analysis of sand and silt from the Irrawaddy.[1] Consequently, C might alter for physical causes rather than conform to a variation in the slope. And the Authors' " reasonable " anticipation that silting in the lower reaches might be prevented by the construction of training-works in the upper reaches did not appear to be a necessary or even a likely consequence; in fact, its improbablity would cast grave doubts on the correctness of the theory advanced, if such doubts did not already exist. The discrepancies pointed out on p. 252 further accentuated the doubt regarding the universal applicability of the formula embodying Mr. Kennedy's theory and the mathematical deductions built on it. Indeed they appeared actually to demolish any such claim.

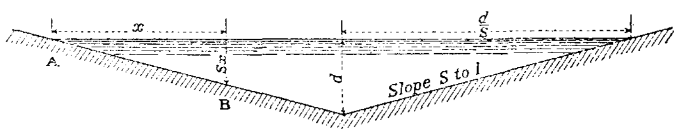
The Author. The AUTHOR, in reply, observed that several correspondents (Messrs. Ballester, Gerald Lacey, Lindley, Sykes, Thrupp, and Trench) had raised the objection that the theory advanced depended for its establishment on Mr. Kennedy's law, which had been proved

[1] R. B. Buckley, "Irrigation Pocket Book," p. 127. London, 1920.

The Author. in relation to a different set of conditions, and which therefore was not applicable. Messrs. Ballester, Lindley, and Thrupp also pointed out that the velocity in a river section flowing under the conditions named had been found to vary not as $d^{0 \cdot 5}$ (as should follow from Chezy's fundamental formula), but as approximately $d^{2/3}$ or not less than $d^{0 \cdot 61}$, and they considered that this vitiated the deductions made. From this it appeared that the Author had failed to make clear the evidence on which he based his theory (pp. 243 to 245); he would therefore endeavour to explain the matter more clearly, even though that would involve some repetition.

As a preliminary, consider clear water flowing in a straight unerodable flume of a uniform shelving section, the stream-lines flowing parallel, and for simplicity's sake assume this shelving section to be triangular (*Fig. 20*), the bed sloping to the centre

Fig. 20.



with a slope of S to 1. Let d be the depth of the section at the centre point. Then the depth at any point B distant x from the side A would be Sx. Assume the velocity at any point in the cross section to vary as the (depth)^K, so that the velocity $= C(Sx)^K$.

The discharge of this flowing section $= Q = 2\int_{d/S}^{o} C(Sx)_K Sxdx =$ $2Cd^{K+2}/(K+2)S$, the area of the section being d^2/S. The mean velocity of flow $= 2Cd^K/(K+2)$. The hydraulic mean radius of the section, $r = \frac{d^2}{S}\,\frac{1}{2\sqrt{\{d^2+(d^2/S^2)\}}} = \frac{d}{2\sqrt{(S^2+1)}}$. If the section was broad and shallow, S was small, and R differed little from $d/2$ and could be considered equal to $d/2$ approximately; so that if the mean velocity of the section varied as r^K, the velocity at any point would vary approximately as d^K.

Now consider the actual case of a river in flood (p. 245), on which this theory was based. Consider any cross section of the river in a straight reach. The section would be found to be shelving (*Fig. 2*, p. 245), but the channel differed from the one just considered, because the bed was erodable, the supply was heavily charged with silt, and the section was not uniform and so flow was not parallel. Observation showed that the section changed with changes in flood-level, but for a steady flood-level it had some stability. The theory advanced was based on the observation that if the velocity of flow at any point in such a section was reduced, the depth automatically decreased, and vice versa. The Author.

It would be conceded that in any flowing section the velocity would vary with r^K, and therefore in a uniform unerodable section the velocity at any point would equal $C\,d^K$ approximately, which might be termed the law of flow. It followed that the law of critical (non-scouring and non-silting) velocity must for all depths follow the law of flow very closely, and it might therefore be written $V_o = C\,d^n$. From the observation that all such rivers adopted naturally a broad shallow section, it followed that n must be greater than K.

The fundamental formula of flow was Chezy's law $V = c\,\sqrt{rs}$ and for uniform unerodable sections it would, the Author believed, be conceded that V would vary as $r^{0\cdot5}$, so that under such conditions, if the section was broad and shallow, the velocity at any point would vary as $d^{0\cdot5}$.

Consider some point in the non-uniform, erodable section as a river-bed under flood conditions. The velocity at the point was not governed by the depth at the point, but by the velocity on the line of approach. On the other hand, the depth at the point was directly governed by the velocity at the point. The ratio between depth and velocity therefore in an erodable non-uniform section must be governed more by the law of critical velocities than by the law of flow.

The contention, therefore, that this ratio was found to vary approximately as $d^{2/3}$ or not less than $d^{0\cdot61}$ appeared to confirm and not refute the Author's theory; and the value of the index agreed closely with the value of m found by Mr. Kennedy in his channel observations, which had been provisionally accepted by the Author in his equations.

The Author's theory would further appear to explain why Chezy's fundamental formula had not proved completely satisfactory as a basis for calculating discharge of river-sections, though satisfactory as a basis for non-erodable uniform sections such as pipes and conduits.

The Discussion and Correspondence were interesting in bringing out the range of values of c and m in Mr. Kennedy's law, applicable to irrigation-channels in different countries, the equation ranging from $V_o = 0{\cdot}52\,d^{0\cdot44}$ (Mr. Ballester, Argentina) to $V_o = 0{\cdot}283\,d^{0\cdot727}$ (Mr. Ghaleb, Egypt), both in metre-second units. The Author attributed these wide variations to the difficulty of selecting channels for observation which had all the same silt-content (that was, the same value of c), because variations in the value of c made the accurate

The Author. determination of m impossible. These two extreme equations gave the same value when d was 1·32 metre = 4·34 feet; that was, the curves crossed at this value of d, which was about the mean point of the range of the observations. That was what might be expected if one or both sets of observations were too irregular to give correct direction to the curves.

It was pointed out by Mr. Ballester that, according to his formula for critical velocities, the greatest depth of a canal that could carry a heavy silt-content without eroding its banks was 38·5 feet, as against 12 feet according to Mr. Kennedy's law. He did not state, however, whether any canal working under those conditions actually existed. On the other hand, in India a depth of 12 feet had been found to be about the practical limit, which indicated the correctness of Mr. Kennedy's law.

The difficulty found in establishing a law of relationship between depth and silt-content did not appear to affect the theory under discussion. The silt-content might vary during the period of a flood, but at a maximum stage the mean content might not differ greatly in different floods.

The series of observations given by Mr. Trench (*Fig. 19*) had been taken at sites where the value of c obviously varied. For example, comparing two observations at sites noted as stable, No. 8 gave a depth of 7·9 feet for a velocity of 1·9 foot per second, and No. 12 a depth of 3·5 feet for a velocity of 2 feet per second. Such observations appeared to be valueless for accurately determining the value of the indices. Since, owing to the draw-off of silt by irrigation-outlets, the value of c might range from 1·09 to 0·84 in a few miles of the same channel, and since alteration in the design of the head might cause a silting channel to scour its bed by reducing the value of c, it was obviously very difficult to select sites on different channels all having the same value of c. That Mr. Kennedy's sites were carefully selected was shown by the fact that the points all fell so near the line, but the value of the indices so found could not be considered as exact.

He considered that it had not been proved that the value of the index varied for river-beds of different materials, but the greater stability of river-beds composed of material varying widely in size pointed to its having a higher value in such river-beds than in those formed entirely of fine light material. The equations for river-control (pp. 254 and 261) had been framed to exclude the variable c, and from the evidence of the application of those equations to practical examples within the range of his experience, he was led to believe that if the value of the index n (or m in Mr. Kennedy's notation) varied, the variation was slight and would lie between a maximum of about 0·64 for sand, shingle, and boulders, and, say, 0·58 for fine silt. The Author.

His observations on the values of c for different river-beds were incomplete, but he advanced the following Table as a rough indication of the variation in these values; these, however, were not based

Serial No.	Nature of River-Bed.	Value of n assumed.	Value of c.	Mean Depth on which Value of c was Based.	Remarks.
				Feet.	
1.	Large boulders, shingle, and heavy sand.	0·64	4·0	6 to 10	Jumna river above Kalsi. (Flood of September, 1924).
2.	Medium boulders, shingle, and sand.	0·64	2·3 to 2·8	10 to 16	Jumna river above Tajewala.(Flood of September, 1924.
3.	Small boulders, shingle, and sand.	0 64	2 0	14	Ganges river at Bhimgoda. (Gona flood.)
4.	Heavy sand.	0·64	1·2 probably.		
5.	Medium sand.	0·64 or 0·60	1·0 1·04	4 to 12	Jumna river N.W. Ry. bridge between Kalanour and Sarsawa.
6.	Fine sand.				
7.	Light silt.	0·64 or 0·58	0·62 0·82	95	Suggested by figure for Hardinge bridge.
8.	Decayed vegetable.	0·64 or 0·58	0·526 0·6	8·75	Figures suggested by Mr. J. A. Spreckley for W. Canada.

on sufficient data for accurate determination. Serial Nos. 1, 2, and 3 were based on observations of river-sections taken after the subsidence of high floods, the flood-volumes having been calculated by Kutter's formula; No. 5 was based on observations of actual depths and of mean velocities deduced from observed surface velocities during a high flood; Nos. 9 and 10 were figures suggested by observations cited in this discussion.[1] He suggested that the

[1] Colonel Sir Gordon Hearn cited a surface velocity of 9·18 miles per hour at the Hardinge bridge as giving a depth of scour of 95 feet. A surface velocity of 9·18 miles per hour (= 13·5 feet per second) corresponded with a mean velocity of $0{\cdot}86 \times 13{\cdot}5 = 11{\cdot}55$ feet per second, and this agreed with either of the two equations $V_0 = 0{\cdot}62\, d^{0{\cdot}64}$ and $V_0 = 0{\cdot}82\, d^{0{\cdot}58}$.

The Author. interesting action creating sand dunes, described by Mr. Claxton, was a secondary action only; the depth of such dunes was only an inch or two, and the action appeared to be unimportant in considering depth of scour.

The equation for critical velocities advanced by Mr. Gerald Lacey ($V_o = 1{\cdot}62\, r^{0{\cdot}5}$) was novel, but did not appear to fit observed facts. According to this formula all channel sections having the same value of r would, on the same bed-slope, carry their silt-content equally well, whether the section was deep and narrow or broad and shallow, which was contrary to common experience of channels carrying heavy silt-content. A channel whose supply was heavily charged with silt must tend to adopt the cross section which gave it the greatest silt-carrying power, and experience showed that under such conditions it tended to adopt a broad shallow cross section and not a semi-circular cross section, which latter would give it the maximum silt-carrying capacity if Mr. Lacey's formula were correct. This point was dealt with also in the Author's reply to the discussion (p. 276). Mr. Lacey's suggestion, that the flattening action in a river-bed which caused it to adopt a broad shallow cross section could be explained by the drift of heavy material to the deep centre where the velocity was highest, appeared to be inadequate, because on account of the higher velocity there it was from the deep centre rather than from the shallow sides that the rolling silt must be mostly picked up.

Mr. Lillie's observation, cited by Mr. J. M. Lacey, that the mean velocity of a river from its source to the sea was approximately constant, if correct, was not inconsistent with the theory advanced on pp. 251 and 252, where it was argued mathematically that a river tended to adopt a mean depth depending on the slope of the bed (the flatter the slope the greater the depth), which depth, based on Kutter's formula and coefficients, was given by equation (1), p. 252. This equation did not completely fulfil Mr. Lillie's law, but if Mr. Lillie's observations were proved to be correct and Kennedy's law were found, as the Author claimed, to be of general application, these two laws might throw light on the correct form of the equation for river-discharges. The Author's contention that the velocity was higher in upper reaches where the bed-slope was steeper, referred to velocities for the same depth, and not necessarily to the mean velocity of the river sections.

He was unable to agree with Mr. J. M. Lacey's suggestion that when a stream was fully charged with silt the vertical eddies might be completely damped out; for, if there were no vertical eddies to support the silt, the stream could not carry it. Neither could he agree with the contention that the velocity of approach did not depend on the width of a weir: if there was no increase in velocity due to contraction, why should there be scour or, as Mr. Lacey described it, the scooping-up of silt in front of the weir. The Author.

With reference to Mr. Lindley's query, the contention that if the waterway was not materially reduced the maximum increase of velocity due to an obstruction would not exceed that due to twice the velocity head of the stream (p. 261), was made on theoretical grounds. When a stream impinged on an obstruction, the velocity was checked on the upstream side and was partly recovered in the form of head (as in a Pitot tube). This recovered head, acting on the flow passing the obstruction, increased its velocity. The maximum head recoverable was that due to the velocity of the approaching stream; the maximum velocity generated in the flow passing the obstruction by this action was therefore that due to twice the velocity head of the approaching current.

The resulting equation evolved, namely, that the maximum total depth of scour (below water surface) due to this action $= 2^{1/2n} d_1$, where d_1 was the depth of the approaching stream, or $1 \cdot 72\, d_1$ assuming $n = 0 \cdot 64$, had however been verified by observation of spurs at Kalsi, on the Jumna river (bed of large boulders, shingle, and sand) and at Narora, on the Ganges river (bed of medium to fine sand). The deduction from this theoretical equation, that the ratio of maximum scour to depth of approaching stream was independent of the nature of material forming the bed (unless n varied), at first sight appeared unlikely to prove correct in practice, but evidence appeared to confirm it.

With reference to Dr. Owens's question, he suggested that the angle of repose of saturated material was the angle of repose that material adopted when deposited in water; and experience indicated that it was flatter than the angle of repose of dry material.

Mr. Purves's data relating to the passage of a great flood over the weir at the head of the Chenab canal were interesting, especially those of the phenomenon of periodic pulsating flow, the theory of which was obscure. The velocity determining the depth of scour below the weir would be the velocity of the stream leaving the unerodable talus and impinging on the erodable bed, which was not given. Mr. Purves mentioned that lateral erosion occurred during this flood, which indicated that the weir was not set normal to the flow of the river; under such conditions the Author's equations could not be applied correctly.

Apparently Mr. Trench had misunderstood the general basis on which the Author's theory was advanced. Clear water flowing

The Author. with a velocity of 1·3 foot per second might in time have some scouring effect on clay, just as a steady drip of water would in time wear away a stone; but such action was too slow to be taken into account in the conditions under consideration, namely, a river-bed changing its section in conformity with rapid changes of flood conditions. Observations showed that the shortening of the course of a river by short-cutting of a loop led to scour at the point and silting of the reach below; and so it appeared reasonable to expect that artificial lengthening of a river in one reach would reduce silting in the reach below.

In conclusion he wished to thank those who had taken part in the discussion for the information they had contributed bearing on the matter under consideration; but he regretted that, save Colonel Anderson, none had applied the equations to practical examples, by which method alone the validity or otherwise of the theory could be satisfactorily decided. The equations had been mathematically deduced from one elementary law. They obviously could not give exact results, because they did not provide for variations in velocity due to changes in direction from unknown causes. If the theory was correct, however, they provided some basis for mathematical calculation of the changes in river sections which might be expected to result from the adoption of artificial control in places where none existed at present.