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Discharge from heavy rainfall †

by

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Discussion

Mr B. D. Richards (Consultant, Sir William Halcrow & Partners) observed that, assuming certain simplified conditions, the Author had found that the maximum flood from a catchment was il where i denoted the rainfall intensity and l the length of the catchment. He had then deduced that that maximum occurred before the water from the farthest point of the catchment reached the outlet and that in fact that water had travelled only about half the distance.

91. The value il implied essentially that the whole catchment was contributing to produce the maximum flood and that could not happen unless the water from the farthest point was reaching the outlet.

92. In arriving at his conclusions the Author had visualized a condition of steady flow extending gradually from the farthest point, or head of the catchment, to the point of outlet.

93. It would appear to be more realistic to consider the discharge at the outlet building up gradually as more and more of the catchment contributed, until the maximum il was reached. If at that instant the rain ceased, the flood would commence to subside. If, however, it continued, then the hydrograph of the flood would have a flat top of duration $T-t$, where T denoted the total duration of the rainfall and t the time from its commencement at which maximum flood had been reached.

Mr A. R. B. Edgecombe (Head of Hydraulics Division, Central Laboratory, Geo. Wimpey & Co. Ltd) pointed out that the Author's estimated flood of 2,642,000 cusecs from the Damodar catchment of 7,200 sq. miles was about four times what would be legislated for on the basis of great floods on other catchments in India over the past century or longer.

95. The normal monsoon discharge of the Damodar was only 25,000 to 30,000 cusecs, with one or two annual floods seldom exceeding 200,000 cusecs. The peaks of these floods lasted 4 to 6 hours, and they dropped rapidly.

96. Floods exceeding 500,000 cusecs had occurred, as followed:—

1840	600,000 cusecs
1913	625,000 ,,
1935	664,000 ,,

The 1935 flood was capable of more accurate measurement than was often associated with great floods, owing to the existence of the Anderson Weir. Although the gauge at Ranigang had been ½ ft higher in the 1913 flood, the discrepancy in discharges had been attributed to the scouring of the river-bed.

97. Large floods of the order of those of 1913 and 1935 were expected to occur about once in 50 years. It was interesting to note that the object to be obtained by the

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† Proc. Instn civ. Engrs, vol. 10, p. 453 (Aug. 1958).

first stage of the Damodar Valley Project was "to control the maximum observed flood of 650,000 cusecs to 250,000 cusecs".

98. Observed freak floods on other catchments indicated a maximum flood on the Damodar of 678,000 cusecs.

Mr J. E. Nash (Senior Research Fellow, Hydraulics Research Station, D.S.I.R.) remarked that the Paper could be divided into three parts: (i) the mathematical treatment of flow off a plane surface subject to rainfall; (ii) the approximate graphical solution of the mathematical equations of (i); and (iii) the application of the results obtained to a real catchment.

100. The Author's treatment was very similar to that of Richards.[6] Both treated the same problem under the conditions of the same assumptions, *viz.* a semi-infinite plane (i.e., extending indefinitely in one direction) of constant slope and roughness, velocity governed by the Chezy formula, neglect of momentum changes in the raindrops, etc. Again both Richards and the Author made use of the formula $h = it$. This formula required some more explanation than was given by either.

101. Considering a plane, extending indefinitely in both directions, subject to rainfall at a uniform and constant rate i, then, since any one point on such a plane was indistinguishable from any other, the depth of water on the plane remained uniform. Consequently the depth at any time was given by $h = it$. On the semi-infinite plane the upper end introduced a complication, but until such time as the influence of that edge could travel down the plane to the point being considered, the depth at that point was still given by $h = it$.

102. Richards and the Author both implicitly assumed that the instant the influence of the upper end reached the point under consideration, a steady state was established at that point, and no further increase in depth could occur. Fig. 26 showed the depth/time relation assumed by Richards and the Author for two points on the plane, one below the other. The steady-state depth was given by the Author in equation (5) as $h_s = (i.x/c_1)^{2/3}$. Further algebraic manipulation produced the conclusion that the time t_s to establish steady state (given by equation (12)) was $t_s = x^{2/3}/c_i^{2/3} i^{1/3}$. This, however, could be derived in a much simpler manner from Fig. 26 and the Author's equation for steady h. Thus:

$$h_s = it_s$$

but $$h_s = (ix/c_1)^{2/3} \quad \text{(Author's equation (5))}$$

$\therefore \quad t_s = (i.x/c_1)^{2/3}.i$, which agreed with the Author's equations (12) and (13).

It was interesting to note that Richards obtained a different value of t_s (see p. 43 of reference 6), despite identical starting assumptions. The difference was worth noting and arose because Richards had erroneously assumed the velocity of the point at which steady state has just been established to be equal to the velocity of the water at that point.

103. Purely as a treatment of the problem of flow off a plane subject to rainfall, the Author's analysis was very inadequate. The assumption that the plane extended indefinitely in one direction simplified the treatment very considerably, but made the results quite impractical. The assumption that steady-state conditions were established at the instant when the influence of the upper end of the plane arrived, was also inaccurate. It could be shown that there was a transitional curve between the two straight lines in Fig. 26. In fact this curve might be asymptotic to the horizontal line. What then became of the time to reach steady conditions? The fact that this curve occurred in the Author's graphical method suggested that the graphical method might be more precise than the mathematical. Mr. Nash believed that, even if that was so, neither the mathematical nor the graphical methods were justified, since an exact numerical solution

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[6] B. D. Richards, "Flood estimation and control" (3rd edn), Chapman & Hall, London, 1955.

could be obtained, even for a finite plane with end conditions, by using the method of characteristics.

104. Turning to natural catchments, the Author's assumption that any catchment could be treated as a rectangular plane of uniform slope and roughness, and of area equal to the catchment, the width-to-length relation (which was obviously important in the formulae) to be determined by no very clear method, was certainly not attractive, particularly when it be considered that most floods were produced, on natural catchments, not by pure surface run-off, but by interflow. In the practical application of the method, however, the analogy with the plane could be neglected and, as in the Author's own example, c_1 could be chosen empirically to obtain the best possible agreement between a calculated and a recorded flood. In this respect it resembled the unit-hydrograph theory.

105. Considering the application of the method to a natural catchment, and assuming, for simplicity, continuous effective rainfall uniform in space and time, then the Author's method simply meant that the discharge would rise in proportion to $(\text{time})^{3/2}$, until it reached an intensity equal to the effective rainfall rate multiplied by the catchment area, at which time steady conditions were established. The relevant equations were $h=it$; $Q \propto h^{3/2} \propto i^{3/2}t^{3/2}$; and $t_s=x^{2/3}/c_1{}^{2/3}i^{1/3}$ (i.e. $t_s \propto 1/i^{1/3}$).

106. Assumption of a greater intensity of effective rainfall leads to a similar rising hydrograph but the time at which steady conditions were established was earlier, in proportion to $(i\text{-ratio})^{1/3}$. Fig. 27 shows the responses given by the Author's formulas for two intensities, one twice the other. It should be borne in mind that c_1 was available, which meant that the time to establish steady state was to be determined, in one actual case, to suit the measured result as well as possible.

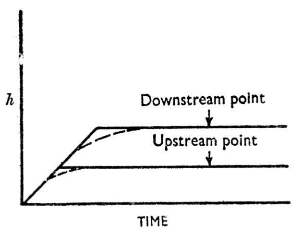

FIG. 26

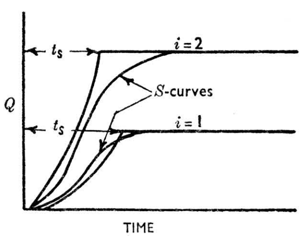

FIG. 27

107. Applying the unit-hydrograph theory to this case, the *S*-curve (the rising hydrograph due to uniform effective rainfall) would be derived in an empirical manner, for one intensity; and it could then be assumed that the *S*-curve for any other intensity was obtained by increasing or decreasing the ordinates in the ratio of the two intensities. All time elements would therefore be the same for all intensities. The Author on the other hand took time elements as being inversely proportional to $i^{1/3}$.

108. The comparison between the Author's method and the unit-hydrograph method in practice was:—

Unit-hydrograph method	*Dawson's method*
Constancy of all time elements	All time elements inversely proportional to $i^{1/3}$
One time element to be chosen arbitrarily	
An empirically derived *S*-curve	An *S*-curve derived by assuming that the catchment behaved as a plane.

109. There could be no doubt that the empirically derived response was to be preferred to one based on such vague evidence as the behaviour of a uniform rectangular plane. It was also known from experience, that on a natural catchment, the response to continuous uniform effective rainfall was not of the general shape given by the Author's method (see Fig. 27). The difference arose from the neglect of storage in the Author's method. The non-linearity caused by $i^{1/3}$ was of small effect. The evidence for it, apart from the suspect evidence of the analogy with the plane, would be very difficult to find in a practical case, simply because the effect was so slight. The complication which a non-linear element introduced made the method vastly more difficult to use than the unit-hydrograph method. Very definite evidence of non-linearity was required before abandoning the simplicity of the unit-hydrograph method.

110. Summing up, the treatment of flow off a plane was imprecise, and by analogy with this, the Author had suggested the use of what might be called a non-linear unit-hydrograph method, in which all time elements were inversely proportional to $i^{1/3}$ and the basic shape[7] was obtained from the plane analogy.

Professor J. C. I. Dooge (University College, Cork) believed that the method used by the Author for predicting the run-off from a catchment subjected to heavy rainfall was open to serious criticism on a number of points. The profile for steady flow was valid only for certain conditions which did not hold for the numerical example given by the Author. The hydrograph of unsteady flow made further assumptions which required justification. More serious than those limitations on the Author's result for the hydrograph of flow off a plane surface was his fundamental assumption that the flow from a natural catchment could be treated as the flow from such a plane surface.

112. In deriving his steady-flow equation the Author assumed that the relation between velocity and depth of flow was given by $V=C_1\sqrt{h}$ where $C_1=C\sqrt{S}$. Since the flow in the case under examination was clearly non-uniform the relation should read:

$$V = C\sqrt{h}\sqrt{S_0-\frac{dh}{dx}-\frac{d}{dx}\frac{V^2}{2g}}$$

which could be transformed to give:

$$\frac{d}{dx}\left(h+\frac{v^2}{2g}\right) = S_0-\frac{V^2}{C^2h}$$

In the present case $q=ix$ and $q=Vh$ so:

$$\frac{d}{dx}\left(h+\frac{q^2}{2gh^2}\right) = S_0-\frac{q^2}{C^2h^3}$$

or

$$\frac{d}{dx}\left(h+\frac{i^2x^2}{2gh^2}\right) = S_0-\frac{i^2x^2}{C^2h^3}$$

giving

$$\frac{dh}{dx}+\frac{i^2}{g}\left(\frac{x}{h^2}-\frac{x^2}{h^3}\frac{dh}{dx}\right) = S_0\left(1-\frac{i^2}{C^2S_0}\frac{x^2}{h^3}\right)$$

For the example given by the Author:

$$i = 1{\cdot}0 \text{ in/sec};\ g = 386 \text{ in/sec}^2;\ \text{and } C^2S_0 = 1$$

thus giving:

$$\frac{dh}{dx}\left(1-\frac{1}{386}\frac{x^2}{h^3}\right)+\frac{1}{386}\frac{x}{h^2} = S_0\left(1-\frac{x^2}{h^2}\right)$$

or

$$\frac{dh}{dx} = \frac{386S_0\left(1-\frac{x^2}{h^3}\right)-x/h^2}{386-\frac{x^2}{h^3}}$$

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[7] J. E. Nash, "Determining run-off from rainfall". Proc. Instn civ. Engrs, vol. 10, p. 163 (June 1958).

This was the differential equation of the plane flow of the Author's example and the shape of the curve was clearly dependent on the value of S_0 and on the conditions at the downstream end. Taking a value of $n=0{\cdot}015$ and putting $C^2S_0=1$ was equivalent to giving S a value of about 1:1,000. A rough calculation, using this value for the slope and inserting a critical depth at the downstream end, indicated that the depth at the upstream end, instead of being zero, would be of the order of 1 in. For greater slopes —say greater than the unit slope 1:1—the depth at the upstream end would tend towards zero and the Author's steady-flow curve would then be approximately correct except in the immediate neighbourhood of the downstream end. It was clear that the steady-flow profile assumed by the Author was not valid for all slopes.

113. Even taking the assumed steady-flow profile as correct, the analysis given of the unsteady flow conditions was open to question. The Author assumed that the water level rose uniformly except within the region of $x^{2/3}>t$. This would appear to be equivalent to assuming that a discontinuity was propagated downstream at a speed of $1{\cdot}5V$ instead of the value of $(V+\sqrt{gh})$ indicated by the unsteady-flow equations. The latter speed would in the case of the Author's example and his velocity formula amount to about $2{\cdot}65V$. The whole question of the manner in which the flow from a plane surface built up to its steady value could be determined numerically by the method of characteristics for given numerical values. Time was not available to check the Author's example in this way.

114. There was a final fundamental criticism of the Author's approach, namely, the use of the flow from a plane surface to simulate the discharge at the outlet of a natural catchment. The two physical systems were quite dissimilar. The major modifications of the rainfall pattern on a natural catchment of any size took place in the channels and not over the surface of the ground. Those modifications were reflected by the unit hydrograph but were ignored in the Author's method. The Author's method did not perform anything which could not be done by the unit-hydrograph method—which was physically more reasonable. The unit hydrograph could be adapted to variations of intensity in both area and time and for certain cases even adjusted for non-linearity.

115. Though unable to accept the Author's contentions, Professor Dooge commended his unusual attack on an important problem. If the Paper stimulated interest in the question and resulted in a sounder approach to the problem of run-off than had been evident hitherto, its publication would be justified.

The Author, in reply, said that Mr Richards's comments had been clearly answered by Mr Nash in § 102; it appeared that the concept of "areas contributing" was misleading and should be discarded, together with the term "time of concentration".

117. The discharge of 664,000 cusecs for the flood of 1935 given by Mr Edgecombe seemed to have arisen from almost the same 3-day rainfall as in the 1913 storm,[8] so if the distributions in time and area were similar in the two storms a greater discharge could not have been expected; but the possibility of rain occurring both in quantity and in distribution in time and area, as shown in Part III of the Paper, still remained, unless further meteorological research indicated otherwise, with the consequential high discharges shown there.

118. From the comments of Mr Nash and Professor Dooge, it was evident that clarification regarding the assumed plane was necessary. The catchment or plane as defined in § 2 had been discovered in attempting to reproduce the hydrograph of the 1913 flood on the Damodar, given details of the rainfall and a plan of the catchment. Since such a plane might be physically impossible, it could be considered imaginary or "ideal"; the assumption that $c\sqrt{s}$ was a constant was indicated by the fact that in a natural catchment there was a tendency for c to decrease as s increased as one travelled up the catchment; no mention had been made of the difference between the slope of the

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[8] Satakopan. Memoirs of the Indian Meterological Dept, vol. 27, Pt VI, Table 5, p. 335. Manager of Publications, Delhi 1949.

water surface and that of the plane, nor of the kinetic energy, for the plane was to be used in investigating natural catchments where these effects were comparatively negligible; this was indicated at the beginning of § 21. The mathematical treatment, §§ 4 to 23, had not been intended as an exhaustive analysis of flow off a plane, but as an investigation into flow off the assumed plane under the other assumptions made. It would be seen, therefore, that Mr Nash's synopsis in § 99 required modification. The method had an entirely empirical justification. No claim was made that it applied to all catchments, but only that it was expected to do so, as stated in § 2. It had been found that it applied to the one or two other catchments studied; § 84 was perhaps not clear enough.

119. Considering the flow on the plane (the ideal catchment) and on the actual catchment, the two processes appeared to the Author to be essentially the same, the velocity in both being proportional to the square root of their depths ("roughly" proportional in the case of the streams); the volumes of rainfall were the same (§ 47) and at the same straight-line distance from the outfall; the chief difference was the fact that, as the streams were tortuous, their volumes had further to go than those on the plane; but this fact was automatically compensated for in the value for c_1.

120. The Author was grateful to Mr Nash for the shorter derivation of equation (12) (see § 102); however, the text covered more fully the state of affairs on the whole catchment.

121. Regarding § 103, the Author did not agree that, under the conditions assumed, there would be a transitional curve between the straight lines in Fig. 26; the curve in the graphical method, Fig. 4, was brought about by the use of twenty elements of equal length to begin with, too long for accuracy (see § 36); infinitely short elements at infinitely short time intervals would give no transitional curve.

122. In §§ 104 to 109, Mr Nash had neglected the transfer of the volumes of rain, strip by strip, on the actual catchment to the ideal catchment, §§ 46 and 47; the depth of rainfall on the ideal catchment would be affected by the shape of the actual catchment; in the case of a diamond-shaped catchment subjected to rainfall of intensity uniform in time and area, the transfer would produce an equivalent intensity on the ideal catchment similar to that in Case III (see § 38) which gave a very distinct S-shape for the rise of the hydrograph, as shown in Fig. 28. It followed, therefore, that only in the

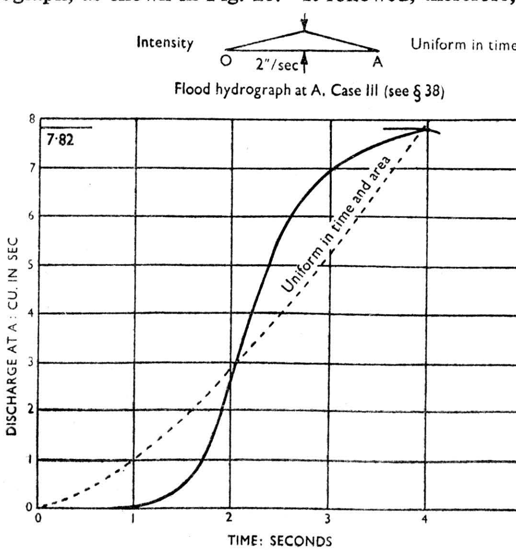

FIG. 28.—CASE III

particular case where the actual catchment was identical with the ideal catchment would Mr Nash's comparison with the unit hydrograph, and Fig. 27, hold good; any other case would give an equivalent intensity on the ideal catchment varying in area, and since the unit-hydrograph method, as described by Nash,[7] was incapable of dealing directly with this type of variation, the comparison became impossible; for the same reason the unit-hydrograph method would have been useless for the investigation of Cases III, IV, V, VII, VIII, IX, and XIV (see §§ 37 to 40); in the latter case it would have given 8 cu. in/sec discharge instead of 12·5, and for Case VII less than 8 instead of 15·7; it would also have been useless for Cases X, XI, and XII, for it took no account directly of what was taking place on the catchment.

123. The references to "neglect of storage" by Mr Nash and Professor Dooge were not understood by the Author; if channel storage was meant then the drawings, Figs 21 to 25 inclusive, were step-by-step drawings of this very storage, called here "residue on the catchment"; it was this residue and its distribution which determined the hydrograph at each step, the hydrograph being derived from the drawing, all in one process; if, however, storage on flood plains due to over-bank spill, or storage in retarding basins, was intended, these would be the subject of a further Paper.

124. The Author was grateful to Professor Dooge for his very interesting equations, but thought that it would now be clear that they were irrelevant. In answer to the criticism in § 114 of the use of the flow from a plane surface to simulate the discharge at the outlet of a natural catchment, the Author drew attention to Figs 13 to 16 where this had been accomplished with reasonable accuracy.