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Discussion
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Discussion: Evaluation of the bearing capacity of footings on slopes

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Contribution by E. T. R. Dean

Castelli and Lentini (2012) have presented interesting and challenging data. In this discussion, the contributor would like to ask some questions that touch on three aspects, and to try to place the data in the context of some of the existing extensive body of relevant literature. The three aspects are: (*a*) the failure mechanism in Figure 1 of the paper; (*b*) the scale effect in Equation 3; and (*c*) the slope effect, relating to the calibration of Table 1 and calculations using Equation 4.

The failure mechanisms

Figure 1 of the paper presents a failure mechanism whose shape appears to be independent of the presence of the slope, and which seems similar to the familiar mechanism for failure for a strip footing and surcharge loading (N_q), loading on flat ground. For later reference, the shape of the latter mechanism for a strip footing is shown in Figure 8.

Mechanisms for footings near slopes are presented by Meyerhof (1957), Brinch Hansen (1970), Bowles (1996), Das (2004) and others. In most cases the shape of the mechanism is affected by the slope. Data of actual mechanisms can be crucial in checking theoretical calculations and finite-element results. Could the authors comment further on the failure mechanisms actually observed in their tests?

The scale effect

The scale effect indicates that the bearing capacity factor N_γ, and therefore the operative average angle of friction at peak bearing stress, reduces with increasing footing size. Equation 3 of the paper implies that the bearing capacity for very large footings approaches zero. The contributor doubts if data support this assertion, and presumes the equation has limits of applicability.

Have the authors considered whether the scale effect might be explained by stress–dilatancy? Bolton (1986) examined data of many sands and found that the maximum angle of friction ϕ'_{max} depends in part on the critical state angle ϕ'_{crit}, which is a constant for a given material, and in part on the angle of dilation ψ, which decreases with increasing mean effective stress p' at failure, and increases with increasing relative density. Bolton found that

5. $$\phi' = \phi'_{crit} + k\psi$$

with k in the range 0·8–1 (with 0·8 corresponding to the proposals by Rowe (1962, 1969) for plane shear). Bolton's equations suggest, among other things, that at the maximum friction angle

6. $$k\psi \approx n\{I_D(10 - \ln p') - 1\}$$

where n is in the region of 3 (triaxial) to 5 (plane strain), and I_D is the relative density. This implies that, if p' were to be increased by a factor of 10, the angle of dilation in plane strain could reduce by as much as $5 \times 0{\cdot}87 \times \ln 10 \approx 10°$ for the given relative density of 87%.

The stress level in the passive wedges of the failure mechanism might be characterised by the product of the unit weight γ of the soil and the footing breadth B. The passive wedge is the part of the failure mechanism that is most immediately affected by the slope, but even so γB (or a slope-dependent multiple of it) might be used to characterise the pressure p'. The peak bearing stress q might be used to characterise the stress level immediately beneath the footing. Consequently there can be different stress levels and hence different friction angles operating in different parts of the failure mechanism, and some way of determining an average might be needed. Whatever way is chosen, a larger footing will imply a larger value of γB, which suggests that the average value of p' at failure will be larger. Equation 6 will then predict a smaller value of dilation ψ, and Equation 5 will predict a smaller value of peak friction angle. In this way, stress–dilatancy predicts what appears as a scale effect.

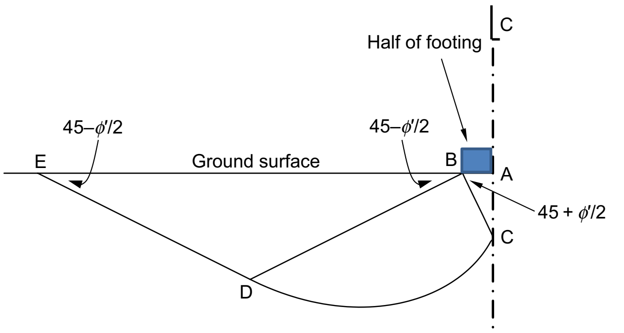


Figure 8 Familiar failure mechanism for a strip footing of full breadth B on flat surcharged ground far from a slope. ABC, half of active wedge; BCD, radial shear zone; BDE, passive wedge. Point D is at depth (B/2)exp[(π/2)tanϕ'] below the ground surface

Stress–dilatancy is one of the reasons why centrifuge modelling is a powerful tool (Muir Wood, 2004; Ovesen, 1975; Schofield, 1980, 2006). The general phenomenon is also incorporated in the critical state models of Schofield and Wroth (1968) and Roscoe and Burland (1968), in the steady-state model for sands by Jefferies and Been (2006), and in many other models in elasto-plasticity. Altaee and Fellenius (1994) suggested a form of density scaling for model tests in earth's gravity. While different approaches have different detailed equations, they all give the same general effect. Positive dilation increases the peak friction angle; but for some combinations of density and stress, negative dilation (i.e. compression) is predicted. The 'failure' mode then changes to one of large, compressive, ductile plastic deformations. The soil moves towards a state at which the rate of compressive volume change reduces to zero and $\phi' = \phi'_{\text{crit}}$. In this way, the stress–dilatancy approach can explain why bearing capacity does not reduce to zero for very large footings.

The slope effect

Equation 3 of the paper expresses a result that, as argued above, may be a stress–dilatancy effect. Could the authors clarify whether Equation 3 has been used to infer the values of N'_γ listed in Table 1?

On the basis of bearing capacity factors N_γ tabulated by Das (2004, table 3·4) and elsewhere, and assuming that the values of N'_γ in the authors' Table 1 can be interpreted as N_γ, the data would seem to suggest friction angles that are significantly greater than the angle of 38° for the drained triaxial tests quoted in the paper. For example, the value of 543·21 would indicate a back-calculated friction angle of over 48°, suggesting 10° more dilation in the footing tests than in the triaxial tests. Could the authors comment on this and its potential impact for practical design of footings that might typically be an order of magnitude larger than those used in these tests?

When computing N^*_γ/N', it seems to have been assumed that the slope had no effect on the footings that were placed furthest from the slope. Thus the second line of the table, for a footing that is closer to the slope, gives N^*_γ = 433·86. It seems that the 543·21 is used as a calibration factor for the square footings with B = 6 cm, and that the ratio 433·86/543·21 = 0·8 is interpreted as the effect of the slope. However, Figure 9 shows the tabulated values of N^*_γ plotted against the distance d for the square footings from Table 1. There is no indication of the plateau which would be expected if the largest values of d did indeed correspond to footings that were sufficiently far from the slope as to be unaffected by it. Also, the responses of the two larger footings appear to be very similar, and noticeably different from the smallest footing. Is it possible to know why?

Equation 4 of the paper predicts that $N^*_\gamma = N'_\gamma$ when d/B = 5·64. (The equation is presumably not to be used from values of d/B greater than this?) If this is applied to a square footing of breadth B = 6 cm, the result is d = 5·64 × 6 = 33·9 cm. Returning to Figure 8, geometry implies that the distance d_0 from B to E satisfies $d_0/B = \sqrt{N_q}$. For φ′ = 38°, this gives d_0/B = 7. d_0 might be interpreted as the closest the footing can be to the top of the slope without this failure mechanism intersecting the slope. The values of d_0 for a 6 cm wide strip footing is 42 cm. Values of the order of 33·9 cm to 42 cm might be consistent with an extrapolation of the data in Figure 9. Another equation is given by Huang and Kang (2008), based on the limit equilibrium theory, and their relation seems to predict limiting values of d/B of the order of 4·5 for ϕ' = 38° and 7·8 for ϕ' = 48°, equivalent to distances of 27 cm and 4·68 cm for B = 6 cm. However, d was only 22 cm for the first footing in Table 1.

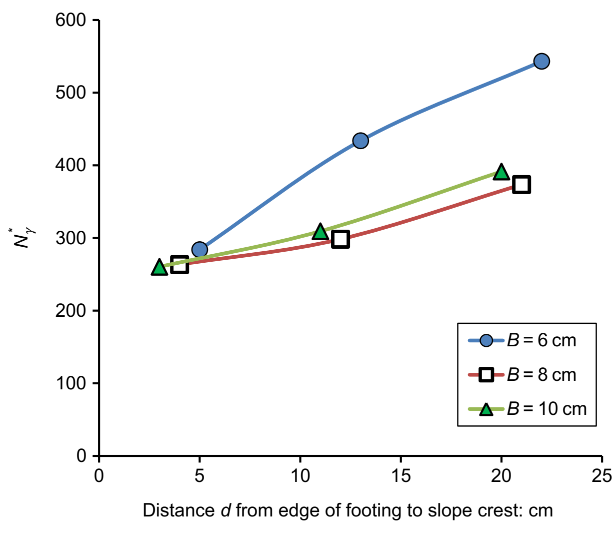
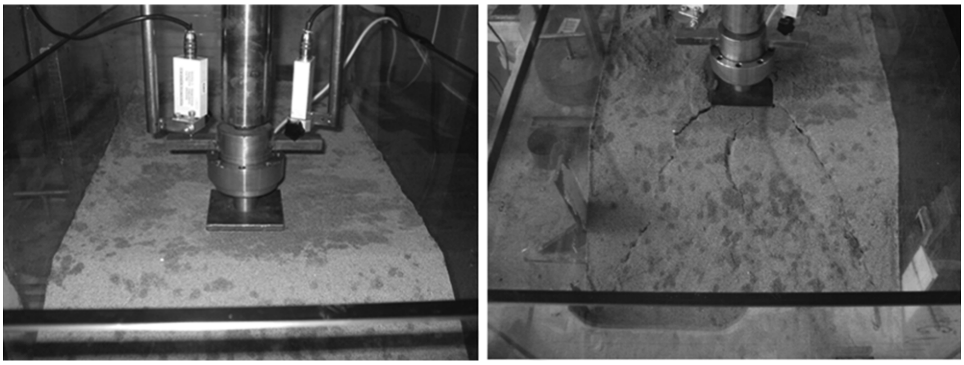


Figure 9 Exploring the authors' results for square footings



(a) (b)

Figure 10 Loading test of a footing on slope: (a) before the test; (b) after the test

Based on these considerations, are the values of N'_γ in Table 1 really appropriate for calibration purposes, or from which to infer values of friction angle?

Authors' reply

The authors thank the contributor for this great interest in the paper and for the considerations regarding (*a*) the failure mechanism, (*b*) the scale effect and (*c*) the slope effect in the evaluation of the bearing capacity of footings on slopes.

With respect to the first aspect, Figure 1 of the original paper was not meant to represent a particular theoretical failure mechanism of a shallow foundation on a slope, but simply it introduces the idea that the failure mechanism is influenced by the slope, as reported by other authors mentioned in the original paper and in the present discussion.

Obviously the bearing capacity of a footing on a slope depends on the failure mechanism and the bearing capacity factor back-calculated from the experimental results depends on the slope effect. Figure 10 reports the failure mechanism observed during one of the experimental tests and it shows clearly that the failure surface propagates also within the slope. Nevertheless, the definition of the position and definition of the geometry of the failure mechanism was beyond the scope of the study.

The focus of the paper is to highlight the effect of the distance *d* due to sloping ground on the bearing capacity of footing, by coupling the results obtained by the numerical model proposed by Castelli and Motta (2010) to those derived by experimental tests on a reduced scale model. With this aim, Equation 4 in the original paper has been proposed to take into account the reduction of the bearing capacity depending on the distance *d* of the footing from the edge of the sloping ground.

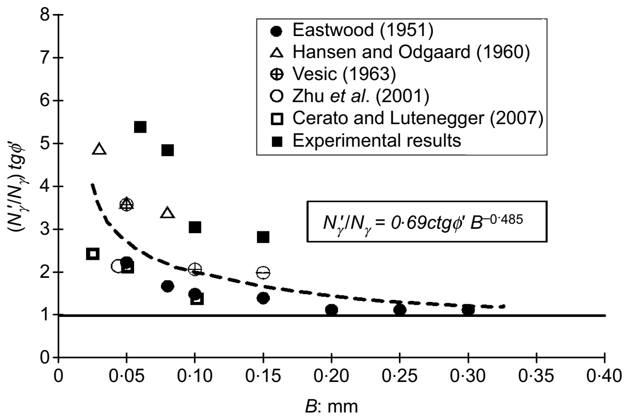


Figure 11 Relationship between the values of $(N'_\gamma/N_\gamma)\tan\phi'$ against *B* (Castelli and Lentini, 2010)

As regards the second aspect, with respect to the values of N^*_γ and N'_γ, in the paper it is clarified that N^*_γ is the back-calculated bearing capacity factor for an inclined slope surface and N'_γ is the back-calculated bearing capacity factor when the footing is far from the slope.

Owing to the scale effect, the values of the back-calculated bearing capacity factors are greater than the quoted values of N_γ reported by Vesic (1975). Equation 3 takes into account this scale effect, represented by the increment of the bearing capacity factor deduced by the experimental test on reduced scale models. As specified in the paper, the bearing capacity increases with the increasing of the foundation width and consequently the back-calculated bearing capacity factors are greater. Equation 3 (Figure 11) has been deduced from the analysis of a large number of experimental tests (Castelli and

Lentini, 2010) and the values of the bearing capacity factor reported in Table 1 have been modified taking into account the scale effect by Equation 3.

The scale effect that was observed in the tests may well have resulted from the stress–dilatancy effect noted by the contributor.

The aspects focused on by the contributor are of great interested and certainly represent a topic for further investigations, as for example, the definition of a suitable distance to be used for normalising the slope effect, and a threshold distance at which the reduction of the bearing capacity due to the sloping ground vanishes.

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