Mr. Criswell. that the limiting rate should be 1 in 20. In Mr. Criswell's view that would be insufficient for curves of small radii and large deviation-angles. On the other hand, there was much to support Mr. Bennett's view that greater rates of banking were required, for curves on moderate or long radii, than those given in Table VI (p. 167). Road speeds had, however, greatly increased since 1929 when the Table had been prepared.

Mr. Bennett rightly pointed out that one of the advantages of the long circular arc with suitable transitions was that the rate of banking was uniform. In the wholly-transitional curve of equal speed-value and with smaller central radius, the rate of banking was constantly changing, necessitating extra labour and expenditure in construction.

Correspondence.

Mr. Adams. Mr. W. F. ADAMS remarked that, although controversy raged regarding the relative merits of various types of transition-curves, Mr. Criswell had submitted comparisons which showed that, for radii and deflection-angles likely to be met with in practice, the differences between the curves themselves (not considering such relatively intangible matters as radii, which had only secondary significance), were to be measured in inches only. Professor Royal-Dawson, in arguing that the spiral curve of Mr. Criswell was a very close approximation to the lemniscate, had also logically proved the converse. Since the differences between the curves were so slight, the reasonable course was to leave the type of transition-curve to be used to the discretion and convenience of the engineer concerned.

The one point of paramount importance was that transition-curves of some type should be provided. No engineer would deliberately propose to make a bottleneck on or at the end of a road-curve, yet that was in effect what was done in joining a circular curve to a tangent. In providing such a road-alignment, which as a matter of simple mechanics could not be followed throughout its length by a vehicle, it was made impossible for the vehicle not to depart from the alignment at some point in its length ; as a result the effective width of the road was reduced by the amount of that departure. Unduly emphatic advocacy of any one form of road-curve might actually tend to defeat the common purpose of all advocates of transition-curves on roads, by making it appear to persons uninstructed in the subject that it was an abstruse one, bristling with difficulties and debatable points. That false impression did undoubtedly exist. Mr. Adams

The history of road-alignment was of some interest in connection with the question under discussion. In the first stage roads simply "grew" under the action of pack-horse and (later) of wheeled traffic. In the second stage motor-traffic appeared, the old alignments were no longer suitable, and organized road-construction became necessary. Sections of the roads were straightened, and curves were put in "by eye." By that method transition-curves were automatically introduced, since the eye interpreted the sudden change from a tangent to a true circle, seen foreshortened, as a "bump." There was thus a tendency for that bump to be smoothed out, giving a crude form of transition-curve. In the third stage the elementary ideas of curve-design were introduced, and the curves were set out as true circles joining the tangents. As pointed out above, the effect was to create a virtual bottleneck, and the result from that point of view was less satisfactory than the curve put in by eye, and much less satisfactory than proper curves of transition, with or without circular arcs. The fourth and desirable stage was the provision of complete transition-curves.

Mr. HERBERT ADDISON desired to offer a few comments from the point of view of a road-user. He would first like to ask whether highway-curve design had been considered from the point of view of facilitating the general flow of the traffic stream by making it as easy and safe as possible for one vehicle to overtake another. Although the maintenance of a uniform speed was an ideal which the road-engineer ought to keep in mind, yet in practice the progress of a normal private car was far from uniform; it alternated between periods at high speed on a clear road, and periods at a much lower speed while waiting for an opportunity to overtake a lorry or omnibus. The greater the acceleration of the overtaking vehicle, the more quickly could it regain its normal station near the left curb or verge (in England), and thus cease to be a potential danger to oncoming traffic. In general, everything should be done to encourage overtaking on straight stretches of road, and to discourage it on curves; for on curves the conditions were in every way unfavourable for manœuvring. Might it not therefore be preferable, in a road which had inevitably to follow a serpentine course, to choose deliberately an alignment involving relatively sharp curves with intervening straight lengths, rather than a succession of gentle curves? Mr. Addison.

Perhaps the Authors would say whether they had studied the

Mr. Addison. relationship between curve-design and the steering characteristics of motor-vehicles. The natural transition-curves which drivers were assumed to execute would be to some extent governed by the rate of turn of the steering-wheel. Supposing that one-quarter of a turn of the steering-wheel was required to set the front road wheels in the proper position for negotiating a curve of given radius, that amount of " lock " could not instantaneously be applied; the driver would prefer to make the movement unhurriedly, and while he was making it the car would be describing a transition-curve. The tendency at present was to supply steering mechanisms in which a large angular deflection of the steering-wheel was required to create a given angular deflection in the path of the car, so that the driver would tend to follow a relatively long transition-curve.

The reaction of motor-drivers to road-curves was also influenced by the degree of " castor action " of the front wheels, for it was that castor action that made drivers sensitive to camber or superelevation. When the vehicle was travelling on the cambered part of a straight road, it tended to run downhill into the ditch, and that tendency had to be consciously resisted by the pressure of the driver's hands on the steering wheel. On banked curves, the car would tend to run up the banking and off the road on the outside of the curve if its speed were above a certain value, and down the banking if the speed were below it. Yet another consideration was the amount of elasticity in the steering connections of the vehicle, owing to which, even on a straight road, a straight course could not be ensured by holding the steering-wheel rigidly. Any change in the road camber or in the transverse wind-pressure on the vehicle would alter its course unless the driver compensated it suitably. On the other hand, it had to be remembered that at high speeds the gyroscopic properties of the front wheels themselves tended to keep the car on a straight course.

He noted with complete approval that both the Authors deprecated an amount of superelevation or crossfall greater than 1 in 10. On roads recently laid out on the Continent, the steeper rate of banking sometimes adopted was apt to be disagreeable or even dangerous. Although they were perhaps suited to the fastest type of traffic, yet, because of the castor action mentioned above, the effect of steeply-banked curves was to give a feeling of insecurity to a driver of a relatively slowly-moving vehicle; he either had to take the bend more quickly than he liked, or else to pull hard at the steering-wheel in the effort to prevent his vehicle running down the banking. The effect was intensified when the curve formed part of a long slope, in which ascending traffic had to take the outside of the curve. A heavy vehicle mounting slowly in an indirect gear required a very conscientious driver if it were not to trespass in the path of fast downhill traffic; for not only had he to make the exertion of hoisting his vehicle (as it seemed to him) up on to the banking, but the superelevation itself might increase the longitudinal gradient to the extent of forcing him to change to a still lower gear. Those possibilities had been forced on Mr. Addison's notice in a sufficiently alarming manner during a recent journey through Spain. He knew, too, of roads in the Near East where the banking was so exaggerated that, when the road became muddy and slippery in bad weather, even a well-found and skilfully-driven touring car would get out of control and literally slide crab-wise to the bottom of the banking. It did, therefore, seem that the lay-out of transition-curves and the provision of banking should be governed by local conditions rather than by universally-applicable rules. Mr. Addison.

Mr. A. J. H. CLAYTON observed that whilst the Papers showed the desirability of providing transition-curves, the actual form of the curves to be used appeared to be purely a matter of convenience. Mr. Clayton.

On curves of relatively small radius, where the sideways force was greatest, superelevation, if kept within the practical limits, could only contribute a small part of the resistance to the sideways force. Nevertheless, it was desirable to employ superelevation, even on large-radius curves, if for no other reason than to avoid having, in effect, adverse superelevation on account of the camber.

If the superelevation were made one-half of that theoretically required for the maximum speed for which the road was designed, then the sideways force outwards on a vehicle travelling at that speed would be equal to the force causing a tendency to slip inwards in the case of a very slow vehicle, and the slope would be ideal for a speed equal to the maximum speed divided by $\sqrt{2}$. That appeared to be an equitable arrangement, and would meet the point often raised that roads should not be designed for very fast vehicles at the expense of the slower ones.

With, for example, a maximum superelevation of 1 in 12 and a maximum speed of 50 miles per hour the maximum superelevation should be used up to a radius of 1,000 feet. Below that radius the speed would have to be reduced for comfort, and above that radius the superelevation would be calculated from the formula $\frac{S^2}{30R}$ (S denoting the speed in miles per hour and R the radius in feet).

Mr. E. H. CORNELIUS, referring firstly to the Paper by Professor Royal-Dawson, observed that his comments would be based on his experience in designing suitable curves for the Rajpur–Mussoorie motor road. That road was situated in mountainous country in the United Provinces, India, approximately 6,000 feet above sea- Mr. Cornelius.

Mr. Cornelius. level, and thus provided unique problems in the layout of lemniscates, which were used for all the curves.

Professor Royal-Dawson's speed–chord formula was suitable only for ascertaining, with respect to each deviation-angle, the location of a curve which would conform to Shortt's standard. In a very large number of cases, however, a curve with such large radii either did not exist or could not be provided at any reasonable cost. In such cases the speed–chord formula tended to confuse the issue when the question of superelevation had to be considered.

With regard to the Toronto traffic diagram, Professor Royal-Dawson had apparently based his comparisons on the 600 south-bound vehicles. The radii of the western half of the curve varied from 123 feet to 150 feet, and the legal speed was stated to be 20 miles per hour. Neglecting friction, the superelevation should have been from 1 in $4\frac{1}{2}$ to 1 in $5\frac{1}{2}$ approximately, but there was actually an "infra-elevation" of 1 in 33. Motorists naturally swerved over from the western half, where such disastrous conditions obtained, to the eastern half with its level table. Professor Royal-Dawson should have excluded that group, and should have used for his purpose the north-bound traffic of 280 vehicles which used the level table of the eastern half.

Professor Royal-Dawson had stated that the difference between the centrifugal ratio and superelevation took the form of a side-thrust balanced by lateral adhesion, and worked out the lateral adhesion by a process of simple subtraction, arriving at 0·11 in his example on p. 139. Mr. Cornelius's experience of the difficulties of the curves of the Rajpur–Mussoorie motor road had driven him to the conclusion that Professor Royal-Dawson, in common with many highway engineers, had not realized that both the superelevation and the lateral adhesion were measured by tangent curves, and therefore could not be separated by the simple process of subtraction. Moreover, his suggested 0·4 rule indicated that he had not yet realized the fundamental equations governing superelevation. Mr. Cornelius had examined the question by the following method.

In *Fig. 8* AGB represented the plane passing through the axle of a vehicle whose wheels touched the road-surface OY at A and B and whose centre of gravity lay on an axis through G.

P_1 and P_2 denoted the pressures of the wheels on the road,
μ ,, ,, tangent of the angle of lateral adhesion,
f ,, ,, outward centrifugal reaction,
W ,, ,, weight of the vehicle, acting vertically downward through G.
θ ,, ,, inclination given to the road surface,
v ,, ,, velocity of the vehicle in feet per second,
V ,, its speed in miles per hour,
R ,, the radius of curvature for which the inclination θ was necessary.

Then Mr. Cornelius.

$$f = \frac{Wv^2}{gR}.$$

In terms of miles per hour, giving the value 32·2 to g, the centrifugal ratio was :—

$$\frac{f}{W} = \frac{V^2}{15R}.$$

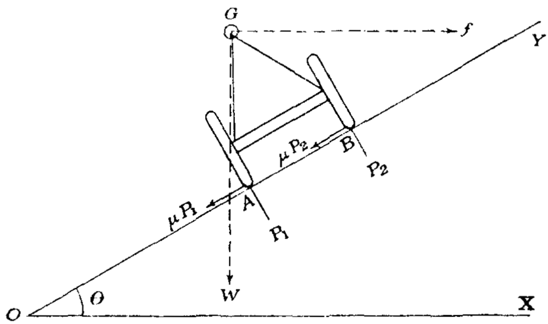
The horizontal components of all forces had to be balanced, whence

$$f = (P_1 + P_2)\sin\theta + \mu(P_1 + P_2)\cos\theta\,;$$

similarly, the vertical components had to be balanced, whence

$$W = (P_1 + P_2)\cos\theta - \mu(P_1 + P_2)\sin\theta.$$

Fig. 8.



Thus

$$\frac{f}{W} = \frac{(P_1 + P_2)(\sin\theta + \mu\cos\theta)}{(P_1 + P_2)(\cos\theta - \mu\sin\theta)}$$

$$= \frac{\tan\theta + \mu}{1 - \mu\tan\theta}.$$

Since $\mu = \tan\phi$,

$$\frac{f}{W} = \frac{\tan\theta + \tan\phi}{1 - \tan\theta\,.\,\tan\phi} = \tan(\theta + \phi).$$

But $\frac{f}{W} = \frac{V^2}{15R}$, as shown above,

whence
$$\tan(\theta + \phi) = \frac{V^2}{15R} \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

That was the first of the fundamental equations for the superelevation of road-curves.

The following points should be noted :—

(i) If the coefficient of lateral adhesion were neglected, $\tan\theta = \frac{V^2}{15R}$.

(ii) If there were no superelevation, then $\tan\phi = \frac{V^2}{15R}$. The coefficient of lateral adhesion was thus, in itself, a superelevation.

Mr. Cornelius.

(iii) Any superelevation given to a road surface was therefore compounded of two superelevations.

To separate the two superelevations which composed any superelevation given to a road, $\tan\theta$ and $\tan\phi$ had to be evaluated :—

$$\tan\theta = \tan(\overline{\theta+\phi}-\phi)$$
$$= \frac{\tan\overline{\theta+\phi}-\tan\phi}{1+\tan\overline{\theta+\phi}\,.\,\tan\phi}.$$

From equation (1), $\tan(\theta+\phi) = \frac{V^2}{15R}$,

whence
$$\tan\theta = \frac{\frac{V^2}{15R}-\tan\phi}{1+\frac{V^2}{15R}\,.\,\tan\phi} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

Similarly,
$$\tan\phi = \tan(\overline{\theta+\phi}-\theta),$$

and
$$\tan\phi = \frac{\frac{V^2}{15R}-\tan\theta}{1+\frac{V^2}{15R}\,.\,\tan\theta} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

Equations (2) and (3) were the second and third of the fundamental equations for the superelevation of road-curves. The following points should be noted :—

(i) Equation (2) gave the superelevation to be given to a road when the coefficient of lateral adhesion was fixed.
(ii) Equation (3) evaluated the amount of reliance placed on lateral adhesion for any superelevation that was provided.
(iii) The equations were left in the form given to facilitate calculation, as the centrifugal ratio $\frac{V^2}{15R}$ was usually evaluated first.
(iv) To find the safe speed, $\tan\phi$ was usually assumed and $\tan\theta$ found. θ and ϕ were then found, and finally $\tan(\theta+\phi)$.

The fundamental equations were thus :—

$$\tan(\theta+\phi) = \frac{V^2}{15R}, \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

$$\tan\phi = \frac{\frac{V^2}{15R}-\tan\theta}{1+\frac{V^2}{15R}\tan\theta}, \quad . \; . \; . \; . \quad (2)$$

$$\tan\theta = \frac{\frac{V^2}{15R}-\tan\phi}{1+\frac{V^2}{15R}\tan\phi}, \quad . \; . \; . \; . \quad (3)$$

θ denoting the angle of superelevation,
ϕ ,, ,, angle of lateral adhesion,
V ,, ,, speed in miles per hour,
R ,, ,, radius of curvature.

Professor Royal-Dawson's 0·4 rule was thus shown to be incorrect because it varied the amount of lateral adhesion for every value of the centrifugal ratio, whereas the coefficient of lateral adhesion depended solely on the nature of the road-surface and of the tires of the vehicle. The fundamental equations demonstrated that clearly, equation (3) showing that its value had to be fixed before superelevation could be evaluated, and equation (2) showing how much reliance was placed on lateral adhesion for any particular amount of superelevation that existed or was being provided.

With regard to the superelevation of circular arcs, Professor Royal-Dawson's insistence on the suitability of his speed–chord formula for all cases obscured its severe limitations for use on existing roads and in hilly tracts. If his speed–chord formula were applied to many existing curves, he would find the shift so great as to be impracticable.

No unit chord such as that used in the speed–chord formula could be the parameter of a lemniscate, which had only one. The equation to the lemniscate was $r^2 = a^2 \sin 2\theta$, where r denoted the ray at the polar angle θ, and a was the one and only parameter, namely, the length of the ray at an angle of 45 degrees. By insistence on the unit chord Professor Royal-Dawson masked the utility of the curve whose use he had so ably advocated. In 1927 Mr. Cornelius gave up using any other curve, and found that all the data he needed were the parameter and the constant relations :—

Tangential angle = twice polar angle.
Intrinsic angle = three times polar angle.
Deviation-angle = twice intrinsic angle and therefore six times polar angle.

The angle relations were extremely useful in the field for obtaining points on the curve, either by intersections or by evaluating offsets.

The lemniscate could well stand on its own merits, and had Professor Royal-Dawson not taken the wrong turning *via* the speed–chord formula he would have been able to convince road engineers of its flexibility. Finally, had he realized the nature of the correct formulas for the superelevation of roads he would have found additional arguments for the curve he advocated in the gradually varying radii of curvature, which alone furnished a scientific basis for gradation of the superelevation between zero and the maximum at mid-curve. Moreover, he would not have found it necessary to

Mr. Cornelius. find the length of transition. The small wheelbase and turning radius of road vehicles enabled the curve to provide a suitable transition-length for all normal cases met with, provided that no attempt was made to join up with a circular arc. There was no necessity in normal cases for circular arcs. Mr. Cornelius had excluded them altogether from the Rajpur–Mussoorie road, and the consensus of opinion among motorists proved that the curve and equations for superelevation were capable of producing a safe and comfortable road in such precipitous country, where the road had at least once to turn back on itself through an angle greater than that of the hairpin-bend shown on p. 146.

With regard to Mr. Criswell's Paper, he proposed to discuss certain points under the headings under which they appeared.

Referring firstly to curves and their angles of deviation, Mr. Criswell, in recommending the cubic parabola, either alone or in conjunction with circular arcs, and in his remarks about the lemniscate, did an injustice to the latter curve. As mentioned in his remarks on Professor Royal-Dawson's Paper, since 1927 Mr. Cornelius had abandoned the use of any other type of curve but the lemniscate as suitable for road-work and had found no cause to regret his decision. He had used no special Tables. The ordinary book of logarithms with natural values of trigonometrical ratios was all that was necessary. In that respect he believed that he went even further than Professor Royal-Dawson in his admiration of the curve. *Fig. 9* would indicate its simplicity and its suitability for road-purposes. The up-to-date road-engineer would adopt the lemniscate readily once he understood it fully.

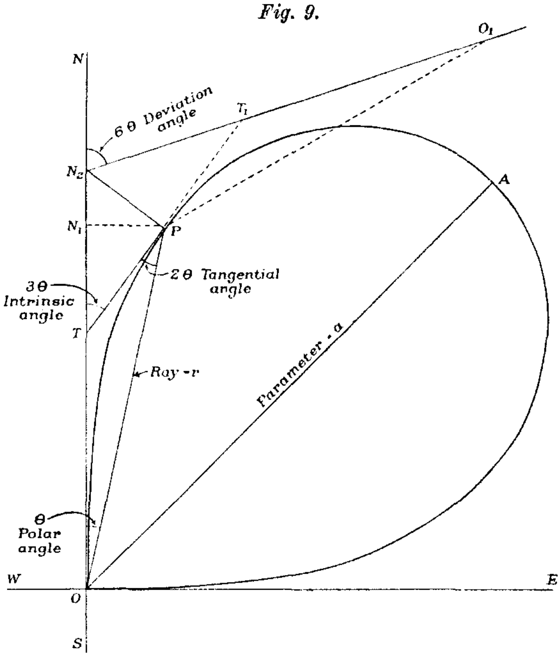
Mr. Criswell had been unfortunate in selecting *Fig. 3* (p. 161) for the purpose of re-alignment. That led to the conclusion that road-engineers had not yet realized the real effect of lateral adhesion, which he would discuss later when considering the superelevation of curves. In *Fig. 3* the radii of the curves were 1,400, 600 and 1,900 feet. Assuming a level cross section, the amount of reliance on lateral adhesion ($\tan \phi$) at a speed of 60 miles per hour would be :—

For the 1,400-foot radius = 0·17.
,, 600- ,, = 0·4.
,, 1,900- ,, = 0·126.

Since, as was proved in his preceding comments on Professor Royal-Dawson's Paper, the coefficient of lateral adhesion was itself a superelevation, and since 60 miles per hour was outside the range of normal cruising speeds, none of those curves required re-alignment. In his opinion the use of lemniscates in conjunction with the superelevation of lateral adhesion, or the use of the latter alone by the provision of a flat table at intersections, was all that was necessary. Mr. Cornelius.

He could not agree with the conclusions of Mr. Criswell that the cubic parabola with its limitations was a suitable curve for use by

Fig. 9.



Equation $r^2 = a^2 \sin \theta$.
P is the mid-point of the curve for the deviation-angle 6θ.
The tangents to the curve are ON_2 and O_1N_2.
PN_1 is used to find the parameter if OP cannot be measured.
$PN_1 = r \sin \theta$.
TT_1 is a tangent to the curve at P.

Diagram for Lemniscate.

road-engineers. *Fig. 9* for the lemniscate would indicate how it could be set out in the field, even when there were obstacles. The pole was on the road, the parameter was easily found, and the polar, tangential, intrinsic, and deviation angles all bore a constant relation to each other as simple multiples of the polar angle. Points on the curve could thus be found either by offsets or intersections. The curve was shown complete on the diagram to indicate how it could be used on hairpin bends.

With regard to superelevation on curves, Mr. Criswell did not

Mr. Cornelius. appear to have realized that the coefficient of lateral adhesion, stated to range from 0·2 to 1·0, might be and should be regarded as a superelevation. That view was confirmed by inaccuracies in Table V (p. 165). The rate of banking, taking a coefficient of 0·2 into account, worked out at 1 in 38; with a coefficient of 0·4 it was 1 in 43.

Conceding that lateral adhesion was a superelevation, and that tire-manufacturers and road-authorities constantly endeavoured to increase the coefficient of lateral adhesion, there was no justification for the extreme conservatism which adopted either zero or the National Physical Laboratory's minimum of 0·2.

Judged by those standards Mr. Criswell's guide to the amount of superelevation required under ordinary circumstances (*Fig. 5*, p. 168) contained several vital defects which resulted in his recommending altogether excessive superelevations. Enumerating those defects:—

(*a*) The curve of superelevation should be a tangent curve and not a straight line, as shown on pp. 229, 230.

(*b*) If a straight line were used, then it would be fairer to use the radius of the outer edge of the curve in arriving at the superelevation of that edge.

(*c*) Road-design was admittedly a matter of compromise between several conflicting factors; a fair compromise would be one in which each factor was proportionately adjusted, and allowance should therefore be made in deciding both the speed and the lateral adhesion. The former at 60 miles per hour and the latter at 0·2 made no allowance, so that the superelevation had to be excessive.

(*d*) A cruising speed of 45 miles per hour would be a fair compromise, though a lower one would satisfy the majority of motorists on curves, especially those in which the radius of curvature was small.

(*e*) Motorists at present habitually went round curves on the "infra-elevation" of a steeply-cambered road, even on wet days. The coefficient of lateral adhesion would have to be considerably more than 0·4 under such conditions, and yet the vast majority of motorists went round safely.

A much more acceptable guide would be obtained if full weight were given to those considerations, but he did not think that such a guide was advisable. If "spoon-fed" in that manner, road-engineers would come to lose interest in their work through being unable to connect cause with effect. It would be better to let engineers work out their superelevations under the Ministry of Transport's guidance by means of a framed policy:—

(*a*) That superelevation should be provided in all cases.

(*b*) That the radius of curvature should be graded to suit superelevations from zero to that necessary.

(*c*) That the *coefficient* of lateral adhesion to be adopted for the type of road-surface in use should be 0·4. **Mr. Cornelius.**

(*d*) That the speed should conform to a scale fixed by the Ministry with due regard to the radius of curvature and graded from 20 miles per hour upward.

Regarding vertical curves, the lengths of the lines of sight proposed by Mr. Criswell were very generous, and would be found impracticable in undulating country. The reason for such generous lengths was Mr. Criswell's assumption that nothing above eye-level came into view. There again, a compromise should be made and the sports car should give up something for the general good. The allowed speed of 60 miles per hour should similarly be reduced. The assumption that the length of the line of sight was governed by the distance within which vehicles approaching each other at speed could be brought to a standstill resulted in an exaggeration, especially on trunk roads where the width was usually ample to enable vehicles to pass in safety.

In conclusion, with regard to widening on curves, Professor Royal-Dawson had shown that widening was automatic when the lemniscate was employed. Its use was therefore to be recommended, and *Fig. 9*, p. 233, showed that there was then no difficulty in setting out the curves for the inner and outer edges.

Mr. WILLIAM DAY observed that Mr. Criswell had given data showing the excessive number of curves existing on some of the most important highways in England, and Professor Royal-Dawson had stressed the advantages to be obtained by using the lemniscate from tangent-point to tangent-point of a road-curve. If it was undesirable that a highway should be deviated every few chains, as shown by Mr. Criswell, did it not appear also that a curve of continually-varying radius was equally to be shunned? Whilst that might be rather an extreme manner of stating the case, nevertheless it revealed the line of reasoning in favour of adopting curves of one radius throughout the major portion of their lengths, employing the lemniscate or cubic spiral for transitions at each tangent-point. **Mr. Day.**

Professor Royal-Dawson had given an example of the recorded movements of traffic on a curve in Toronto. It was admitted that no superelevation existed, and further it had to be assumed that the roadway was cambered in the normal manner, as no statement was made to the contrary. Experience as a motorist, and still more as a motor-cyclist in the past, had led Mr. Day to believe that drivers of fast-moving vehicles cut their corners, not only to take advantage of the camber of the inner half of the roadway on a curve, but still more to avoid the adverse effect of the reverse camber on the outer

Mr. Day. side of the roadway. It appeared, therefore, that the use of superelevation was the factor which would best serve to improve conditions for fast traffic. Further, it was usually easier to carry out the modification of level required, whereas the re-alignment of a curve was often more difficult, especially in urban areas, owing to various limiting factors. The use of transition-curves would follow the use of superelevation when and where possible as a secondary improvement.

A further advantage to be gained by the use of superelevation was the reduction in the component of wheel-thrust acting parallel to the road-surface. Engineers who were responsible for water-bound macadam or gravel roads, treated only with bituminous surface-dressings, would be familiar with the constant ravelling of the road-surfaces on corners.

With further reference to the Toronto road-diagram, assuming that the existing road-limits did not permit of the modification of the curve, it would seem that a great improvement in the situation might be obtained by the use of superelevation or at least by the removal of the camber on the outside of the road, and finally by the laying-down of a traffic line, studded or painted, with a trace modified to a central circular curve with introductory transitions and displaced at its centre towards the inside of the bend. Such a course would only be possible on roads of width sufficient to carry, say, five lines of traffic, if undue congestion of traffic on the inner side of the roadway were to be avoided.

Finally, it would seem that delay in the introduction of such improvements as had been described would be due, not to lack of conviction on the part of executive engineers, but to the difficulty of persuading public authorities, more especially in the smaller and more backward countries, that the slight extra outlay would be justified.

Mr. Doak. Mr. W. J. Doak observed that his experience of nearly 40 years as a railway-engineer had interested him greatly in the subject of transition-curves. On the Queensland Railways it was the standard practice to employ cubic parabolas 2 chains long for all circular curves of 20 chains radius or less. Such short transitions were especially convenient and often necessary when re-aligning old lines which originally had simple circular curves. It was necessary, therefore, to examine the principles governing the choice of length, and, after considerable study of Mr. W. H. Shortt's Paper [1] and of American text-books on railway engineering, he had reached the following conclusions :—

(i) A particle following a tangent experienced no more shock on

[1] "A Practical Method for the Improvement of Existing Railway-curves." Minutes of Proceedings Inst. C.E., vol. clxxvi (1908–9, Part II), p. 97.

entering a circular curve than it experienced at any point on that curve, the centripetal force being the same all round. Mr. Doak.

(ii) A vehicle following the same track without superelevation would heel over, owing to its centre of gravity being higher than the point of application of centripetal force; the amount of heeling-over would depend upon the amount of the overturning moment and the elasticity of the springs, and oscillation of the vehicle would then take place until damped out by friction if the curve were long enough, when the vehicle would be in a condition of lateral equilibrium in a canted-over position.

(iii) A transition-curve might or might not obviate that oscillation, according to its length, but clearly it could not remove discomfort entirely, since the vehicle would be more or less canted all the way.

(iv) If superelevation were provided, the cross slope being $\frac{P}{W}$, the ideal condition would be secured; that could be done by using a transition-curve (such as a cubic parabola) and providing it with cant equal to $\frac{P}{W}$ at every point.

(v) The length of the transition-curve should be such that undesirable oscillation would not ensue. American practice had fixed the rate of rise of superelevation at 2 inches per second, and Mr. Shortt's rule gave $1\frac{3}{4}$ inch.

It should be clear from those considerations that the results of railway experience might not be applicable to road practice because the characteristics of the vehicles were radically different. The comparatively low centre of gravity of most road vehicles reduced the overturning couple, but, on the other hand, the greater flexibility of springing might increase the amplitude of oscillation.

In his opinion, Professor Royal-Dawson had over-rated the importance of "rate of comfortable turning" and under-rated the importance of banking. As a result, apparently, he would provide transitions suitable for maximum speeds, while banking only for average speeds. The correct procedure, however, was to bank for average speeds and to provide transitions suitable for those speeds, and he believed that Mr. Criswell would agree with that suggestion. The Toronto traffic-diagram merely showed that the average motorist was avoiding the reversed banking on the western side and endeavouring to take as short a cut as possible; every driver knew that the most comfortable way to get round a corner was to follow the curve of greatest radius by starting to turn as early as possible and going as close to the corner as possible.

Too much refinement was entirely out of place in road-curve design, on account of the variety in types of vehicles, speeds, con-

Mr. Doak. ditions and types of surfaces, and weather-conditions. The greatest satisfaction would probably be secured by keeping transitions as short as possible, and for that reason experiments should be made to determine whether the railway constant of 2 inches of cant per second could not be increased for road purposes. Professor Royal-Dawson's predilection against circular curves was probably based on a misconception regarding comfortable turning; there was no discomfort provided speed, curvature, and cant were correctly related.

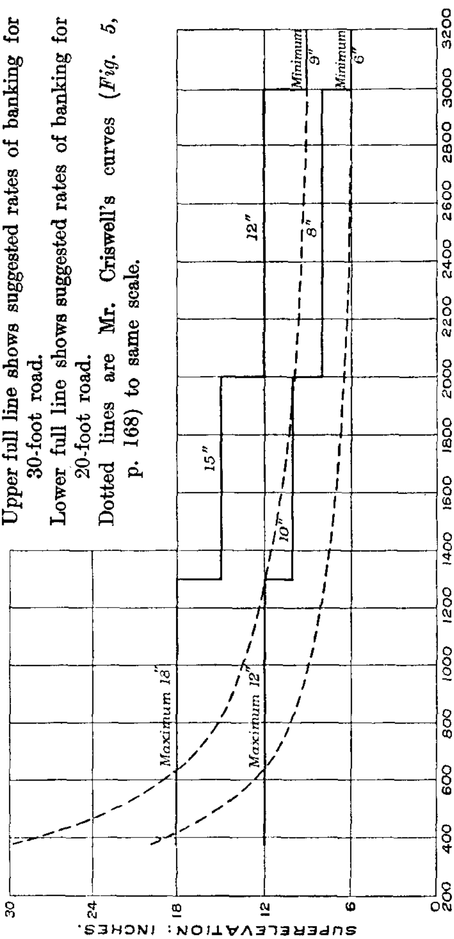
Mr. Leeming. Mr. J. J. Leeming, whilst fully in agreement with Mr. Criswell's general conclusions regarding the banking of road-curves, did not

Fig. 10.

Upper full line shows suggested rates of banking for 30-foot road.

Lower full line shows suggested rates of banking for 20-foot road.

Dotted lines are Mr. Criswell's curves (*Fig. 5*, p. 168) to same scale.



Note.—If a curve is approached by any great length of straight, it should be given more banking, even if an ample transition-curve is provided.

Banking of Road-Curves.

Mr. Leeming. entirely agree with the rates of banking given in Table VI and *Fig. 5*, p. 168. In the first place, he considered that a banking of 1 in 12 was far too steep for any curve. In frosty weather, or if the road became covered with dirt from farm carts, it would be dangerous to slowly-moving traffic, and in any case it was uncomfortable to horses. He thought that 1 in 20 was the maximum permissible banking. He had known of a case of a slowly-moving lorry, travelling round a right-hand curve, slipping down to the right-hand curb on a morning after a slight ground frost on a curve of 660 feet radius with a banking of 1 in 20. Further, he thought that the rates of banking provided for the flatter curves were not enough, especially for curves of about 1,300 feet radius, which were very deceptive and often led motorists to over-estimate the speed at which they could be taken. *Fig. 10*, p. 238, showed the rates of banking that he would suggest, compared with those given by Mr. Criswell. The proposed rates were based on those used in the reconstruction of the Oxford–Cheltenham Road, carried out under the supervision of Mr. G. T. Bennett, Assoc. M. Inst. C.E., County Surveyor of Oxfordshire, on which Mr. Leeming had acted as Resident Engineer. The road was satisfactory for high speeds, and the rates of banking employed seemed to give very good results.

Mr. Lloyd Jones. Mr. D. T. Lloyd Jones observed that the writings of both Authors on road-engineering were familiar to him, but when the subject was treated in its bearing on road-safety, and remembering the conditions of road-transport by reason of political expediency, he was reluctantly compelled to part company with them at the point where they assumed that consistent high speeds were to be admitted as a feature of English road-transport. Until the attainment of complete canalization of all the conflicting modes of transit now using the road, the case of the fast vehicle was only one of many, and, considered solely from the view-point of safety to all users, it was by no means the most important.

Notwithstanding the convincing expositions of both Authors, he could not but feel that intensive concentration on one feature of road-design only was liable to give an exaggerated impression of its value as a cure for, at the least, most troubles. The application of a principle would then be carried beyond the realms of practical issues, to the exclusion of even more vital considerations which were to be learned only at the wheel. He had had the privilege of reviewing Professor Royal-Dawson's Book "Elements of Curve Design," and, whilst giving it unstinted praise, he had questioned the limits of its application to an old-established road-system which was now required to handle many conflicting modes of transit. In the present scientific age there was a tendency to apply exact science

Mr. Lloyd Jones.

to problems such as that of road-design, forgetful of the fact that roads were made and used by a diversity of men and machines. Were road-transit by fast vehicles a new mode of travel and were new roads provided exclusively for it, there would be justification for adopting every device known to science for the improvement of road-efficiency.

The analogy of the railway used by so many, including himself, could only be carried to a very limited extent; England being such a town-ridden country, its roads were never intended to be and could not become speedways, and a definite and reasonable speed-limit ought to be imposed upon them so long as the attempt to accommodate many fundamentally different forms of locomotion in the same track was persisted in. The roads had grown from wandering tracks, incapable in the main of suitable alignment or curvature by reason of the developments of the past which had rigidly followed them; the engineer, attempting to design the ideal road, was left with only a few by-passes to which he might apply his methods, and even while he worked their frontages were built up and he had all his old troubles to face.

In his observations on the relation of efficiency and safety over a period of 15 years, he had found it necessary to bear in mind that the widely different interests of road-users could be only to a very limited degree reconciled, and that the refinements which high speeds would suggest, unless they could be universally applied, encouraged a sense of security at speed from which a driver might be suddenly confronted with a less favourable position and might be unable to recover control. There could be little question that the "element of surprise" which road-travel provided was the forerunner of practically all road-accidents, and that the remedy lay in giving the required visibility in terms of distance and time for every moving thing on the road, each at its own speed and in its own track, so that one or more might in case of need be brought to a standstill in time to avoid collision. The criterion obviously was the fast vehicle, and on the basis of visibility he had evolved formulas for curves and vertical curves giving a visibility of 4 seconds at the then maximum legal speed of 20 miles per hour. The factor of "time visibility" was introduced to allow for the human element, which had since figured so largely in statistical returns of road-accidents, and the period of 4 seconds was decided upon after experiments in mental reaction, application of brakes, and stopping-distances following surprise. The work of the National Institute of Psychology in the investigation of mental reaction, and exhaustive tests carried out by motoring organizations to establish braking distances, had since confirmed with agreeable accuracy the value of those early crude experiments.

Mr. Lloyd Jones.

Expressing the visibility of 4 seconds in terms of distance, he had prepared formulas and Tables to show the minimum curve radius on the inside of the curve which would give, with varying widths of footwalk, the safe visibility in the track of the vehicle. Mr. Criswell in his treatment of vertical curves had naturally accepted visibility as the prime factor, and followed the same line of argument with regard to mental inaction of drivers and braking distances as Mr. Lloyd Jones had previously done, but Mr. Lloyd Jones had applied the same reasoning to both horizontal and vertical curves, with the single objective at that time of establishing the minimum safe radius of curve in each case for the maximum legal speed. (Incidentally, the apparatus used in driving tests at the National Institute of Psychology, where the screen was in motion and the car stationary, gave to the practised driver the inverted illusion of the landscape rushing towards him ; as a test with the accelerator down it was valueless, and the machine was remarkable only for its ingenuity.)

At such moderate speeds as 20 miles per hour circular arcs only were used. He did not deny that transition-curves were beneficial, even at low speeds, but his sole object at the time had been to place in the hands of the average road-foreman information which he could readily understand, to enable him to carry out effective works within the limits of practical road-making operations at a time when many emergency relief works were in prospect.

The value of transition-curves was shown by an experience he had met with during the War as a railway engineer in the Near East. A 60-centimetre-gauge line was hurriedly laid to convey ammunition to a forward dump in preparation for an attack and advance in the difficult broken country to the north of Jerusalem. Curves were perforce of small radius, the ruling grade was 1 in 50, and the speed set by the military authorities was 8 miles per hour, so that only four trains per day could be run. It was decided to lay the tracks as accurately as for a main-line passenger service, introducing transition-curves. The principal curves were selected from twelve standard radii and four transition-curves were designed with a length of 1 chain which could be used in whole or part to provide a constant rate of change on approaching the main curve. Each transition was designed as a "false spiral" (the spiral of the Ionic volute), and was set out with very small unit chords to radii decreasing by arithmetical progression from the tangent to the constant radius of the curve. It was then possible to calculate the offset-angles for the transition and the main curve and to set out by theodolite, in place of the more cumbrous field method of setting

Mr. Lloyd Jones.

off the lateral displacement or "throw" and laying down the transitions by the offset method. The results were so satisfactory that eighteen trains per day were run, and speeds of 25 miles per hour attained with safety. Professor Royal-Dawson was the avowed champion of the lemniscate (admittedly the best transition), but for all practical purposes, with the relatively small radii of road-curves, the several forms of spiral differed little, and Mr. Lloyd Jones would suggest that the simpler method that he had outlined would encourage highway-engineers to make more general use of transition-curves.

Mr. MacGregor.

Mr. William MacGregor remarked that the subject of transition-curves, including the lemniscate, had been included for many years in the course of civil engineering at Glasgow University. He considered that the chief criticism against the lemniscate as a transition-curve was the difficulty met with in setting it out, which entailed a certain amount of laborious calculation and chaining unless special Tables, such as those prepared by Professor Royal-Dawson, were used. The length of the curve was not readily determined, and as most engineers preferred to have the running chainage maintained throughout the entire length of new road work, setting-out, using equal chord lengths, was not altogether satisfactory. The clothoid or spiral and the lemniscate were very similar. In the former the curvature was proportional to the length along the curve, whereas in the latter it was proportional to the length of the polar ray. The length of the clothoid was easily calculated from the equation $L = m\sqrt{\phi}$. The setting-out entailed calculation for deflection-angles unless special Tables were used. Such Tables were now available [1] and were prepared in such a form that the whole process was exactly analogous to setting-out a circular curve. Using those Tables no difficulty was experienced when the instrument position was at an intermediate point on the spiral, and, furthermore, a transition-curve between two circular arcs of a compound curve was easily inserted.

The length of the vertical curves used on the Glasgow–Edinburgh road was made NG feet, N being 120 feet wherever possible, with a minimum of 50 feet at bridge crossings, and G denoting the algebraic difference of the percentage grades. Expressing the length of the curve as NG feet had many advantages. The equation of the curve, the minimum visibility, the chainages of the highest and lowest points, and the extent of channel grading were readily determined.

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[1] A. Thom, "Standard Tables and Formulæ for Setting out Road Spirals." London, 1935.

The notation was as given in *Fig. 11*, and the following method of calculation was employed, dimensions being in feet :— Mr. MacGregor.

$L = N(a + b)$
$d = aN$ = distance of highest point from start of curve.

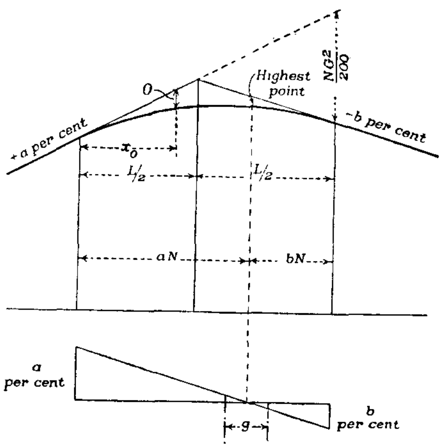
The equation of the curve was $y = \frac{x^2}{200N}$,
where y denoted the offset from tangent at highest point,
x ,, ,, distance from the highest point.
The approximate radius of curvature $R = 100N$.

$V = 10\sqrt{30N}$, where V denoted the visibility based on a height of eye of 3 feet 9 inches.

The length of channel grading on gradients flatter than 1 in f

$$= g = \frac{200N}{f} \text{ feet.}$$

Fig. 11.



Gradient at any point on curve

If V were equal to 600 feet N would be 120 feet.

In certain cases the visibility was greater than that given by the formula $V = 10\sqrt{30N}$.

If either aN or bN alone were less than $5\sqrt{30N}$,

$$V = 5\sqrt{30N} + \left(\frac{bN}{2} + \frac{375}{b}\right) \text{ or } 5\sqrt{30N} + \left(\frac{aN}{2} + \frac{375}{a}\right)$$

respectively.

If aN and bN were both less than $5\sqrt{30N}$,

$$V = (a + b)\left[\frac{N}{2} + \frac{375}{ab}\right].$$

Mr. MacGregor. The visibility V for various values of a, b, and N was as shown in the following Table :—

a: per cent.	b: per cent.	N: feet.	V: feet.
+ 3	− 3	120	600
+ 3	− 2·5	120	600
+ 3	− 1	120	735
+ 2·5	− 2·5	120	600
+ 2	− 2	120	615
+ 2	− 2	112·5	600

If 3 feet 9 inches were taken as the standard height of eye and the minimum visibility were 600 feet, no vertical curve would be necessary as far as visibility was concerned when both grades were flatter than 1 in 80. The minimum length of curve, however, should be about 200 feet. The finished road-levels were calculated either by offsets from the grade-lines or from the equation $y = \frac{x^2}{200N}$. The latter was very useful when the controlling levels at a bridge crossing were known. Any offset O from the grade line could be obtained from the equation $O = \frac{x_o^2}{200N}$, where x_o denoted the horizontal distance measured from the tangent-point (*Fig. 11*).

Mr. Nimmo. Mr. W. H. R. Nimmo observed that he found himself in such general accord with the views expressed by Mr. Criswell that such comments as he had to offer were occasioned mainly by the differences between the conditions experienced in Great Britain and in a sparsely-settled country such as Australia. The ideal to be aimed at was undoubtedly that curves should be built for the same speeds as the adjoining tangents, the grading of which should be such as to permit of the maintenance of a uniform speed; that ideal, however, was usually unattainable in built-up territory, and even in country districts, where the density of population was low, it might be economically unwarranted. It was a sound principle that no road should be improved beyond its traffic-value; under Australian conditions the engineer, although perhaps free to deviate from existing routes, was frequently forced by economic considerations to adopt curves of smaller radii than he would otherwise deem desirable, and, in order to reduce the cost of earthworks, he had often to undulate his grades. Since many drivers did not realize that a blind vertical curve was as dangerous as a blind horizontal one, sight-distance was of even more importance in the former case than in the latter. The sight-distance of 600 feet proposed by Mr. Criswell was none too great for important highways. Unfortunately, on many roads in hilly country the engineer had to be satisfied with a sight-distance of 200 feet on vertical curves, and often less on horizontal ones. As traffic, and consequently revenue, increased, the horizontal sight-distance could often be improved by benching back the sides of cuttings, but the improvement of vertical sight-distances might involve discarding an expensive section of pavement, and therefore all vertical curves should be designed in the first instance for the greatest practicable sight-distance. Horizontal curves of 100 to 200 feet radius were not uncommon in Queensland, and on bituminous pavements a central white line in conjunction with a widening of the pavement had been found to work well in separating the lanes of traffic. Such white lines were also particularly effective on vertical curves. In some cases of blind curves on roads having water-bound pavements, two distinct one-way tracks had been provided, either with or without separation of the grades. In difficult country such double-tracking might appreciably reduce the cost of construction, but from the standpoint of safety he would favour the single pavement liberally widened. **Mr. Nimmo.**

On high-class pavements all vertical curves should be carefully worked out. The parabola, owing to its property of changing grade at a uniform rate, was convenient and was usually adopted for vertical curves, but in regard to passengers' comfort it possessed no special virtues compared with other curves. The designing of vertical curves was greatly facilitated by using celluloid templates made to distorted scales for a few standard parabolas.

A special problem was encountered when a road on a comparatively steep grade crossed a railway-line, which might be on a skew to the road and might be also superelevated against the grade of the road. In such a case, reverse parabolic curves, separated by a tangent between the rails, should be carefully worked out for the centre-line and for each edge of the pavement, which became a warped surface as it approached the rails. Experience had shown that such crossings carefully constructed, with parabolas having a rate of change of grade as great as $\frac{1}{3}$ per cent. per foot, might be negotiated at speeds up to 40 miles per hour without appreciable discomfort.

There should be no need to stress the necessity for superelevation on curves, but practice varied widely as to the rate of banking to be adopted. In Queensland, banking was based upon a coefficient of friction of 0·2, which occurred quite frequently on wet or greasy pavements. For curves of 1,000 to 3,000 feet radius, banking ranged from 1 in 24 to 1 in 36, which represented the normal crossfalls for water-bound and bituminous surfaces respectively. Flatter slopes than 1 in 24 on water-bound macadam or gravel gave rise to excessive maintenance, and curves over 2,000 feet radius were not

Mr. Nimmo. usually banked. For similar reasons, only the higher types of bituminous pavements on important routes should be banked on curves of radii exceeding 3,000 feet. Concrete surfaces should be banked 1 in 48 for radii from 3,000 to 5,000 feet.

Transition-curves should be provided to all curves on high-class pavements carrying fast traffic. Theoretically, the curve adopted for a transition should be a spiral, for which the change of curvature was proportional to distance and for which superelevation would be also correct if applied in proportion to distance. Actually, however, drivers did not handle their steering-wheels in accordance with any mathematical formula, but more or less made their own transitions, so that mathematical precision in the selection of transition-curves was not called for. For many cases the cubic parabola, which was simple to set out, was quite satisfactory. For water-bound pavements it was doubtful whether transition-curves were worth the trouble of setting-out, as drivers would not follow them exactly; moreover, such pavements usually became gradually wider owing to maintenance, and the transition-curve lost its shape.

Widening of the pavement on curves should, as Mr. Criswell pointed out, be based on the additional width occupied by a vehicle plus an allowance for the increased difficulty in judging clearance. In Queensland twice the extra width occupied by a vehicle 20 feet in length was allowed for each traffic lane; that was satisfactory on banks, but in cuttings it was frequently necessary to widen the pavement to the foot of the batter owing to drivers cutting into the water-table. Widening should be applied throughout the curve, usually on the inside, and should have a simple transition-section extending beyond the tangent-point at each end.

Compensation of grade on curves was not necessary for motor-traffic, but on mountain roads the ascending grade following a blind curve should be eased because a cautious driver would lose momentum by slowing down on the curve.

Curves in the same direction joined by a short straight were very objectionable, and should be eliminated by using a compound curve. In cuttings in rough country it was sometimes impracticable at reasonable cost to avoid reverse curves with only a short straight between them, and in such cases the warped surface of the transition-section required careful designing to ensure that water from the water-table on the high side would not run diagonally across the pavement.

Most road-engineers, who were accustomed to drive cars, had probably realized that liberal superelevation, even when applied gradually throughout the kind of transition-curve usually encountered, did not necessarily produce comfortable riding on curves, Mr. Nimmo.

and they would therefore welcome the valuable analysis of the problem presented by Professor Royal-Dawson. The Paper confirmed the opinion, long held by Mr. Nimmo, that the practice of introducing angles instead of curves, even at very small changes in direction, was to be deprecated. In such cases a curve of long radius should be constructed, but, since the transitions theoretically required were short, their omission did not seem to be of practical importance if the pavement were suitably widened. The Paper also established the fact that the shorter the radius of a curve, the greater should be the length of the transition; unfortunately, however, it was frequently impracticable to comply with that requirement in rough country or in built-up areas. Since it was improbable that the average driver handled his steering-wheel in such a regular manner as to produce a uniform rate of change in radial acceleration, it did not seem that the lemniscate possessed any real superiority over the other curves in common use, provided that the length of the transition was sufficient.

Reliance upon the sense of discomfort produced by an inadequate length of transition to prevent excessive speed, and thus to enable superelevation to be reduced, would be conducive to accident. Upon approaching a curve having a transition which was too short for the speed at which he was travelling, a reckless driver would, instead of slackening his pace, endeavour to lengthen his transition by cutting across the pavement; unless his judgement was particularly good, he would be likely to reach the outer edge of the pavement before he had completed the necessary length of transition. He would then have to endeavour, by a sudden manipulation of the steering-wheel, to negotiate a curve of even smaller radius than the actual curve of the centre-line of the road, and in the absence of sufficient superelevation the car would skid or overturn. Especially was that liable to occur if the view of the driver were obstructed so that he could not judge the total deviation-angle. On such curves considerations of safety appeared to indicate that superelevation should, if anything, be increased on the latter portion of the curve, since overturning due to excessive speed seldom occurred near the commencement of a curve. A white line properly located would produce good results, but such lines were frequently so misplaced as to constitute a danger.

Mr. Opie.

Mr. O. R. Opie agreed with Professor Royal-Dawson that the heavy annual toll of road-accidents might be due largely to the shortcomings of human nature, particularly in "road-sense," but that there was great scope for engineers to help to reduce accidents by improving roads. Every day, ordinary modern cars attained speeds up to 60 miles per hour on main roads through open country without

Mr. Opie. unduly increased risk of accident, and any appreciable general reduction in maximum speeds seemed unlikely. The importance of the design of curves on main roads had thus become increasingly great, but, as Mr. Criswell pointed out, the task of the British road engineer was mainly limited to re-designing existing highways, which frequently prevented any considerable deviation from existing curves. One of the greatest sources of danger to fast traffic was the variation in the coefficient of friction on road-surfaces and its great reduction after comparatively short periods of use, or under adverse weather conditions. The attainment of an uniformly high coefficient of friction on road-surfaces at curves would be of greater value than superelevation to prevent skidding, and special cast-iron sections for road-surfacing had recently been used as a means to that end. In the Mersey tunnel, where a permanently non-skid surface was essential, the whole of the road-surface had been covered with cast-iron setts made by the Stanton Ironworks Company, Ltd., having studded surfaces of non-skid pattern, comparable to the treads of motor-car tires, which had given very satisfactory results. The sections were laid in bitumen on a reinforced-concrete foundation, and their cost was usually less than that of granite setts.

Mr. Urquhart. Mr. G. F. URQUHART observed that the following notes and Table in connection with the superelevation of highways had been compiled in order to standardize as far as possible the setting-out of road-curves with transitions, and the application of superelevation to the reconstruction of existing roads and to the general improvement of existing surfaces where no capital expenditure was contemplated and the work was being executed as ordinary maintenance, in addition to the design of roads for entirely new locations. The type of country traversed by the roads under consideration was comparatively hilly, gradients of 5 per cent. being common even on the main trunk roads, and curves greater than 20 degrees occurring frequently. He agreed with Mr. Criswell that the cubic parabola used as a transition in conjunction with a circular curve met the requirements of the majority of road-alignments, had distinct advantages over the lemniscate with regard to the practicability of the field and office work, and did not essentially differ from it in practical results; the cubic parabola had therefore been used as the means of applying the transition from the tangent to the circular curve.

(i) A ruling degree of curvature of 20 degrees for a road 20 feet wide, and 15 degrees for a road 30 feet wide, was adopted, and maintained wherever possible. In mountainous country that condition was often difficult to maintain, except where entirely new roads were being constructed.

Mr. Urquhart.

Radius of curve: feet.	Curve: degrees.	Superelevation (crossfall of road).	Superelevation recommended by Mr. Criswell.	Superelevation by Prof. Royal-Dawson's speed-chord method.	Length of transition for cubic parabola: feet.			
					Original. (A.)		Modified. (B.)	
				Calculated for 35 m.p.h.	Road 20 ft. wide.	Road 30 ft. wide.	Road 20 ft. wide.	Road 30 ft. wide.
287·94	20	1 in 12	1 in 12	—	200	—	100	—
302·94	19	1 in 12	—	—	190	—	95	—
319·62	18	1 in 12	—	—	180	—	90	—
338·27	17	1 in 12	—	—	170	—	85	—
359·27	16	1 in 12	—	—	160	—	80	—
383·07	15	1 in 12	1 in 12	1 in 11·6	150	225	75	150
410·28	14	1 in 14	1 in 14	—	140	210	140	140
441·68	13	1 in 15	1 in 14	—	130	195	130	130
478·34	12	1 in 16	1 in 16	—	120	180	120	120
521·67	11	1 in 17	1 in 16	—	110	165	110	110
573·69	10	1 in 18	1 in 18	1 in 14·5	100	150	100	100
637·28	9	1 in 20	1 in 20	—	90	135	90	135
716·78	8	1 in 22	1 in 22	—	80	120	80	120
819·02	7	1 in 24	1 in 24	—	105	140	105	140
955·37	6	1 in 26	1 in 26	—	90	120	90	120
1,146·28	5	1 in 28	1 in 27	1 in 27	100	125	100	125
1,432·69	4	1 in 30	1 in 32	—	80	120	80	120
1,910·08	3	1 in 32	1 in 36	—	90	105	90	105
2,864·93	2	1 in 36	1 in 40	—	80	100	80	100
5,729·65	1	1 in 36	1 in 40	—	80	100	80	100

Mr. Urquhart. (ii) It appeared to be a logical conclusion that the length of the transition-curve should increase directly as the degree of curvature (as the greater the change of curvature, the greater length would be required to effect that change), and also directly as the amount of superelevation (thus, for a constant degree of curvature and increasing width of road, the length of the transition should increase). The lengths of transitions shown in the accompanying Table were intended to meet those conditions as far as possible, but practical considerations arising in the field work rendered a uniform increase impracticable.

(iii) In cases where the tabulated value of the transition-length was greater than 150 feet, the tabulated value should only be used when the length of the original circular curve was greater than $1\frac{1}{2}$ times the tabulated transition-length. Where the length of the original circular curve was less than $1\frac{1}{2}$ times the transition-length, or where reverse curves occurred whose tangent-points were closer together than the sum of half the transition-lengths, modified transition-curves would be required.

(iv) The rate of banking provided agreed closely with the rates recommended by Mr. Criswell, and, in the case of the 15-degree, 10-degree, and 5-degree curves, the rate of banking provided agreed favourably with the required rate calculated by the formula advocated by Professor Royal-Dawson.

(v) The lengths of transitions shown in column A of the Table which were first adopted, were found to be too long for general use with the higher degrees of curvature, which were generally used in locations tending to restrict the application of long transition-curves.

(vi) A number of curves had now been constructed on the basis of the values shown in the Table, and both for appearance and for ease of driving a vehicle round them, they had proved very satisfactory. Criticism had been levelled at the small rate of banking provided for the curves of 10 degrees and less, but the critics admitted to driving at 45 to 55 miles per hour on the curves, which were not designed for such high speeds; he did not consider that it was necessary to design highway-curves, which had to accommodate all classes of traffic, for greater speeds than 35 miles per hour.

Professor Royal-Dawson. Professor Royal-Dawson, in reply to the Correspondence, observed with satisfaction that the general consensus of opinion was in favour of transitions in some form, although there was diversity of opinion regarding their value and method of application. Mr. Doak, whose opinion as a railway-engineer was that no more shock was experienced on entering a circular arc from a tangent than at any point on the curve itself, and who employed short transitions merely as a convenience, represented what perhaps might be called the zero point of view. At the other end of the scale, Mr. Cornelius, from his experience on the Rajpur–Mussoorie hill-road, was a whole-hearted advocate of the lemniscate, used transitionally throughout, without the insertion of a central circular arc, in all his curves. That should allay the apprehension of Mr. Day that drivers might be puzzled by the absence of a circular arc. Professor Royal-Dawson.

To Mr. Lloyd Jones, Mr. Nimmo and others, who argued against the scientific treatment of transitions on the ground that the driver was unscientific and made his own transitions, also that the same road was used for a diversity of traffic, it might be pointed out that a driver in motion was subject to inexorable dynamic laws which controlled his movements whether he knew it or not, and that it was the business of the engineer to design roads so as to compel the fastest driver to follow the course and the speed designed for him under the operation of those laws. It was not claimed that scientific curvature could solve all traffic problems, but it was the first and fundamental essential of road-reform, without which no other improvement could function efficiently.

He was glad to note that the speed–chord formula had not been challenged, although Mr. Cornelius had doubted its applicability to sharp mountain curves. It could in fact be usefully applied to the sharpest curves by using the sliding scale for moderate speeds referred to in the Paper (p. 130), whereby 10 miles per hour by the formula might be increased to 13, 17 to 20, and so on. There might also be diversity of opinion as to what constituted comfortable turning on a hill road. Many travellers looked for, and enjoyed, a mild sensation when going round a sharp curve, provided no sense of insecurity was felt.

Mr. Cornelius's contention that Professor Royal-Dawson's unit chord had masked the utility of the lemniscate would probably be overcome by further acquaintance with it. The unit–chord method of setting out the curve had been specially devised to meet situations where the direct polar-ray method might be found inconvenient. The Table of polar rays given in Professor Royal-Dawson's book [1] in terms of such units (the parameter or major axis being 10·3648 units) enabled the curve to be set out by the direct polar-ray method, for any required speed-value, merely by multiplying those units by the actual length of the unit given by the speed–chord formula. Nothing could be simpler.

Several correspondents, including Mr. Cornelius, saw in the Toronto diagram merely the tendency of motorists to avoid the camber or "infra-elevation" of the western half, although Mr.

[1] "Curve Design," Table V, London, 1932.

Professor Royal-Dawson.

Cornelius elsewhere observed that motorists habitually went round curves on the "infra-elevation" of a steeply cambered road, even on wet days. Mr. Day also assumed that the eastern half was cambered, but that was precluded by the road-junction on that side. To put the case conversely, and looking at the lay-out of the curve, he would ask whether it could be seriously believed that if the western half had been level, or even superelevated, those particular motorists would have pursued any other course than that shown. In any case, even assuming the *locale* to have been thus transferred two lane-widths westward, such a hypothesis would not explain the actual shapes and dimensions of the tracks in relation to their respective speeds, in whatever area they might have been located. That was the outstanding lesson of the diagram to those who had the mental vision to perceive it.

Mr. Nimmo's warning that an unduly short transition would be dangerous to a reckless driver might be commended to the notice of those who were inclined to apply disproportionately short transitions to "designed" speeds, involving the use of a higher factor than 1 foot per second per second in a second in the central acceleration formula, on which the natural transition and its derivative, the speed-chord formula, were based.

There was a tendency throughout the discussion to relate superelevation to radius only, without any other consideration, or by assuming speed-values without reference to the existence, or otherwise, of transitions. The method of Mr. Cornelius in basing speed-values on the minimum radius, maximum superelevation, and assumed coefficient of lateral adhesion was not very convincing. His objection to Professor Royal-Dawson's 0·4 rule on the ground that it varied the amount of lateral adhesion for every value of the centrifugal ratio, whereas the coefficient of lateral adhesion depended solely on the nature of the road-surface and of the tires of the vehicle, appeared to be due to his treating the coefficient of friction as a constant force, whereas the actual amount of frictional force exerted at a given moment depended on the centrifugal force. The lateral coefficient of adhesion on a straight road might be 0·5, but the actual side-thrust would be nil until the vehicle entered the curve. It would then function by degrees as the centrifugal force developed. The 0·4 rule was based on a maximum centrifugal ratio of 0·25 and a maximum superelevation of 0·10. The application of that ratio throughout the curve was logically correct.

Mr. Clayton's suggestion that the ratio should be 0·5 while limiting the superelevation to 1 in 12 implied a centrifugal limit of 0·166, which was unnecessarily restrictive.

Regarding the theory that lateral friction was superelevation, Mr. Cornelius was at pains to show that tan $(\theta + \phi)$, which represented the maximum centrifugal ratio, was not the same thing as tan θ + tan ϕ. But when tan $(\theta + \phi)$ was limited to 0·25 and tan θ to 0·10, as working maxima, the difference was negligible. Greater refinement might be necessary if those maxima were materially exceeded. Professor Royal-Dawson.

Mr. Day had rightly stressed the need for superelevating existing curves, and it was in view of the importance of applying such superelevation rationally that the Author's skeleton Table (Appendix IV, pp. 150–51) had been prepared. That also disposed of Mr. Cornelius's contention that the speed–chord formula could not be applied to existing curves as a basis for superelevation or realignment.

Mr. Addison had made some useful observations from the user's point of view. Of special value was his reference to the danger involved in the extra lift imposed on the driver of a heavy lorry proceeding up a steep hill on the outer side of a heavily-banked curve. That supported the Author's contention that superelevation should be applied about the centre-line, and not from the inner edge as a datum, in order to minimize the amount of lift on the outer side.

A further valuable remark of Mr. Addison was that a vehicle on a camber tended to run into the ditch, that tendency having to be consciously resisted by the pressure of the driver's hands on the steering-wheel. It would be well if that circumstance were borne in mind by those who favoured a side-slant as the normal profile of a carriageway in the case of a dual roadway with a central dividing strip. Regarding his general experience of steep banking on continental roads, individual cases could not be considered in detail without reference to the local alignment and speed-potentialities, the latter being influenced by the nature of the gradients in the localities concerned.

Of the various forms of transition discussed, Mr. Lloyd Jones's claim that his Ionic volute was simpler than the lemniscate might be set against Mr. Cornelius's whole-hearted advocacy of the lemniscate for its simplicity and suitability for road-purposes, a view which Professor Royal-Dawson fully endorsed.

In conclusion, Professor Royal-Dawson wished to express his gratitude to the various correspondents for their appreciation of the main points of his Paper.

Mr. Criswell. Mr. CRISWELL, in reply to the Correspondence, entirely agreed with Mr. Adams's statement regarding the choice of transition-curves that the differences between various types of transition were so slight that the only reasonable course, so long as transition-curves were provided, was to leave the type to be used to the discretion of the engineer concerned.

Mr. Criswell. The problem of providing for overtaking vehicles, referred to by Mr. Addison, could best be dealt with by the provision of ample vision by keeping verges clear of undergrowth and cutting back the slopes of banks. Ease of passing would be greatly facilitated also by the display of road-courtesy. Too frequently selfish drivers refused to draw over to the left to allow an overtaking car to pass, with the result that the latter passed on the extreme offside of the road thus risking collision with an oncoming vehicle.

He was entirely in agreement with Mr. Clayton's advocacy of the employment of superelevation on large-radius curves, if only to avoid the negative-superelevation effect of the camber.

Mr. Cornelius would probably get little support from the County Surveyor of Buckinghamshire (who had reconstructed the curves) for his statement that the original reverse curves illustrated in *Fig. 3* did not need re-alignment. Neither could Mr. Criswell agree with him, as he had often found difficulty in negotiating the original curves even at 35 miles per hour.

With regard to superelevation, Mr. Cornelius stated that Table VI (p. 167) gave altogether excessive rates. On the other hand Mr. Bennett, County Surveyor of Oxford, considered that many of the rates of superelevation given were insufficient. Such a conflict of opinion seemed to justify Mr. Criswell's purpose in submitting the Paper. Lateral adhesion of 0·2 had been taken fully into account when calculating Col. 5 of Table VI.

Regarding vertical curves, Mr. Cornelius described the length of vision provided in Table XI (facing p. 176) as very generous and impracticable in undulating country. American standards of vision were, however, very much greater than those recommended in the Paper. Were the country not undulating, no standards of vision would be needed for vertical curves. Mr. Criswell was entirely at variance with Mr. Cornelius in regard to the speed for which vision should be provided. However strongly it might be felt that roads should not be designed for 60 miles per hour, the fact remained that that speed was quite common on major highways, and it seemed to him that engineers should design new roads for the requirements of present-day traffic to be satisfied with safety. Widening on curves became automatic with the spiral as with the lemniscate if the same curve were used on both inner and outer edges of the carriageway.

There was a great deal to be said for Mr. Day's suggestion that it might be advantageous to adopt a curve of one radius throughout the major portion of its length, with transitions at each end. So long as it had the same speed-value, its advantages over the wholly-transitional curve were that (*a*) the total distance between the same two given points on the tangents was less, (*b*) the rate of banking was constant on the circular arc, instead of constantly changing as on the wholly-transitional curve, (c) the maximum rate of banking required was less because the centrifugal ratio acquired was lower. Mr. Criswell.

He agreed that the greatest stumbling-block to the execution of road-improvements, such as banking and the provision of transition-curves, was not reluctance on the part of engineers to put such works in hand, but the difficulty of obtaining financial provision for the works. That difficulty was common in all countries, however.

He was unable to follow Mr. Doak's conclusion that a particle following a tangent experienced no more shock on entering a circular curve than it experienced at any point on that curve. He would have thought that the sudden application of centrifugal force at the tangent-point would inflict a much greater physical shock than that which obtained once the curve had been entered, as, once a certain centrifugal ratio had been attained, it remained constant so long as the speed and radius remained constant.

With regard to his deprecation of the design of roads for high speeds, Mr. Lloyd Jones appeared to have overlooked the fact that if roads were designed for a reasonably high speed, then vehicles of lower speeds found no inconvenience in using them. If, on the other hand, roads were designed only for low speeds, vehicles capable of high speeds were liable to become a source of danger to themselves and others when their capabilities were utilized. The tendency of all motor manufacturers was still further to increase the speed-potentialities of their vehicles. It seemed, therefore, that all new roads should be designed for a reasonably high speed, and that existing ones should be similarly reconstructed when that became possible.

Mr. J. J. Leeming's view that superelevation should not exceed 1 in 20 coincided with that of Mr. Bennett, the County Surveyor of Oxfordshire. Mr. Criswell was surprised to hear of the case of a lorry slipping sideways down a cross slope of 1 in 20, but, of course, when a carriageway was ice- or frost-bound anything might happen. Vehicles were just as likely to skid down a longitudinal gradient under such conditions unless the surface were well gritted.

He was delighted to hear that the subject of transition-curves, including the lemniscate, formed part of the curriculum at Glasgow University. He gathered from Mr. MacGregor's remarks that on the whole he preferred to use the spiral because both the distance and the radius at any point on the curve were so easily ascertainable.

The length of vertical curves used on the Glasgow–Edinburgh road appeared to conform closely to those that he recommended for trunk routes in Table XI (facing p. 176).

It was always both interesting and valuable to have records of

Mr. Criswell. the views of engineers such as Mr. Nimmo who dealt with similar problems in Australia, their treatment being modified by the difference in conditions. Mr. Nimmo's experience lent support to the view that for trunk routes a sight-distance of 600 feet was none too great, and for practical reasons he favoured the parabolic form. Mr. Criswell differed from Mr. Nimmo as to the limiting radius of curve requiring banking, in that Mr. Nimmo put the maximum at from 3,000 to 5,000 feet whereas Mr. Criswell would bank all curves above those radii by providing the normal camber in one direction right across the carriageway.

With regard to the speeds attained on modern highways, Mr. Opie confirmed his view that 60 miles per hour was safely attainable on properly transitioned and banked roads.

The Table furnished by Mr. Urquhart showed remarkable agreement between the rates of banking that he adopted and those recommended by Mr. Criswell. Mr. Urquhart's view that highways should not be designed for speeds greater than 35 miles per hour was strangely at variance with modern conditions, when the legal speed-limit for large heavy buses and lorries was 30 miles per hour. It would be unreasonable to expect passenger cars to travel at such a low speed as 35 miles per hour on the open road. In the built-up areas where, for reasons of safety speed was restricted to 30 miles per hour, a design speed of 35 miles per hour had more to recommend it.

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26 February, 1935.

Sir RICHARD AUGUSTINE STUDDERT REDMAYNE, K.C.B., M.Sc., President, in the Chair.

THE discussion on the Papers by Professor F. G. Royal-Dawson and Mr. Henry Criswell on Road Design was continued and concluded.