Correspondence.

Mr. Blyth. Mr. C. E. Blyth remarked that, although he quite understood that, in studying the stresses in reinforced-concrete structures by means of experiments with transparent models, results could be obtained which might be taken as truly indicative of the stresses in the respective members of the actual structures, he failed to see how such methods could be used with the same reliability for ascertaining the stresses in the various members of a riveted structure. Models to be handled in a research-laboratory must necessarily be of small scale, and in the cases of the junctions of members, the smallness of the details of riveting, by reason of the scale, would render it exceedingly difficult to obtain reliable indications of stresses in members at the position of rivet-holes; and it was at such positions that it was most difficult to calculate stresses with any accuracy. He was of the opinion that instruments for the measurement of stresses in members of actual structures by recording the vibrations in them could by improvement be made to give more reliable results than hitherto; the Thomas instrument particularly was capable of development. Possibly apparatus could be evolved comprising two separate parts, one a transmitter or vibrator to be attached to any member of a structure, and the other a recording portion connected to the transmitter or vibrator by electric wires, permitting it to be placed in positions where there would be no external disturbing influences. The greatest difficulty to be overcome would be in determining the scale for measuring the records accurately.

Mr. Shaddock. Mr. W. T. Shaddock remarked that the agreement between the calculated and the observed stresses shown in *Fig. 6* appeared to be very good, especially in the important cases of the sections subject to maximum stress, with the exception of the left-hand abutment, where there was apparently a difference of about 25 per cent. in the stresses for the intrados. Such satisfactory results of the experimental determination of stress might justify its application in practice to the open type of reinforced-concrete bridge, as had already been done, for example, with Professor G. E. Beggs's method for the Grandfey viaduct, Switzerland.[1] But could similar results be obtained with models of the filled type of arch, where there was such a large element of doubt about the distribution, on the actual

[1] *Bulletin Technique de la Suisse Romande*, vol. 51 (1925), p. 301; Journal Franklin Inst., vol. 203 (1927), p. 375. (Engineering Abstracts, **26-27**, 178; **32**, 83).

arch, of the transmitted live load? A not inconsiderable number of small-span (100 feet downwards) reinforced-concrete bridges of this type had recently been constructed, and an analysis of their design led to the belief that many erred somewhat heavily on the side of safety, with consequent waste of material, which would probably never be called upon to bear stresses approaching what would be permissible for concrete and steel of the quality used in the structures. With respect to small arches, this objection had frequently evoked the reply that, although theoretically the thickness of an arch might be less than 12 inches, that dimension was about the minimum to which concrete could be poured with satisfactory results. But even in the case of short spans there appeared to be no insurmountable difficulty in the way of evolving a more economical thickness, based upon accurate information as to the maximum stresses which might be produced. For, apart from reductions of section justified by stricter grading of the aggregate and closer supervision during pouring, with the development of rustless steels the concrete normally introduced as cover for the reinforcement, and assumed to do no useful work in bearing tensile stress, might in time be eliminated by using special forms of reinforcement as centering. It would therefore be not only interesting but also probably of value to designers of filled arches to have the results of the application of the Author's methods to that type, or necessary modifications of those methods. They would at least form useful data for checking a design based on rules such as those given[1] by Sir E. Owen Williams, M. Inst. C.E., which, being built up on the elastic theory, had done much to reduce the labour involved in the orthodox design of the indeterminate arch. He could not quite reconcile *Fig. 7* with the Author's statement (p. 46) that the method of plotting the stresses showed to an exaggerated scale the positions the members took up under load. An examination of *Figs. 7* and *8* seemed to indicate that, to produce that result, compressive stresses would have to be plotted inwards, and not outwards, from the boundaries of the members, and vice versa for the tensile stresses. Mr. Shaddock.

Mr. W. H. Thorpe remarked that the chief value of the Paper to engineers, apart from its interest as a carefully conducted study, lay perhaps in its connection with stresses developed in concrete structures. Long experience showed that in steel bridgework the existence of localized super-stress due to the distortion of diagonally braced frames did not lead to serious results, so that it would be Mr. Thorpe.

[1] "The Philosophy of Masonry Arches," Inst. C.E. Selected Engineering Paper No. 56 (1927).

Mr. Thorpe. difficult to find any instance of failure from that cause. Structural steel was so ductile that in cases of angular distortion of diagonally braced structures it could suffer stretch in particular places—provided the stretch was not progressive—without harm. With open frames, however, of the type shown in *Fig. 7*, having no diagonals, it was essential to know and provide for the maximum bending-moments induced, as in such cases yielding did not give relief. There was, indeed, a vital difference between the two types of frame with respect to that matter. Reinforced concrete, lacking as it did ductility in any useful degree, was liable to develop tension cracks which, though objectionable, were fortunately of minor importance, as the tensile strength of concrete was not commonly taken into account; but there were cases in which it was desirable to limit the tension in concrete in order to avoid such cracks, and the Author's methods of measuring stress might then be of much real value. The use of rubber models in the study of stresses in dams, in which average values of stress were determined across circles of appreciable size, would seem to be fully justified, because the material of dams was itself liable to variations in ultimate resistance which might be perhaps 25 per cent. above or below the mean value, so that any great precision would seem to be unnecessary.

Mr. Vivian. Mr. A. C. Vivian asked the Author to be good enough to give stress-strain curves of the nitro-cellulose he used, which he had no doubt already determined by direct measurement in tension and compression. He considered that it was important to emphasize that the measurements of stress given in the Paper were really measurements of strain, translated therefrom into stresses by means of very ingenious apparatus. But the stress-strain curves of most materials of construction were not straight lines, and although the curvature of the lines might be only slight, it was by no means certain that the resulting translation from strain to stress was not appreciably in error unless the curvature was taken into account. Professor Love's mathematical theory of elasticity, cited by the Author, was founded on Hooke's law. It was, therefore, important to qualify the Author's proviso on p. 52 with the statement that his comparisons were only true for materials which obeyed Hooke's law. It would be a great help to engineers to establish the extent of the errors which would be produced by applying the method of stress-analysis employed in the Paper to materials of construction, such as concrete or high-tensile steel, which did not obey Hooke's law within their elastic limits. A proof that such deviations in the cases of the materials mentioned were negligible would remove a doubt as to the efficacy of the stress-analysis advanced. Mr. Vivian.

The AUTHOR, in reply, remarked that the examination of stress-distribution in members such as occurred in riveted structures had already been dealt with in the oral discussion. It was therefore only necessary to deal with one further matter of criticism by Mr. Blyth, who had remarked on the apparent difficulty of examining detailed distributions round rivet-holes and similar discontinuities. If the total forces expressed as direct loads, bending-moments, and shears were known, and could be applied to a plate, then it was not very difficult to determine the stresses in a model at points 0·001 inch apart, and it was for such cases as rivet-holes that photo-elastic methods were especially useful. When stressed plates were superposed, the measurements were usually not practicable. He was not able to offer any opinion on the measurement of stresses by the aid of measurements of vibrations in structures, or the possibility of improvements in apparatus of this type, nor was any reference made to that kind of investigation in the Paper. The measure of agreement between calculation and measurement shown in *Fig. 6*, as obtained by Mr. Field, was satisfactory, as Mr. Shaddock remarked, and especially so having regard to the fact that the model used was very small and the measuring apparatus less perfect than that used in later investigations on other bridge models. It would not be difficult to find the stress-distribution in the arch-ribs of bridges of the filled-in type, as that form of loading could be applied by the aid of very fine shot piled up on the rib to give the required intensity of pressure. It would probably be worth while to investigate the stresses in small bridges of the type mentioned by Mr. Shaddock to find whether they could be designed and built more economically. The remarks on plotting the stresses (p. 46) were intended to apply only to the vertical members of *Figs. 7* and *8*. There was no doubt that the mild steel used in bridges had been remarkably accommodating in taking care of the very high stress-concentrations that some designs produced, owing to its valuable plastic properties; but such properties were feeble in concrete and in the comparatively high-tensile steels now coming into use, so that more care in design would probably be required in the future. From a general point of view it seemed very unsatisfactory to regard experimental errors of ± 30 per cent. as of no real importance, whatever the nature of the material, and in the Author's view ± 5 per cent. was a fairly wide margin of error, and it was still better to reduce this to ± 2 per cent., or even less, if possible. The stress-strain properties of nitro-cellulose had been examined with great The Author.

The Author. care and detailed results had been published,[1] so that this information asked for by Mr. Vivian was already available. The investigation mentioned showed that, provided the stress did not exceed 2,000 lbs. per square inch, the stress-strain curve could be regarded as a straight line without sensible error, and care was taken to keep the stresses below that intensity as far as possible, although occasionally at places where discontinuities occurred the stress exceeded that limit locally, and the measurements at such places were therefore slightly lower than the true values for a perfectly elastic body. It was not necessary to qualify the statement on p. 52, since the generalized Hooke's law was implied or expressed everywhere in the Paper. The deviations from a straight-line law observed in various kinds of steel used in bridgework were so very small for the range of the working-stress that it appeared to be an unnecessary refinement to take these deviations into account in practical applications. The term "concrete" was, however, a generic name for a variety of materials differing in their physical properties and their behaviour under load according to the materials used and the methods employed in manufacture and moulding. He believed that it was perfectly true to say that all engineers with a due sense of their responsibilities used a high grade of material, which in a few weeks after moulding might be regarded as substantially an elastic solid, under the stresses it bore, and that the stresses might be taken to be those which could either be calculated —as in fact they were in practice—or measured by aid of the elastic properties of other bodies, such as were described in the Paper. The doubt expressed by Mr. Vivian whether the stresses could be found by one or other of these ways was, possibly, purely subjective.

[1] "The Stress-strain Properties of Nitro-cellulose and the Law of its Optical Behaviour." See Appendix, p. 53.