

brought too near the extremities of the voussoirs; and in brick arches, particularly those turned in separate rings, a much greater latitude must be allowed. It must be evident, however, that it is desirable to form brick arches as much as possible in one bonded mass, using the best cement.

In abutments, a still greater variety of considerations will arise. To render this part of the subject tangible by theory, the abutment must be assumed as standing alone, the foundations being perfect, and the point of rupture being at the base of the abutment. In practice, they are rarely, if ever, without earth behind them, aiding more or less in their support. Some cases, such as in arches under embankments, the force acting to push in the abutment, exceeds the horizontal thrust of the arch, and a tendency has been frequently exhibited in arches so situated, to rise in the crown.

The foundations, the wing walls, the spandril walls, the backing, the nature of the materials employed, and many other practical considerations, all tend to affect the stability, and modify the results of theory; each of which circumstances, acting pro or con, will be made to fulfil its duty to the best advantage, by the skilful engineer. The utmost that theory can do, is to show the conditions of equilibrium, under certain fixed practical data, and there can be no doubt, that the line of equal horizontal thrust in an arch, is analogous to a vertical line drawn through the centre of gravity in a column. That this line should fall within the mass at every joint, and that the position of each joint should be such, that the direction in which the pressure acts should be within the limiting angle of friction, are conditions common to both structures.

The paper is illustrated by three sheets of diagrams, (Nos. 4293 to 4295,) of the arches treated of, and by a series of models.

Mr. CUBITT, *V.P.*, in answer to a question from the President, said he felt the propositions in the paper were so conclusive, that they scarcely afforded an opportunity for remark, much less for discussion. The great merit of the communication, and of the illustrations, was the adaptation to practice; in most of the treatises on arches, the theory alone was considered; Mr. Barlow had, however, very properly pointed out the possibility of constructing arches of certain forms and dimensions, strictly within the theoretical rules, so that they should stand well alone; but that when any pressure was imposed on them, they would fail. These were points of great importance, which should never be lost sight of by the engineer, and demanded not only great attention to the proportions of the structure, but also to the quality of the materials employed, the situation, the nature of the foundation,

and of the backing and numerous other considerations, in order to adapt the arch to the use for which it was intended.

Mr. SOPWITH agreed in the value of the paper. He viewed it more particularly in its application to the construction of arches in mines, where solidity and permanence were of such importance, on account of the unequal pressure to which they were subjected. He was much pleased with the models used for illustrating the propositions, and from their simple and inexpensive character he thought them particularly worthy of imitation. He had used the same method in his dissected geological models,* and in those of the coal-field of the Forest of Dean, and other mineral districts, and always found they conveyed to unprofessional persons more accurate impressions than the most elaborate plans. He hoped that as the Institution progressed, space would be obtained for the display of such models as those before the meeting.

Mr. BRUNEL had endeavoured, during the reading of the paper, and the remarks of the preceding speakers, to find some point whereon to found observations, but it was very difficult, as the author's practical experience appeared to have constantly directed his theoretical investigations. He thought, however, that the compressibility and elasticity of materials of construction had not been sufficiently insisted upon. This did not generally obtain enough consideration, yet it was of great importance to the stability of a structure; all materials, even to granite, possessed an amount of elasticity, and it did not suffice to have the line of pressure fall merely within the mass; it should be sufficiently within it to allow for any yielding from elasticity, without endangering the building.

Mr. W. H. BARLOW said he had intended to insist especially on that point, when he stated that "In practice an arch would exert more pressure, and resist less than theory would denote; because unyielding material and a mathematical adjustment of the joints was incompatible with practice. Even in arches of the hardest stone, and with the best workmanship, the lines of resistance and impression must not be brought too near the extremities of the voussoirs; and in brick arches, particularly those turned in separate rings, a much greater latitude must be allowed."†

He was flattered with the encomiums passed by the excellent judges who had spoken; his intention had been to produce a paper pointing out the practical application of certain theoretical propositions, and he anticipated its principal utility would be, in ascertaining where the lines of pressure would fall, when described upon the profile of any

* Vide Minutes of Proceedings, 1840, Vol. i. p. 49.

† Vide ante, p. 172.

projected arch; thus probably enabling errors to be avoided. He had found this of great practical advantage in the course of his professional career.

Mr. PELLATT thought this communication was one which peculiarly demonstrated the advantages to be derived by the Associates of the Institution, from mingling with the Members. The illustrations were confined to practical examples, falling within the daily professional engagements of the members; but Mr. Pellatt perceived in it also very valuable information, which might be rendered available in the construction of the vaults of furnaces, the duration of which was of great importance to him in his glass manufactory. It was desirable, that the crown of the arch of a glass furnace should be so low as to keep the heat well down, and yet if it was too flat, it was soon destroyed by the impinging action of the flame, or else the expansion of the materials by the heat destroyed the equilibrium of the arch, and it fell. At present the practical judgment of the workman was alone depended upon for the proper form, and the consequence was, that although a well-built furnace arch might last fourteen years, it might not last longer than fourteen months. It was most desirable to adapt theoretical propositions, such as had been given, to practical cases connected with the constructions used in every branch of manufacture. The Institution numbered among its Associates, manufacturers of almost every kind, and if every one contributed a little of his practical experience, the discussions would obtain an increased importance and value.

Mr. INMAN said that the ruins of ancient buildings would afford many striking lessons of the correctness of the principles laid down in the paper. Numerous examples of remains of arches standing without other support than the stones of which they were composed, might, he believed, be found, which could corroborate the views of the author, and he recommended such examples being sought for as illustrations.

Mr. R. STEPHENSON wished to express his conviction of the useful character of the paper, which, he was convinced, would remove many difficulties hitherto felt in examining the subject by the process laid down by Professor Moseley, whose formulæ, though highly scientific, and no doubt very beautiful, were much too abstruse for the use of the practical man. Anything which tended to elucidate these formulæ, and render the subject more popular, must be received with great interest by the Civil Engineer, whose labours would be materially facilitated by such clear adaptations of theory to practice.

It would appear, that the principal novelty consisted, in describing

by a simple process from two given, or assumed, points, a curve of equal horizontal thrust, falling within such points in the voussoirs, as should demonstrate the stability or instability of the structure. There could be no doubt of the value of such a process; but he would suggest to Mr. Barlow the desirableness of giving, in somewhat more precise and simple terms, the mathematical demonstration of that which must be universally admitted in practice. He would suggest whether Moseley's term of the 'line of pressure,' as contradistinguished to the 'line of resistance,' did not convey the meaning of the proposition better than the term 'curve of horizontal thrust.' It was accepted as perfectly true, that, as stated in the paper, the horizontal force at any part of the curve was opposed by a horizontal force of equal amount, exerted in an opposite direction, and that the horizontal force or thrust was equal throughout the curve, and hence the equilibrated arch; yet this had not been hitherto clearly and simply laid down, in such a manner as to be practically used.

Mr. BIDDER accorded with Mr. Stephenson in his appreciation of the value of the paper, he had seldom heard one of greater utility, and he trusted so good an example would be followed in the Institution. The proposed mode of describing the curve or line of pressure, showed the impropriety of constructing brick arches in separate superposed rings; the line would in almost every instance, travel out of the ring in which it commenced, and in case of fracture, the rings would fail consecutively; but if the arch was well bonded together throughout its entire depth, the line, or curve, would be traced within it, and it would possess the requisite strength. All the best brick arches were now built in that manner with full bond. An arch had recently been so built by Messrs. Grissell and Peto over the River Lea, with a span of 87 feet, and a rise of 6 feet; the centres were struck within an unusually short time after the arch was keyed; but it stood perfectly, and with very little subsidence.

Mr. Bidder was tempted to consider an arch constructed of rectangular bricks set in a matrix of cement, as a bent trussed girder, the tension rods of which were represented by the abutments of the arch. Very flat arches, such as the Maidenhead Bridge, were examples of what he meant.

Mr. BRUNEL could not agree with Mr. Bidder's comparison, or what he might be permitted to term his amusing theory; on the contrary, he must contend that there was no analogy between the arch and a trussed girder. In the former the main force was pressure, in the latter the force exerted was tension; the abutments of the one had to resist a horizontal thrust, at a given angle, whilst

the wing walls, under the other, had to support only a vertical pressure ; any tendency towards horizontal thrust, which might have arisen from deflection of the beam or girder, was prevented by the tension rods which connected the opposite extremities. If an arch could be considered as a bent trussed girder, it must follow, that it would stand equally well whether the curve was upwards or downwards, which certainly did not accord with his notions of the properties of an arch.

Mr. BIDDER replied that his views were misapprehended ; what he contended was, that a brick arch being formed of rectangular pieces, set in a matrix of cement, having great adhesive properties, upon which it in a great measure depended, it should be considered as a homogeneous mass, assuming the nature of a curved trussed girder, the resistance of the abutments acting as the tension rods of a girder. He must still contend for his position, and that the bridges of great span and small rise, erected by Mr. Brunel, were excellent examples of the construction he meant.

Mr. R. STEPHENSON conceived that Mr. Bidder only meant to put forward the position for the sake of argument. It was certain, that the arch and the trussed girder, being supposed to be formed of the same materials, the former would be supported by the resistance of the abutments to compression, and the latter by the tension of the tie-rods ; the adhesive properties of the materials not being in either case taken into consideration. The arch, *per se*, should always be considered as composed of separate masses, not set in a matrix ; but combined in a certain form, the only adhesion being the friction of the surfaces.

It would be desirable if Mr. Barlow would give a more perfect mathematical formula for describing the curve ; the rule which he had given had too much the character of being empirical and of being made to fit given cases.

Mr. W. H. BARLOW was unable to perceive any deficiency in his definition, or in the method by which he ascertained the curve. The line of thrust, as obtained by the construction given in the paper, was practically given in the models.

It was not a necessary condition of stability, that the line of pressure should intersect the surfaces of contact at right angles, it was sufficient that the direction of the pressure should meet the surfaces of contact, within the limiting angle of friction. The same condition was exemplified in a column ; there the line of pressure was a vertical line, but the surfaces of contact of the stones might be inclined, without occasioning the upper part to slip, provided the inclination was within the angle of friction of the material employed.

Moseley's formulæ, although theoretically perfectly accurate, were too complicated, and involved too much mathematical knowledge for the general use of practical men.

A deep or thick arch contained more than one 'line of pressure;' the line of pressure to be dealt with in practice was, in effect, the centre of a surface of pressure.

Mr. R. STEPHENSON said, that mathematicians always considered the line of pressure to be at right angles with the supporting surfaces or the abutments. It would appear, however, from Mr. Barlow's explanation, that instead of drawing a series of lines at right angles to the surfaces through given points, thus forming what might be termed the Polygonal theory, he described a correct curve through the same given points. Mr. Stephenson could not understand how the voussoir could give a line differing from the line of force treated of by mathematicians.

Mr. BRUNEL said, the subject was one of great difficulty, as it embraced so many considerations; it might, however, he thought, be rendered simple, by considering an arch not as composed of separate voussoirs bound together by cement, thus involving other principles, but as a homogeneous, and, he might almost say, an elastic mass. If viewed in that light, the pressure would be found to extend more or less over the whole surface. The 'centre line,' or 'neutral axis,' might in such case receive the denomination of the 'line of pressure.' If this idea were followed up, there would be less difficulty in explaining the principles laid down by Mr. Barlow.

Mr. W. H. BARLOW said, that Mr. Brunel's 'line of neutral axis' expressed more nearly what he understood by the 'line of pressure,' and that line described by the impinging points of the curved surface of the voussoirs of the model (fig. 1, plate 1). Mr. Barlow thought, that Mr. Bidder's experience, as to arches turned in one entire bond, being stronger than those composed of separate rings, bore out the deductions of the paper. The rings could not separately contain the curve of equal horizontal thrust; but when bonded they did so. An arch turned in separate rings depended too much on the adhesive strength of the cement or mortar.

Mr. CUBITT, *V. P.*, said, it appeared to him that the whole question was contained in the proposition demonstrated by the model with curved voussoirs (fig. 1, plate 1), where the points of contact, and consequently the curve of pressure, varied according to the spot where the pressure was imposed. In practice, this pointed out the necessity of adapting the form of the arch to the service it was intended to perform. For instance, if the roadway over an arch were level, and the pressure equal, the fracture would take place by depression of the

haunches and the opening of the extrados of the crown ; but if the roadway were curved, the pressure being thrown upon the crown, the crown would fall, and openings would occur at the extrados of the haunches. Different calculations must therefore be made for the different constructions.

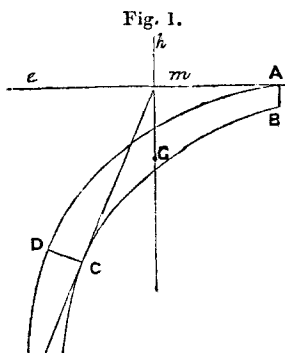
Mr. G. SNELL observed, that Mr. Barlow, in his geometric construction of the line of resistance, assumed, that there were already two points found. Now two points in the line of resistance were determined, when the points of rupture were known ; for, at the time when rupture was about to take place, when the arch was balancing between standing and falling, the line of resistance touched the extrados or intrados of the arch, at the points of rupture. One of these points of rupture was determined by the conditions of the question, the other might be determined by a geometric construction, founded on the principles set forth in Moseley's works, and which he had demonstrated in his lectures. The process was one of approximation, and he supposed three trials would be sufficient to determine the point of rupture, with perfect accuracy. The process would apply to all shapes of arches under pressure, in any direction, in any position, or of any amount ; but Mr. Snell would at present confine himself to the simple case of an arch which was loaded equally on either side, and the voussoirs of which were equal each to each, on either side of the centre line.

In such a case, the one point of rupture would be at the crown of the arch, which would be on the point of turning on one of its edges, at its extrados, if the arch was about to fail by the sinking of the crown ; and at its intrados, if it was about to fail by the rising of the crown.

In the first case, some stone at the haunches would be on the point of turning on its edge at the intrados.

In the second case, some stone at the haunches would be on the point of turning on its edge at the extrados.

He would confine himself to the first of these cases.



Then, to find the point of rupture, choose some point C (fig. 1), which was considered to be near the point of rupture, and which, in this case was at the intrados ; draw CD the joint of the voussoir.

The arch being about to fail, by the turning of the key-stone on its edge at A , the resultant of all the forces, at that point, must touch the curve of the extrados at A ; its direction was therefore horizontal, and was represented in position and direction by the line Ae .

The resultant of the weight of the mass A, B, C, D , was represented in position and direction, by a vertical line passing through the centre of gravity, G , such as GH .

Now, the only forces acting on the point C (in this case) were the resultant of the forces at A , and the weight of the mass A, B, C, D , and these, being represented in position and direction respectively, by Ae and GH , which intersected at the point m , the resultant of all the forces acted through m ; it also acted at C , and therefore mC represented the resultant of all the forces acting at C .

Now, as before stated, if C was the point of rupture, the line of resistance touched the curve of the intrados at C , therefore a tangent to the line of resistance at C was also a tangent to the intrados, and the resultant of all the forces, acting on any point in the line of resistance, was in the direction of a tangent to that line.

Therefore, if C was the point of rupture, mC was a tangent to the line of resistance, and therefore mC was a tangent to the intrados, as in fig. 1.

If C was not the point of rupture, but if the point of rupture was above C , mC would cut the intrados, as in fig. 2 ; but if the point of rupture was below C , mC would cut the intrados, as in fig. 3.

Fig. 2.

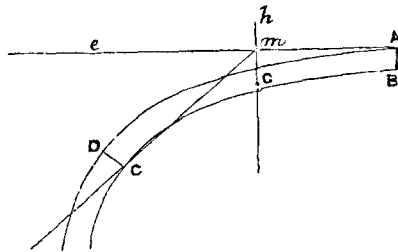
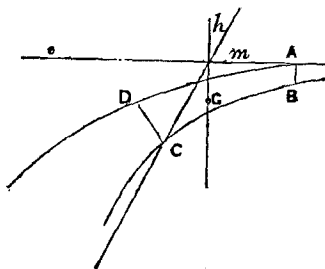


Fig. 3.



Mr. W. H. BARLOW said, he assumed two points in order to facilitate the investigation. As far as practical utility was concerned, the line of thrust might thus be obtained at one operation, instead of pursuing the laborious process necessary for determining the theoretic line of resistance; indeed, excepting for the most regular geometric forms of the arch, Moseley also adopted a tentative process. Moseley's line of resistance touched the intrados and extrados at the points of rupture. Now a practical eye could detect very closely which would be the point of rupture, and a curve of equal horizontal thrust drawn through these points, though it might not produce the line of resistance with mathematical accuracy, was sufficiently near it for all useful purposes.

Moseley's theory was undoubtedly very perfect; in fact he was the only mathematician who had treated the subject consistently with its practical requirements. The difficulty in his mode of investigation was in those arches which did not partake of regular geometric forms, and in these cases Mr. Barlow's method would be found easy of application.

Mr. BRUNEL still thought, that Mr. Barlow had scarcely met the objections which had been raised. It was true, that in practice some points might be assumed; but it was more satisfactory to have positive rules for finding these points, and assuring the mind as to the correctness of the basis of the proposition. In a very large arch, with a small rise, the line of pressure must be confined within very narrow limits, and in such a case a formula giving the points definitively was essential for inspiring confidence.

Mr. W. H. BARLOW replied, that the limits which confined the line of resistance, depended on the thickness of the arch and not on the ratio of the rise to the span; the points of rupture in ordinary forms of arches were well known; they were at the extrados in the crown, and at the intrados in the haunches; there was, therefore, no

difficulty in finding the line of resistance in these cases. If the mind was as perfectly impressed with the direction of the forces in arches, as in the case of columns, both could be built with equal security.

Mr. G. SNELL stated, that in all cases of equal thickness of voussoirs throughout, Mr. Barlow's rules might apply; but if the thickness was less at the crown, as in the case of an arch with a keystone of limited depth, but of which the voussoirs increased towards the abutments until they came to an extreme length, he did not see where Mr. Barlow could assume his points in the line of resistance.

Mr. W. H. BARLOW replied, that in reference to that particular form of arch, it was evident many curves of equal horizontal thrust would be drawn within the thickness, so that it was unnecessary to entertain the question; because, if any one curve of equal horizontal thrust was contained, it proved that the theoretical line of resistance was also contained. It would be observed, on referring to the paper and consulting the drawings and models, that the rules were general, and applied to every form of arch and archiform structure, loaded or unloaded, and whether of equal thickness or otherwise. The model, with the rectangular voussoirs leaning together at the apex, was selected as an extreme case. He wished to remove an impression, which might have been produced, by his stating that his mode of treating the subject of arches was not so mathematical as that of Professor Moseley: he only alluded to the use of geometric construction instead of algebraic formulæ; the principle or theory was the same in both cases.

The misapprehension as to assuming points in the curve, which Mr. Stephenson alluded to, as not having been sufficiently explained, arose from the modification which was necessary in applying theory to practice. If perfect hardness of materials and mathematical accuracy of workmanship were attainable, the pressure would be transmitted in the line of resistance, as laid down by Moseley, and described by Mr. Snell. On the other hand, if the materials were in the softest state in which it was possible for an arch to sustain itself, the pressure would be transmitted in that curve of equal horizontal thrust, which corresponded most nearly to a line drawn through the centre of the thickness of the arch; because in that state of the arch, the whole available surface of the voussoirs must be acting, to support the insistent pressure; practically, therefore, the pressure would be transmitted in a curve of equal horizontal thrust, somewhere between these two limits.

Now, in the case of large brick arches, particularly when the centres were first struck, the state of the arch approached that which had been just mentioned, and it was for that reason he had stated in the

paper,* that in determining abutments for arches of large dimensions, the points $p p'$ should be taken in the centre of the thickness of the arch, though the extreme limit of stability, if the materials were hard, would be when the points $p p'$ were in the theoretical line of resistance.

Assuming the points $p p'$ to be in the centre of the thickness, and making the abutments accordingly, was in effect nothing more than providing abutments of such dimensions as should resist the thrust of the arch, when it was in the most disadvantageous state in which it was possible for it to exist.

An arch constructed with abutments only just sufficient to contain the theoretical line of resistance, would possess the same degree of stability as a column placed so far out of perpendicular, that a vertical line drawn through its centre of gravity would just fall at the extremity of its base; but an arch, with abutments built so as to contain the curve of equal horizontal thrust, which accorded nearest to the centre line of its depth or thickness, would be under the same condition of stability as a column placed perfectly vertical.

February 10, 1846.

SIR JOHN RENNIE, President, in the Chair.

The discussion upon Mr. Barlow's paper (No. 728), "On the existence (practically) of the line of equal horizontal thrust in arches, &c.," was continued, and extended to such a length as to preclude the reading of any communication during the evening.

February 17, 1846.

SIR JOHN RENNIE, President, in the Chair.

No. 732. "On Water for Locomotive Engines, and its Chemical Analysis." By William West, Assoc. Inst. C. E.

When locomotive engines, in consequence of the invention of the tubular boiler, first became generally applied to railways, it appeared as if engineers were contented with obtaining a sufficient quantity of water at each station, omitting all inquiry as to its quality.

This continued to be the case on some lines for several years. In

* Vide ante, p. 172.