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Statistical approach to structural design†

by

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Discussion

Professor A. W. Hendry (Department of Building Science, University of Liverpool) observed that the Paper was an interesting contribution to the fundamental examination of concepts of structural safety which had been going on since the end of the war. The Author had derived a formula connecting the conventional factor of safety with certain statistical parameters, and had illustrated the use of this formula in design calculations. It was, of course, quite possible to base a design on statistical data without introducing such terms as factor of safety, load factor, or permissible stress at all, but as engineers were accustomed to thinking in terms of these quantities, it was no doubt advisable to retain them rather than to attempt to introduce the idea of designing on "probability of failure".

44. The Author had not discussed the definition of "safety" or "serviceability" in any great detail, but it was clear that the basic aim in structural design must be to proportion a structure in such a way that excessive deformation or cracking or outright collapse were highly improbable under any reasonably foreseeable circumstances. This statement called for the elucidation of three very complex problems:—

(*a*) When did deformations become "excessive".
(*b*) How was "highly improbable" to be defined?
(*c*) What circumstances were "reasonably foreseeable"?

If safety and serviceability were defined by reference to the onset of large deformations, it would appear essential to correlate the statistical approach with the known behaviour of various kinds of structures, otherwise any definition of "excessive deformation" became rather meaningless. This question had been discussed in an article[12] by Professor Hendry some years before in relation to mild-steel structures. It had been shown there that a very low probability of complete collapse was not in itself a guarantee that objectionably large permanent deformations might not occur under circumstances which, statistically, could quite conceivably occur within the lifetime of the structure.

45. The Author had, with some ingenuity, looked for an answer to question (*b*) in the statistics of road accidents. It was certainly worth looking at these grim figures, but statistics for hazards directly related to buildings and structures (e.g. those for fire or earthquake damage) would form a more useful guide. At all events, it would be impossible to justify any proposal which would increase the incidence of serious structural failures to a figure in any way comparable to the accident rate on the roads.

46. "Reasonably foreseeable circumstances" in question (*c*) included those connected with the probable loading and strength of the structure and any factors, such as temperature effects, which might tend to cause failure. The application of statistical methods was greatly hampered at present by the scarcity of data, both on loading and on the strength of structures. The Author had used the yield stress of steel and the

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crushing strength of concrete as index values of strength in his illustrative examples, but in a series of tests on welded steel frames, nominally identical, Professor Hendry had found[12] that the variation in strength of the frames was considerably greater than would have been expected from the variation in the yield point of the steel used in making them. While no general conclusion could be drawn from a limited series of tests, it was clear that whatever parameter was adopted as a measure of structural strength, it must include an allowance for variations arising in construction, in addition to those inherent in the materials used.

47. While it was quite easy to see how statistical allowance could be made for variations in loading and structural strength, there remained a category of freak occurrences, some arising from human fallibility (such as gross mistakes in design or construction), or from what the insurance companies termed "acts of God". Such occurrences had a statistical rate of incidence, but presumably they were left out of account in assessing safety factors. It would seem that the most that could be done here was to accept that there was a chance of one in x million that any particular structure would fail as a result of pure accident, and to guard against it by the exercise of careful supervision and judgement.

48. The Author's comment in § 41 on the role of plastic theories in design did not seem to be consistent with the general philosophy of the Paper, which in effect suggested that safety factors should be explicitly related to the probability of failure. It therefore seemed reasonable to use methods of calculation in which attention was focused on failing loads. In conventional elastic design of steel structures attention was confined to the "safe" condition. What was done in effect was to compare the computed "working" stresses with the yield stress: if the ratio between the two did not exceed about 2/3, experience showed that the structure was "safe"—but how safe remained unknown. It seemed, therefore, that the attempt to rationalize the problem of structural safety would in itself tend to extend the use of ultimate load theories in design.

Mr S. C. H. Robinson (Bridge Engineer, Howard Humphreys & Sons) observed that though it had not been the Author's intention to advocate a particular probability of unsafety, in his first example it was easily calculable that he had in fact used in the design of the steel girder a safety parameter of 2·5, since he had had to pick a number in order to compare his choice of the 13-in. × 5-in. × 35-lb. R.S.J. with the 15-in. × 5-in. × 42-lb. R.S.J. called for by the plastic theory. It was interesting to note that the safety parameter chosen signified a probability of unsafety of 0·7%, the very figure given earlier in the context as the probability of any of us being killed on the roads. Since he had supported this statement with detailed statistics on road accidents (see Table 1), it was logical to assume that by "probability of unsafety" he meant "probability of unlife". Despite the disclaimer that it was not his intention to initiate a new philosophy, the Author showed a definite tendency to accept a 1 to 142 risk of death as unlikely to cause a public outcry.

50. It was possible to criticize the statistical approach without involving philosophy unduly by referring instead to the law of the land, because by the law of probability, the more the odds were shortened, the sooner the engineer would have the opportunity of explaining the approach in question to the jury.

51. If the first example in the Paper were taken one step further by assuming the girder in question to have been used as a supporting structural member for, say, a theatre balcony that by the 143rd unlucky chance fell in, there would be a case of forensic engineering in point. Moreover, there was little doubt that for many years to come there would be eminent expert witnesses available for the prosecution or for the defence.

52. The case for the prosecution would be that the engineer in the box had undertaken to provide the theatre owner with a cheaper structure than would be obtainable from others adhering to more conservative or orthodox methods of design, had calculated himself by what he considered a reliable method that a financial saving of about 16% could be incurred by substituting the joist in question for a needlessly safe joist, knowing full well that by so doing he would be increasing the chance of failure from one

in many millions to one in 143, and that he had been remunerated for his work—the motive. The prosecution would prove that many other engineers designed with a view to perfect safety.

53. The case for the defence would be that engineers claiming perfect safety in structural design could be held to be deluding the public and themselves, there being no such thing as perfect safety; that, given a mathematical analysis that could be relied upon to reveal the probability of unsafety in any structure, it would be more irresponsible to neglect it than to use it; that by using it the engineer became aware that construction must come to a complete halt if perfect safety be demanded every time; and that it was clearly the engineer's duty to move with the times and apply science. The work of Sir Alfred Pugsley could be cited (referred to in the Paper), who in wartime advocated encroaching on the factor of safety of aircraft to the extent that a pilot incurred a definite risk of death through structural failure, in order to enable him to carry sufficient armament to match that of the enemy and avoid the greater risk of being shot down. It could be shown how A. M. Freudenthal (also referred to in the Paper) after the war advocated a retention of a small calculable risk based on accident statistics and insurance claims in order to permit structures to be designed that would be more favourably considered by the entrepreneur. Finally, it could be shown that the law of probability, based on the Gaussian distribution, was invaluable in many fields of science generally considered to be far more exact than that of the strength of materials.

54. It would be necessary to explain pedantically to the jury that the Gaussian distribution is the theoretical graph of the frequency of a population of random variables tending to possess a mean value, such a distribution as might be experimentally obtained by recording heads and tails resulting from a great number of tosses of a coin. Strictly speaking, the curve tails off continuously to a point infinitely distant from the mean on either side, signifying theoretically an infinite sequence of heads on one side and of tails on the other. Thus, strictly speaking, it has never been possible to verify the Gaussian distribution to the limit, because of the obvious impossibility of recording an infinite number of observations. The jury would want to know what the plot of the frequencies of a population of steel strengths looks like, and would find out that, like most test results plotted in this way, the experimental curve is by no means continuous to infinity, but tails off erratically at a finite distance from the mean into irregular isolated peaks and then, in any real case, ends in some definite upper and lower value. Under the necessity of providing a fair trial and a decision affecting human life, the jury would not feel obliged to consider hypothetical cases like the probability of finding any values beyond the limits of those actually recorded. If the engineer could prove that he had chosen a value lower than any obtained in tests on samples and on similar materials elsewhere, then the jury would recognize him as more knowledgeable about his materials than they were, and could not require him to show that he had taken any particular lower value that they might think of. But if the jury could find a single test result showing that a fair sample had been found to break at the very value he had used in the design, or lower, then the jury would find him guilty of contempt for safety. In the first case above, they would not be concerned at all with probabilities, in the second case any person who could count could calculate the odds against unsafety. In short, there was all the difference in the world between a policy of suspecting danger to the innocent bystander and using any method, however rough, to dodge it, and a policy of calculating the risk by any method, however accurate, and courting danger. In the last resort it was a jury of ordinary people who must be satisfied with any proposed factor of safety, and they were bound to decide on empirical grounds, not theories. The engineer might be absolved despite his theory, not because of it.

55. It was therefore proposed that statistics in the O.E.D. sense: "numerical facts systematically recorded", be studied carefully in relation to structural design by all means, but that laws of chance based on extrapolations of any curve beyond the limits of experimental observation were extremely questionable science, and so should be avoided.

Mr John Knibb (U.K. Atomic Energy Authority) felt that while considerations of scatter of "factors of satisfaction" might, after the collection of suitable statistics, prove of value, yet currently they might just be an additional theoretical complication to the art of the civil engineer.

57. The backcloth to any considerations of "factors of safety" must be one of ethics and the greatest care must be exercised in using "statistics of suffering" (such as road casualties) in any connexion which is not to be directed to the alleviation of that suffering.

The Author, in reply, said that Professor Hendry's remarks and questions reflected faithfully the present situation of the design philosophy. He was quite correct in thinking that in the statistical approach to design problems, such terms as factor of safety, etc. might be dropped altogether. These terms, like many others in any branch of science, were used merely for the sake of convenience and clarity of expression and communication.

59. To the three questions Professor Hendry raised, there could be quantitative answers provided that sufficient data were available. Question (*a*) was physical and familiar to engineers. The other two questions were probabilistic and could not be answered completely in a few pages.

60. Safety, serviceability, or usability could be referred to any criterion controlling a design such as fatigue, buckling, etc. "Excessive deformation" was one of such criteria. Examples on the use of deflexion and stability criterion have been produced in ref. 13.

61. It was of course desirable to base a structural design on some statistics more closely related to structures, as suggested by Professor Hendry. However, such data were still rare at present and it was hoped that efforts would be made in future to develop this neglected field. The primary object of introducing statistics of road accidents was to show that (i) there was no such thing as "perfect" safety in reality and (ii) safety could only be appreciated through the concept of probability.

62. The variation in strength of the frames encountered by Professor Hendry could be explained by equation (16) or equation (17) if ϕ was considered as a function of effects of construction too. Then an extra term would appear in the σ or γ equation. It should be noted that additional terms in such equations always increased the resultant variability, which was the measure of variation commonly used in statistics.

63. The "acts of God" were connected with the concept of probability, and had in fact been included in the safety factor defined by equation (9) through the safety parameter. A comprehensive discussion on this was again beyond the scope of the Paper.

64. As to the role of plastic methods of design, it was fair to state that the foundation of these methods was on the verge of collapse. It should be pointed out that plastic theory of structures was a heterodox development of the concept of allowable stresses, and there could be criticisms of plastic design at all levels as revealed recently [14, 15, 16].

65. A structure, even designed in accordance with plastic methods, should behave elastically, or at least not plastically. If such a philosophy was acceptable, why should not a viscous method with a load factor of say 3 or 4 be used? In the atomic age, why should not a sublimation method be used, to assume a structure to sublime in the design stage and to condense it to an elastic material in the construction stage? In both cases, the extraction of experimental evidence to support the theory was no more difficult than in the plastic case.

66. Plastic theory was useful where plastic phenomena actually appeared. Therefore, in the ceramic industry, in food processing, in sheet rolling, in metal cutting, in wire drawing, etc. to which the yield mechanism was indispensable, the plastic theory was indeed valuable. Structural engineers obviously could not enjoy a similar "collapse" treatment.

67. Mr Robinson seemed to have remained steadfast in fact of the impact made by the "Report on Structural Safety", in which the use of different factors of safety was advocated. One of the advantages of statistico-structural design was incidentally, flexibility. For a major member in a structure, the safety parameter should be raised

to 3, 4, or even 5. On the other hand, 2 may be more than sufficient for an unimportant member, say, in a barn.

68. The designer using a safety parameter of 2·5 for a theatre balcony was no doubt very ignorant of the statistical approach and hence deserved severe penalty indeed. It should be noted that, although not explicitly stated in the Paper, the example on a steel girder had also been intended to expose the myth that plastic design gave an economic result. Why should a load factor of 1·75 be used but not 2·75 or 1·01?

69. In any design method previously proposed, nobody knew quantitatively how safe a designed structure was. The designer was indeed deluding himself, his client, and the public when claiming that his design was perfectly safe. How could he know that the chance of failure was one in many millions? Furthermore, how could he use the same load factor for all beams, which might be used for very different purposes? It was worth pointing out that a safety parameter of 4 had been used for a foundation in the example, and there was no reason why 4 or any other appropriate value should not be used for a certain type of steel girder.

70. If the public, having been cheated for decades, was suddenly alarmed and awakened by statistically minded engineers, that was a very great step forward. Reality existed despite the ignorance of human beings. It was more sensible to face than to evade reality, and the sooner it was faced the better. This was precisely why the statistics of road accidents had been chosen to demonstrate the probabilistic significance of the statistical approach.

71. It was reasonable to assume that roughly one-sixth of the members of the jury had experienced some road accidents (if their average age was, say, 35), and no doubt every member appreciated the "acts of God" implied in such happenings. It would seem not merely pedantic, but idiotic to give a series of lectures on the theory of probability and statistics to the jury, which after all was not composed of snowmen.

72. Statistics and engineering were both applied sciences. If an applied science had been developed to such a stage that no scientist could explain what it was intended for, then it was no longer "applied" nor "science" and all the scientists in that field must have either evaporated or become eccentrics. This was the very reason behind the idea that a jury should consist of ordinary people so that common sense, but not anything cloaked under the name of "knowledge", would prevail. If the defence failed to speak a language the jury understood, he had no others to blame even if he had used a safety parameter much greater than 2·5.

73. Returning to the question of safety, it should be noticed that probability of safety or usability necessarily contained probability of life, but probability of unsafety did not necessarily mean that of no life. For example, buildings with an expectation of life of say 100 years were quite common, but do they always stand there for 100 years? They might be bombed, or demolished to widen a road, or replaced by new buildings, or flooded because a dam was erected downstream, and so forth. It was such uncertainty in the life of the structures, not just the settled question of life or death of occupiers, which should receive the attention of modern engineers. This was partly the reason why the concept of "safety" was outdated and should be replaced by that of "serviceability" or "usability", which contained the former as well as other modern concepts.

74. As to the distribution curve, the use of the Gaussian curve was only a suggestion. In fact, the proposed method was quite non-parametric and did not depend upon the applicability of any particular distribution curve. The impossibility of infinity was well known and did not constitute a great obstacle in this case. On the other hand, the extreme case would now be dealt with by statistical methods quite satisfactorily.

75. The problem, therefore, seemed not so much whether the engineer would have the opportunity of explaining the approach in question to the jury, but whether he could appreciate the philosophy behind such a design approach himself. It was of course ridiculous to increase exposure to danger indefinitely. However, there was no harm, but a great deal of benefit, in finding out the lower limit of "usability" in order to avoid it in the design.

76. Mr Knibb's view on the art of the civil engineer was very interesting. It seemed that when a "science" was still in a qualitative stage, it should be called an art. On the other hand, an exact science should be defined as the same subject in a quantitative stage. Civil engineering at present was neither a pure art nor an exact science, but in an intermediate stage. It should be every civil engineer's aim to continuously advance his art to a scientific level. In fact, the remaining artistic aspect of structural engineering seemed to be precisely the statistical side of design problems. Why should they remain stagnant and loyal to their grandfather's conception of artistic "safety"?

77. The art had so far been touched only in its primitive form. It could become a refined art through a series of scientific evolution. The primitive art existed because of ignorance of the objective laws implied in it, whereas the refined art could appear only after a realization of such laws and utilization of them. No doubt the refined version was desired.

78. When any new theory was first proposed it always appeared to be "an additional theoretical complication". It had taken 1,000 years for fire crackers to become modern rockets, which might never have appeared if "theoretical complications" had always been avoided.

79. Conscientious designers would certainly agree with Mr Knibb that the greatest care must be exercised in using "statistics of suffering". Statistics of accidents were useful because probability of "safety" and that of "unsafety" were complementary. Since (i) it was much more difficult to collect the "safe" statistics than the "unsafe", and (ii) it was possible to obtain the probability of "safety" from that of "unsafety", the "statistics of suffering" would play a great and increasingly important role in future.

80. However, the statistico-structural designer was by no means interested in exposing his clients to such suffering. On the contrary, one of his prime motives was the alleviation of such suffering, and that was one of the reasons why he should divorce himself from the philosophy of the collapse-happy designers.

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