

Discussion.

Sir Frederick
Bramwell.

Sir FREDERICK BRAMWELL, President, said that the Paper having been read only in abstract, there had been no mention whatever of what the Author himself had done. The members would no doubt, under the circumstances, allow him considerable latitude in personally explaining the apparatus on the table. He would not, however, ask them to defer the expression of their thanks to Professor Shaw for his valuable Paper until this explanation was given; but he felt sure that after this was done it would be still more clear that those thanks were well deserved.

Professor H. S.
Hele Shaw.

Professor H. S. HELE SHAW said that the Paper had been only read in the form of a brief abstract because from the nature of the subject, and its method of treatment, it appeared advisable that he should personally give a short account of its contents. The engravings, of which there were a good many, could not be prepared in time to be sent to those who were likely to take part in the discussion, and therefore he would explain the principal points which he believed to be original, and which he hoped would be thoroughly discussed. He would take this opportunity of thanking Professor Amsler for kindly lending him several instruments, some of which were now shown for the first time in this country, having been sent from Switzerland for the purpose, at Professor Amsler's own expense. He also wished to thank Mr. C. V. Boys for lending him models of his tangent integrators, and also two of the actual instruments to exhibit; and his friend and colleague, Mr. C. D. Selman, for several valuable suggestions and assistance in the preparation of some of the diagrams. The Author then proceeded to explain, by means of the diagrams on the walls, and by models which had been constructed on a large scale, the principles of the classification adopted in the Paper and the various instruments exhibited.

Mr. Anderson.

Mr. WILLIAM ANDERSON (of Erith) observed that he had had considerable experience with continuous integrators in measuring work done by agricultural implements. There was a good deal to be said about the use of those instruments and the defects to which they were liable, about the personal error, which was an important point, and errors from imperfect adjustment. The conclusion at which he had arrived with regard to continuous integrators in which the space passed over and the strain were multiplied together and registered continuously, was that they were exceedingly good for comparative results, but were not altogether to be trusted for positive indications. In comparing,

for example, a number of machines working in a field under Mr. Anderson, similar circumstances as to weather and everything else, with the same operator, the comparative results would be trustworthy; but if there were any variation in any of the conditions, they would not. There was always a good deal of doubt about the positive results. The causes of error were these. In the continuous integrators the integrating wheel was attached to a train of wheel-work which possessed a considerable amount of inertia and friction. In planimeters and integrators of that class, where the observations could be made slowly, at a steady speed, and where the conditions did not vary, inertia did not count for much; but in the steam-engine, and in agricultural implements or in traction indicators, there were great and rapid variations of speed. The sudden strains which were put on by the tractive force shifted the integrating wheel along the disk suddenly, and the speed changed in a similar manner. The force necessary to accelerate the movement of the integrating wheel and its mechanism tended to cause a slip, which was partly counteracted by a slip produced by the force necessary to arrest its motion when the speed changed, but the friction of the mechanism always acted in the same direction, tending to augment the error. With one implement, for example, if there were a tolerably steady pull with no sudden variation in the velocity, there would be one amount of error, whereas if another implement were worked in a jerky fashion there would be a totally different amount of error. With regard to personal error, to which the Author did not attach much importance, he had reason to think that it was of great consequence. The degree of care and skill exhibited in adjusting the instrument and taking the measurements had an important influence; he was therefore always careful, in a series of experiments with agricultural implements, to have the same observer throughout, if possible, because the results aimed at were rather comparative than positive. Still he was bound to say that, with care and experience, satisfactory positive results could be attained. The Author did not appear to be aware of the extensive use which the Royal Agricultural Society had made of the integrator which had always been known as Morin's, but it appeared that honour had been ascribed where it was not strictly due; probably the reputation of the great French mechanic, who had done so much to introduce it, had obscured the claims of the real inventor. For the last thirty-five years the Royal Agricultural Society had used continuous integrators, and he thought that there was no one more competent to speak of their action than Sir Frederick

Mr. Anderson. Bramwell, who had himself conducted many of the experiments. The issues had often been very important, involving the fortunes of manufacturers of agricultural implements, which had, in a great measure, hung upon the indications of the dynamometers. Most of the apparatus used by the Royal Agricultural Society had been designed and constructed by the late Mr. C. E. Amos, M. Inst. C.E., and by his son Mr. J. C. Amos, who for many years filled the office of Consulting Engineers. With reference to continuous indicators for steam-engines, he had little or no experience. He had tried them, but the results had not been, so far, satisfactory. One of the chief defects was the difficulty of keeping the little integrating wheel perfectly free from flats. It was not easy to find any metal perfectly uniform throughout; and if it were not uniform, a flat would soon form, and then all the results would be utterly untrustworthy, because the wheel tended to hesitate at the flat place. Formerly integrating wheels were made of gun-metal, cast with great care, and under a great deal of head. Latterly they had been made of steel, and a better result had been obtained. But even when the wheel wore uniformly, if the width of the surface in contact with the disk varied, then again there was a source of error, because the surface of the integrating wheel lying upon the disk became greater, and then there was uncertainty as to the true diameter of the periphery of the disk on which the integrating wheel was working. The only way of eliminating these errors was by repeated testing of the apparatus.

Mr. Boys. Mr. C. VERNON BOYS said the first part of the Paper on which he desired to say a few words was the division of the subject into different classes. The Author had referred to one system of classification which Mr. Boys had adopted in a Paper published in the 'Philosophical Magazine,'¹ a division into three classes. The Author had rightly shown that the two classes which Mr. Boys had called the "radius class," and the "Amsler class," were in a mechanical sense, that was so far as the connection between the surface of the integrating wheel or roller and the surface on which it worked was concerned, absolutely identical. But though that was undoubtedly the case, he thought the division might still hold good, for an inventor could not have contrived machines in one class or in the other without having had some such system of division in his mind at the time of the invention. There was one considerable omission in the Paper, and that was the only point on which he felt it necessary to find

¹ Vol. xiii. Fifth series. 1882, p. 77.

fault. There was no mention of a very large series of most beautiful machines, designed and he believed partly constructed by the Author himself. As those instruments had been fully described in a Paper by Professor Shaw before the Royal Society, possibly he thought that they were so well known that it was unnecessary to describe them again; but it would certainly have rendered the Paper far more complete and valuable if that large amount of work had been incorporated in it. Of all the instruments brought before them he thought that the new precision planimeters and that extraordinary spider-looking instrument crawling on the sphere were those which called for the utmost admiration. It was impossible to look at them or to use them without being impressed with the extreme mechanical beauty of their construction and design. But though that was undoubtedly the case, he thought that no one could see some of the combinations of a sphere with three, eight, or six little wheels round it, inventions of the Author, without classing them even above those instruments in point of beauty.

The Author had spoken of the tangent principle as being a particular case of machines which worked without slipping. This was perfectly true; but there was a very great distinction between integrators depending upon the tangent action and integrators depending upon a rolling action of the radius class, to which alone the other machines belonged. There was one accidental mistake in the wall diagrams representing two spheres¹ which he desired to point out, as it was a little puzzling. In the instrument of Clerk Maxwell, one of the non-slipping radius class, the axes of the two spheres were shown in such a position that rolling would ultimately make them coincide. In reality the equator of one rolled over the pole of the other, as was obvious to those who had anything to do with integrators. The distinction between instruments of the tangent class and those of the radius class might be represented by the little wheel of the instrument. The accuracy of integrators of the radius class depended upon the exact size of the wheel, and the exact size of the surface upon which it rolled; and, as Mr. Anderson had remarked, it was very much impaired when flats formed upon any of the surfaces in contact, for there was then a little hesitation. The mere action of integrating-machines in which there was slipping was sure to produce those flats at some time or other, so that the time they were likely to last and the amount of work they were capable of doing were limited. The actual size of the roller was

¹ This has been corrected in Fig. 33.—SECT. INST. C.E.

Mr. Boys. of importance, for as it wore and became gradually smaller, the number of rotations were affected, and therefore the recorded result gradually increased. These were the objections to instruments of what he had called the radius type. In the class of instruments at which he had worked exclusively, and which he believed he had originated until he found that to a certain extent Mr. B. Abdank-Abakanowicz had preceded him, the wheel was allowed to roll along just in the direction in which it was pointing. Any one who had been in the streets of London must have noticed with what extraordinary persistence and power the wheels of all vehicles went in the direction in which they were pointing. A butcher's cart driven round a corner with tremendous fury would often, in spite of its jumping on the ground, still continue its course with very little side slip. If the springs were sufficiently good, or the road was sufficiently smooth, so that the wheel kept in contact with the ground, it would apparently not slip at all. In the case of the first instrument which he had made, which he had called the cart integrator, he was astonished to find with what extraordinary accuracy results were obtained by its use, such for example as the area of a circle and other things that were known. In that instrument he had depended entirely upon the fact that the steering-wheel of a tricycle would go along in the direction in which it pointed. The instrument had to pull a heavy brass-cart after it and slide in a comparatively roughly made groove, but even so he found the value of π on squaring the circle came out 3.14 when using a very rough home-made instrument. It seemed, then, that if the sources of side-slip which were undoubtedly present, due to the great friction that had to be overcome in dragging the cart, could be removed—that was, if the ground under the cart could be made easily movable, which was the case when it was converted into a cylinder, the cylinder would follow exactly in the direction in which it should go. For that reason an integrator of that class was free from the objections which had been raised, that in time integrators wore out, the little wheel got flats upon it, and as the size varied the record varied. In the case in point there was no slipping and no tendency to make flats, and if the wheel was made half or twice the size the record was the same, for it only depended on the direction in which it wanted to go, not upon the amount that the wheel turned in going along in that direction. He desired to say a word or two with regard to an instrument, his engine-power meter, which was not so well known as he had hoped it might be. There were certain sources of error apparently present in it, but which were

really imaginary. In the first place, it would seem that if the Mr. Boys. position of the piston-rod, and, therefore, the angular position of the little tangent wheel G varied in the least, then as the cylinder H travelled along there would be an error due to the angular want of true precision. But that was not so, for supposing the spring to be too long or too short, and the tangent wheel to be permanently deflected, when there was no steam-pressure, at an angle, say, of 2° or 3° , then as the cylinder moved in one direction, the wheel would run up a certain slope, and when the cylinder moved in the other direction, it would run back along the same slope, and so far as that was concerned, no error would be introduced. The wheel might be set at a permanent angle, and the roller work backwards and forwards, and nothing would be recorded at the end of the operation. He could show it with a rough paper and wood model. If he permanently compelled the tangent wheel to assume a certain angle by holding it with his thumb, as the cylinder travelled in one direction it would rotate a certain amount, and when it travelled in the other direction it would rotate back again, and on the dial there would be no permanent record. The disk-cylinder integrator had one serious defect; it was very difficult to apply it to cases in which growth of time or of motion was continuous. If it was desired to have a time-integral and a motion-integral, when the motion was continuous, it could not be easily applied, because when the cylinder had got to one end of the stroke, there was nothing for it but to stop and come back again. It was, therefore, necessary to apply a mangle-motion, so that as the motion was continuous the cylinder went backwards and forwards; then by causing the cylinder to work between two tangent wheels alternately, or by letting the mangle-motion work ordinary reversing gear between the cylinder and the recording mechanism, continuous integration could be effected. But of course the instrument, where continuous integration was concerned, would not compare with the spherical integrators designed by the Author. On the other hand, he did not think that any instrument, in the peculiar case presented by the automatic integration of an engine-diagram, could compare with it, for those peculiarities of motion which interfered with the ordinary form of radius machines by causing a perpetual scrubbing, to which reference had been made, produced no trouble at all, for there was no side-slipping, and because the very large moment of the radius integrator was replaced by the extremely small moment of a little cup containing a wheel not the size of a three-penny bit, and by a little bar of steel. In fact,

Mr. Boys. the moment of the piston and its attachments was nothing like so great as that of an ordinary Richards' indicator, because there was no multiplication of motion.

Mr. Anderson. Mr. W. ANDERSON wished to state that he had been able to bring to the meeting the integrating part of one of the old dynamometers of the Royal Agricultural Society. The date of its construction was unknown, but he believed it was about the year 1848.

Mr. Mair. Mr. J. G. MAIR observed that the Author had not given himself sufficient credit for the machine he had invented, and he would like to ask him if the reading of the counter gave such accurate results as the vernier on Amsler's planimeter. To read by means of a counter was of great advantage, and especially so where large numbers of indicator-diagrams had to be taken out; on some of the engine trials he had made, three hundred diagrams had to be averaged, and constantly reading the vernier on Professor Amsler's instrument was trying to the eyes. The application of the integrating machine as a power-meter was a most useful one, and he had made several trials with the one invented by Mr. C. Vernon Boys; on three trials the readings were 68·5, 68·2, and 73 by the meter, against 67, 67·5, and 72 in the pump. The pump-power was taken from the displacement of the pump-piston, and the head as shown on a mercury gauge. He thought those results were a proof of the correctness of the instrument. He did not think that reading a counter was more difficult than reading the dial of a watch. One other measurement had to be made, namely, the distance between the stops, and as that could be measured with an ordinary rule, a child could almost make the simple calculation, so that he did not think personal error very much affected such an instrument. There was naturally a good deal of diffidence in using a machine, the details of which were not thoroughly grasped. With an indicator-diagram, there was something to see which was readily understood; but where compound measurement was shown on a counter, it was at first difficult to realize the reading as correct. As soon, however, as the instruments were better known, he had no hesitation in saying that, where the absolute shape of an indicator-diagram was of no consequence, the continuous integrator would entirely supersede all other means of measuring power.

Mr. Halpin. Mr. DRUITT HALPIN said that the Author had (p. 88) referred to one of the minor improvements by putting a locking-spring to the frame. Mr. Mair had noticed the great difficulty there was in reading the instrument, on account of the small scale to which it was graduated, but the Author had not completely followed the

idea of adding the locking arrangement. The convenience of Mr. Halpin. reading the instrument was doubtless very great, for it could be taken up and put in a good light, but the real object of the locking gear was to make the instrument do its own addition, and also carry over the decimal places now lost. Instead of taking each diagram by itself, and writing down the result of each separately, and adding them up, the instrument was set at 0 and locked; the instrument was run over the diagram, and it was locked, and so on. So that by taking ten diagrams successively, and putting the decimal point one point back, the instrument was made to do its own addition. It had the further advantage that, whatever the last decimal might be, it was carried on by the instrument. If it was 2·38, it could not be said whether it was 2·389 or 2·381. With regard to Mr. Boys' application of these instruments, he had had an opportunity of testing it on an engine of 120 or 130 I.H.P., and he found the results coincided within from 1 to 2 per cent. of the indicated power which was obtained with standard indicators. With Mr. Webb's permission, he also put one of his power-meters on the large rail-mill at Crewe, and took a series of observations there of the exact power required to roll a rail, from the moment the ingot touched the rolls to the moment the next ingot touched them, which gave the true power, correction being made for any difference of velocity in the fly-wheel. It gave the power both while the engine was running with the bloom in the mill, and when it was running empty. Whether the machine in its present form would be suitable for locomotives, he was hardly prepared to say, because he feared that the attachment which was provided for taking diagrams, was perhaps so heavy that its inertia would interfere with the correct action of the instrument. But, if that was lightened, or the attachment left out, he was sure the best results would be obtained from it.

Mr. H. CUNYNGHAME said that his attention had been called a great deal to Mr. Boys' machines, in the development of which he had assisted, as they were first designed for practical application to steam-engines. He had made many attempts to apply continuous integrators for the purpose of integrating electric power, and he certainly could safely bear out all that Mr. Anderson had said about the imperfections of integrators of the slipping type. If a wheel was running in any direction in which it was likely to slip upon the surface, it would be rubbed into facets, and in like manner a sphere would be rubbed into a polyhedron. In this state both wheel and sphere would be worse than useless for integration. But if these machines had an integrator of the roller type, even if

Mr. Cunyng-
hame.

Mr. Cunyng- it had facets already, the rolling would take the facets out of it
hame.

just as an apothecary rolled his pills. Instead of an integrating wheel of hard steel, as was necessary in all integrating machines of the slipping type; in those of the rolling type, soft metal might be used, and the more it was used, the better it would get. Moreover in such machines, since what was counted was not the revolutions of the wheel, but the revolutions of something caused to roll by means of the wheel, the accuracy of shape of the rolling wheel became of minor importance, and even if it "skated" over the surface instead of rolling a fair result would be obtained. He thought Mr. Anderson's remarks were somewhat unjust to planimeters of the rolling type, which he did not think had been brought before the public so as to be within the experience of any one. That led him to say a few words on Mr. Boys' steam-power meter. He believed that Mr. Boys and he were the first in this country to make a trial of those machines. They made a trial at the works of Messrs. Ransome and Jocelyn at Battersea, and for that purpose they had an extremely good indicator made by Messrs. Elliot. It would be quite understood that Mr. Boys' indicator was not in any way calibrated by mere trial. They took the measurements of the engine and the diameter of the cylinder. Then they took the amount by which a certain pressure of steam would cause the spring to rise, just as was done before taking the HP. by means of a Richards' indicator. They thought that if those two machines were used upon the same engine, and if the results given by the two machines, namely, Richards and Boys, were independently calculated from the constants of the two machines, then if these results nearly corresponded, a very remarkable coincidence of testimony would be obtained. They therefore put Boys' machine on the engine alternately with Richards', and he thought they must have alternated twenty times, and the results of Boys' machine were found to be uniformly during the first series of experiments about 23 per cent. too high. That puzzled them extremely, but upon examination they found that the manufacturer, instead of making the piston 1 inch in diameter, had made it 1 square inch in area, and when they made the proper correction they found the results to agree to about $1\frac{1}{2}$ per cent., and with those stated by Mr. Halpin and Mr. Mair. Mr. Mair's tests were made in this way: he raised, by means of a pump, a given weight of water through a number of feet, and then he estimated the foot-pounds and compared them with one another, and the result showed a remarkable degree of accuracy, the discrepancy of 1 or 2 per cent.

being accounted for by the loss of work owing to the raising of the pump-valves, which were large and heavy. Mr. Cunyng-
hame.

Dr. WILLIAM POLE, after testifying to the high character of the Paper, offered a few remarks on the very early example of mechanical integration with which the Author had connected his name. It had come about in the following manner. Some half-century ago, the engineers of the centre and north of England became aware of the reports published from time to time of the extraordinary economy of the pumping-engines of the mines of Cornwall. These reports at first obtained no credence, and even when they were found to have some foundation, the most singular attempts were made to explain them away. In the midst of the controversy, the late Mr. Thomas Wicksteed, M. Inst. C.E., the Engineer to the East London Water-works Company, determined to throw light on the question by buying an engine in Cornwall, and setting it up to pump water on his own premises at Old Ford, where it could be thoroughly tested and examined. Dr. Pole.

The subject had previously been brought before the notice of the British Association for the Advancement of Science, and had attracted the attention of Professor Henry Moseley. He, in writing his excellent work on "The Mechanical Principles of Engineering and Architecture," had become acquainted with a principle of dynamometrical admeasurement proposed by Mr. Poncelet, and carried out in 1838 by General Morin; and it occurred to him that a machine might be contrived on a similar principle, applied to record the work done by a steam-engine, by a species of mechanical integration, combining the pressure exerted on the piston with the space moved through; and it was seen that such a machine would be most usefully applied in testing the performance of Mr. Wicksteed's Cornish engine.

At the meeting of the British Association in 1840, a grant was made for the purpose, and a committee, consisting of Professor Moseley, Mr. Eaton Hodgkinson, and Mr. J. Enys, was appointed to carry it out. The machine was constructed under Professor Moseley's direction, and a full account of it was given in the Report for the following year. The trials were then made on the engine, and the results were exceedingly satisfactory. The integrator worked for a month without intermission, and its indications, when calculated out, were found to agree closely with the results obtained, as accurately as they could be, by other means.

The fixed data of the engine had all been well ascertained, but, to render the comparison complete, it was found desirable to get, if possible, an accurate measurement of the velocity of the piston

Dr. Pole, at various parts of its stroke; this velocity was very variable, depending not only on the mass in motion, but also on the ever-varying force of steam acting on the piston, and no means had hitherto been devised for ascertaining the velocity experimentally. The attention of the Committee had, however, been directed to an admirable chronometrical instrument contrived by Messrs. Poncelet and Morin, and Professor Moseley undertook to adapt a machine on this principle to the Old Ford engine.

At this time Dr. Pole (who had been occupied independently in investigating the action of the Cornish engine) had the honour of being invited to join the Committee, and the two succeeding Reports, in 1843 and 1844, were written by him.

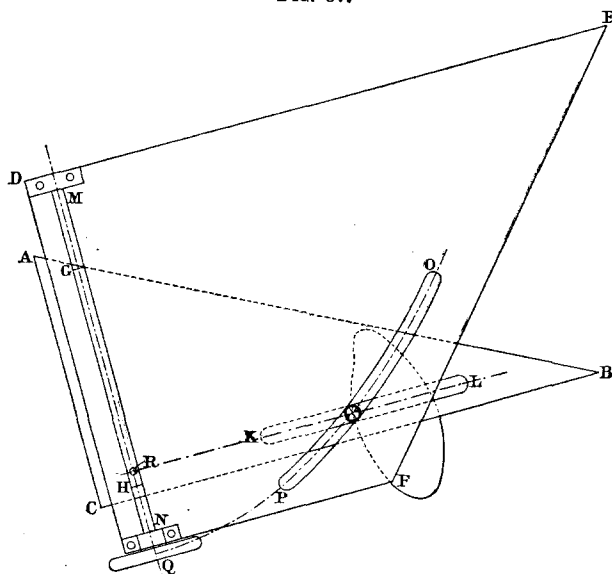
By means of these two instruments, and by an ordinary indicator, aided by careful observations as to the consumption of coal and water, and other variable factors, the working of the Old Ford Cornish engine was investigated, both theoretically and practically, in a most complete and accurate manner, and the peculiarities of this form of engine, as compared with the ordinary engines in use at that time, were thoroughly brought out,¹ so as to clear up all the doubts that had been so long entertained by engineers. This application of a mechanical integrator to real work of magnitude and importance was probably the earliest made. It was intended to follow it up by adapting the machine to other purposes, especially to ocean-going steamers, and it was actually fitted to the engines of the "Great Western" steam-ship, on her first voyage, but by an accident which it was difficult to repair at sea, the experiment was rendered useless, and the attempt was never renewed.

Mr. Bousfield. Mr. W. R. BOUSFIELD said he had not had the advantage of reading the proof of Professor Shaw's Paper, but he had heard it read in its short form at the previous meeting, and he thought possibly it might be of interest, and useful in connection with a Paper which dealt, more or less, with the different types of mechanical integrators, if he mentioned a principle which, so far as he knew, had not been alluded to. It occurred to him some years ago that, by the use of various curves in connection with planimeters, various products and squares might be readily integrated. A simple form of area-planimeter of this kind might be made by combining a T square and a set square, or two set squares. Referring to the

¹ This investigation was afterwards published in "A Treatise on the Cornish Pumping Engine. By William Pole. Part iii., Analytical Investigations. Weale. London, 1848."

illustration, Fig. 57, the lower T square or set square A B C would have in it a slot K L and upon it a rib G H, and would be capable of motion about a point R in line with the slot. The upper set square would have a slot M N capable of sliding upon the rib G R, and also a parabolic groove O P. A pointer—in practice a ring with cross wires—would slide in the slots K L and O P. In using this planimeter a needle point would serve to pin the lower set square through the point R, at a convenient pole in relation to the area to be integrated. The cross wires or pointer would be carried round the curve enclosing the area to be integrated, the two set squares at the same time sliding on one another by means of the rib G H and slot M N. The measurement would be made by the

FIG. 57.



AREA-PLANIMETER.

roller Q placed at the vertex of the parabolic curve. The principle of the apparatus was very simple. Taking polar co-ordinates, with R as the pole, the area to be integrated could be expressed as

$$\int \frac{1}{2} r^2 d\theta.$$

If now the equation of the parabolic slot were

$$y^2 = 4ax$$

Mr. Bousfield. and the parabola moved, as in the apparatus, so that y was always $= r$ the expression for the area would become

$$\int 2 a x d \theta$$

This showed that the area was measured by the roller Q at the vertex of the parabola. This proof rather suggested that possibly the use of polar co-ordinates in some of Professor Shaw's proofs would shorten them. By the use of other curves, instead of a parabola, the principle could be applied to the mechanical solution of various problems in integration.

Professor W. H. H. HUDSON remarked that the classification adopted by the Author coincided, if not entirely, very considerably with this, that in the instruments in which slipping was bound to take place, when it was attempted to express the element of area, in mathematical language $y dz$, dz corresponded to the angle turned through and y to the radius, so that the area corresponded to the length. That was certainly so in some of the earlier instruments; but those in which no slipping was allowed to take place seemed to correspond rather to a different mode of translating the mathematical expression. There was a certain amount of turning which was proportionate to dz , and then by a suitable mechanism that was turned into something else proportional to $y dz$ by using a multiplier proportional to y . One wheel was made to turn so many times faster than another, the number of times corresponded to y instead of, as in the first class of instruments, the radius being proportional to y . That multiplier occurred in the various forms of the instrument, sometimes as a sine, and sometimes as a tangent. Those of the sine class would have this disadvantage, that the multiplier must of necessity be a proper fraction. That would limit the range, whereas those dependent on the tangent seemed to have no limitation whatever, and the multiplier might be anything. This seemed to be a great advantage of the sphere instrument of Professor Shaw, that there was, apparently, no limitation at all, that which represented y being a multiplier which represented a tangent. He had always taken great interest in mechanical illustrations of mathematics. In most cases the mathematics had been incidental, but in this case it was of the very essence of the whole business. These were instruments designed to illustrate not only mathematical results, but even mathematical processes, and he was very much struck by the way in which the Author had developed Amsler's planimeter out of the simple form of the disk and roller. It illustrated a mathematical conception which was sometimes received with in-

Professor
Hudson.

credulity by those who met with it for the first time, namely, treating parallel straight lines as meeting in a point at infinity. This was done in pure mathematics, and he found that the Author had been obliged to do it in order to bring his instruments all under one head. It seemed to him that this was a striking illustration of the inter-action of mathematics and mechanics, and the benefit which each gave to the other. Professor Hudson.

Mr. W. E. RICH said the integration of the areas of plane surfaces, which these instruments were ordinarily designed to do, was much less difficult than the integration of work done on a dynamometer in the manner spoken of by Mr. Anderson. One of the great desiderata in dynamometrical measurements was to get a spring which followed Hooke's law; but unfortunately in practice they never got one that followed it precisely. The next thing was to adjust the instrument for zero. If the instrument were not accurately so adjusted, complex formulas had to be used for reducing the results, and as in many instances zero was not accurately determined, and these formulas were at the same time overlooked, the results recorded were in such cases very erroneous. One point of practical difficulty with integration for a dynamometer was the slipping of the small disk upon the large one. In the field, when it was raining, the observers were much hampered in that way, and on a stony road when travelling at a high speed with a small integrating disk, the contact between the disks was frequently broken by the shake and jar of the whole instrument. These observations were perhaps not directly pertinent to the question of integration; but one of the most useful objects of integration was for dynamometrical purposes, and therefore he thought it worth while to mention them. Mr. Rich.

Professor H. S. HELE SHAW in reply said that the question of Mechanical Integrators was a very much wider one than many would at first think; but the discussion upon the Paper had embraced two principal points: Planimeters such as were shown upon the table, and the subject of Continuous Integrators. The slipping class of Planimeter had been brought to a great state of perfection, and their accuracy was, as he had shown, remarkable. It seemed to him that the only objection to them—an objection which several speakers had adverted to—was the difficulty of reading them. Most of the remarks had been devoted to Continuous Integrators. Perhaps that was not altogether unnatural, because they had, as Mr. Rich had pointed out, such a directly practical application. Mr. Anderson had given valuable testimony on the subject of the defects of such instruments Professor H. S. Hele Shaw.

Professor H. S. Hele Shaw.

as were in use for dynamometrical purposes, and his experience agreed with that of Mr. Rich and of all who had much occasion to use them. Professor Shaw thought the general conclusion was that none of the Continuous Integrators in use for this purpose were really satisfactory in their action. To say of an instrument of measurement that it was not an accurate instrument, that it was merely capable of giving comparative results, was after all to give it very poor praise, for mere comparative results simply meant that no constant for the instrument could be obtained. He had no doubt that, for temporary purposes, the difficulties might be overcome, but it was the general opinion that an instrument like the disk and roller, which, by the way, in spite of what Mr. Anderson had said, *was* first applied by Morin after its invention by Poncelet before 1840, was not reliable when in continuous use. An instrument that worked one day differently from another day, because the conditions were slightly changed, must certainly be pronounced to be very imperfect. It was because of this very difficulty that he was led, as he had no doubt Mr. Boys and others had been, to work at the subject, and to endeavour to overcome the difficulty of the slipping action. Mr. Mair, Mr. Druitt Halpin, and Mr. Cunyngame had all testified to the satisfactory action of the tangent-integrator of Mr. Boys, which was one solution of the problem of avoiding the slipping action in question, but had been employed in a far more trying application than with a dynamometer, viz., as a steam-engine integrator. This he was very glad to hear, because the instruments of his own exhibited on the table depended upon the same principle for their successful action, and it was just in this particular that they differed from the integrator referred to by Dr. Pole. His instruments had been alluded to in complimentary terms by one or two of the speakers, and perhaps he might be allowed briefly to describe the principle upon which they worked, and also to call attention to two new instruments which were shown for the first time on the previous Thursday at the Royal Society. The difficulty which he endeavoured to overcome was to make use of those mathematical properties of the sphere, which he had previously discovered and published, and which Professor Hudson had alluded to as to sine and tangent forms. As was not surprising, he had found, when he came to collect information on the subject, that the same properties had been employed by others, amongst whom were Professor Mitchelson and Professor Amsler. But what the Author had endeavoured to do was to avoid the slipping of the measuring-roller over the integrating sphere. That was his first thought. There was generally

a germ in all inventions, and that was the germ of his. If instead of moving the measuring-rollers round the sphere they could keep them in contact with the sphere, and move the axis of the sphere, or, as it at first occurred to him, have the two rollers in contact and roll them round the spheres, they would hold it in such a way that they would allow it to rotate round the different axes in the solid. At first he did not think it would work, but he tried it with a rough boxwood ball, and two disks with india-rubber rings around them, and he was surprised to find that the rollers did move round, and to see the ball rolling round on a different axis. He thought at first it was merely a question of rolling centres, but after some time he saw that it was really a question of geometrical principle, and having certain rollers in contact round one equator it did not matter where they were placed so long as the others were perpendicular to the first, acting in fact on the same principle as that by which two bevelled wheels would work together. The axis of rotation round which the sphere must turn, because of the contact of the other set of rollers, was in the vertical plane; there was but one intersection of the two planes, and that intersection was the axis of rotation of the sphere. That principle once understood enabled him to construct various other models based upon it. He was indebted almost entirely to his brother, Mr. Edward Shaw, Stud. Inst. C.E., and to Mr. W. E. Kerslake, for working out this in a practical form, and constructing the various instruments exhibited. One of these instruments was a continuous integrator, which had been designed for application to the dynamometer of the Rev. F. J. Smith, of Taunton, who intended to have been present and take part in the discussion, but from whom he had just received a telegram stating that he had tried it most carefully and exhaustively, and had found that it had given absolutely uniform results. He had tried it himself, but it was a very satisfactory thing that it went to Mr. Smith straight from the hands of the maker, not adjusted in any way. It was remarkably simple. According as the arm was moved into a more or less inclined position, so there resulted a larger or smaller travel of the indicator. No appreciable effort was required to turn it; it was done with a minimum of power, and it entirely got over the difficulty of unequal wearing. He had every reason for thinking that if anything it would rather tend to wear the ball round than otherwise. Another instrument that he had on the table was the moment-integrator. Professor Amsler's moment-integrator was an expensive one, and at the time of writing the Paper he had not any idea of constructing one; but

Professor H. S.
Hele Shaw.

Professor H. S.
Hele Shaw.

directly he had made a small area planimeter, he saw that a much simpler instrument could be devised by employing the wheels in a very small compass, and he should be happy to explain after the meeting to any one who might be interested the performance of the operation by the application of these small wheels. Mr. Mair, he believed, asked whether the readings of the instrument were accurate. It had not been tried so exhaustively as Amsler's instruments, and therefore he could not compare it with them; but he saw no reason why they should not expect the same accuracy from it as from those instruments in the non-slipping class concerning the accuracy of which they had already heard testimony. He greatly regretted that it had not been possible to prepare the engravings in time to be sent round with the Paper, or some points he had been anxious to have discussed would no doubt have been dealt with more fully by the various speakers. For instance, the points which Professor Hudson had referred to were actually treated in the Paper, and he ventured to think were, by means of diagrams, made perfectly clear. He would remark that with reference to the range of multiplier alluded to, although it was quite true that this was much greater in the case of the tangent form of integrator, in fact was infinitely great, yet for all practical purposes the sine form enabled quite sufficient range to be obtained, while the latter had the great advantage of allowing of the polar instead of the linear form of construction. He wished to say in conclusion that the point about which he felt extremely gratified was that none of the speakers, although many of them had had considerable experience concerning integrators, had taken any objection to the division of the subject he had made. When he began to write the Paper, his only object was to endeavour to classify and put in more concise form the details of the subject, but he found the utmost difficulty in doing so. He found out that the other classifications in use would not embrace all the instruments, and were very unsatisfactory; directly, however, the mode of treating the whole question, which he had employed in the Paper, occurred to him, the matter began to assume a clearer aspect, and he had found little trouble in grouping and explaining all known integrators by its means; and inasmuch as the classification had led him to certain modifications of his own instruments, he hoped that it would not be altogether without results to other workers on the subject.

[CORRESPONDENCE.]