CORRESPONDENCE

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"Recent Developments in Highway Bridge Design in Hampshire" †

by

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Professor A. D. Ross proposed to make only a brief comment on the test.

Anomalous strains had been recorded during pre-stressing and that had been his own experience. It was plausible to add the stress distributions resulting from pre-stress and dead load but he doubted if the nominal combined distribution was ever, in fact, realized initially. Because of the usually small eccentricity of the cable at the ends of a beam, the change in curvature there was small and, although the centre lifted clear, the beam was probably supported along a considerable length at each end owing to deformations of the base and of the beam itself. In effect, the dead load was operative on a reduced span and since the moment was proportional to the square of the span the reduction in dead-load stresses could be quite marked. He believed that it was wise in design to assume that only a fraction, perhaps one-half, of the dead-load moment was acting after post-tensioning while the beam still rested on its base. When the indeterminate effects of longitudinal restraint of the base and friction of the cable were added, the resulting stresses might be very different from those envisaged by the designer, but it was encouraging to find that the strains approached their expected values when the beam was placed in the test rig.

The Author had certainly used good judgement to obtain experimentally a load factor against cracking of 1·65 in a beam with a nominal factor of 1·5. From the figures in the Paper the tensile stress in the lowest fibre due to the

† Proc. Instn Civ. Engrs, Part II, vol. 1, p. 461 (June 1952).

cracking load appeared to be $1{,}700 \times \frac{4\cdot 95}{3\cdot 06} = 2{,}750$ lb. per square inch. Thus, combining stresses, the following Table could be drawn up. The modulus of rupture was difficult to estimate with precision and in predicting the *cracking load there was the added uncertainty of the actual loss of* pre-stress which had taken place. In the case of that beam, tested early in its life, the modulus was probably in the range 600–700 lb. per square inch, a figure which agreed with Professor Ross's experience for that class of concrete. Tested after a long period, the load factor against cracking might have been somewhat less than 1·5 since the probably small increase in the modulus of rupture would not compensate for the reduction in pre-stress.

Stress due to	Initially	After relaxations
Pre-stress plus dead load	+ 2,145 lb. per square inch	+ 1,750 lb. per square inch
Cracking load (4·95 tons)	– 2,750 ,, ,, ,, ,,	– 2,750 ,, ,, ,, ,,
Nominal modulus of rupture	– 605 ,, ,, ,, ,,	– 1,000 ,, ,, ,, ,,

It was a little surprising that the ultimate load factor was only 2·40. That was the kind of figure to be expected with unbonded or poorly bonded beams, but the crack-pattern and the photographs in *Figs 30* provided ample evidence of good bonding. *Fig. 29*, however, strongly suggested that shear had been an influential factor in the final failure. Under large point loads near the centre of the span it was difficult for a heavily cracked beam to develop its full flexural strength because the greatly reduced compressive area was called upon to carry a heavy shear in addition to the high bending stresses. It seemed probable that a much higher load factor would be obtained under uniform loading since the large shears at the ends were unlikely to be troublesome because of the freedom from cracking and the beneficial inclination of the cables.

Dr K. Hajnal-Kónyi observed that it was of interest to analyse the test results given in the Appendix.

The ultimate moment was composed of the dead-load moment

$$= \frac{34^2 \times 17 \times 12 \times 1\cdot 04 \times 12}{8} \qquad = \quad 367{,}890 \text{ lb.-inches}$$

together with the moment resulting from the weight of testing equipment plus applied load

$$= \frac{574 + (7\cdot 23 \times 2{,}240)}{2} \times 16 \times 12 \qquad = 1{,}609{,}840 \quad ,, \quad ,,$$

Therefore, ultimate moment (max. M) $= 1{,}977{,}730$,, ,, The design moment was composed of the moments due to the weight of beams including grout

$$= 367{,}890 \times \frac{18}{17} \qquad = \quad 389{,}530 \text{ lb.-inches}$$

and to the surfacing, together with the moment resulting from the Ministry of Transport load as assumed by the Author $= \quad 700{,}000$,, ,,

Design moment, therefore, $= 1{,}089{,}530$,, ,,

The factor of safety related in conventional manner to the total design moment was $1{,}977{,}730 : 1{,}089{,}530 = 1{\cdot}81$. Even if the assumption for surfacing was not quite correct, the result would not be substantially affected.

The steel stress at failure was worked out on the basis of a cube strength of 8,000 lb. per square inch (as stated in the Paper), and assuming a rectangular stress block of $c_p = 0{\cdot}6 \times$ cube strength, it would be seen that:

$$\frac{\max M}{bd^2c_p} = \frac{1{,}978{,}000}{16{\cdot}5 \times 10{\cdot}375^2 \times 4{,}800} = 0{\cdot}232$$

Lever arm: depth ratio $= \frac{1}{2}(1 + \sqrt{1 - 2 \times 0{\cdot}232}) = 0{\cdot}866$

Lever arm $= 0{\cdot}866 \times 10{\cdot}375 = 9{\cdot}0$ inches

Steel stress at failure $= \dfrac{1{,}978{,}000}{9 \times 36 \times 0{\cdot}0314} = 194{,}500$ lb. per square inch.

The initial pre-stress had been $\dfrac{69{,}000}{12 \times 0{\cdot}0314} = 183{,}000$ lb. per square inch.

It would be useful if the Author could supply data on the physical properties of the wires, particularly a stress/strain diagram and data on relaxation.

With an assumed ultimate strength of 100 tons per square inch for the 0·2-inch-diameter wires (which was usual for the material available in Great Britain), the initial pre-stress was 82 per cent of the ultimate strength. That was very high and had surely caused a considerable loss by relaxation. It should be noted that the calculated stress at failure had exceeded the initial pre-stress by only 11,500 lb. per square inch, which did not indicate a great efficiency of the grout.

With the conventional method, assuming $m = 15$, the ratio of lever arm to depth would be 0·881, and with $m = 5$ (in view of the high concrete strength) that ratio would be 0·925. That second assumption would result in a lever arm of 9·58 inches and a steel stress of only 182,600 lb. per square inch, that was to say, about the same as the initial pre-stress.

On the basis of a calculated stress with rectangular stress-distribution (which gave the highest value) and with an assumed ultimate strength of 100 tons per square inch, the stress at failure represented only 87 per cent of the ultimate strength of the material. Had the beams been pre-tensioned, there was no doubt that the ultimate strength of the wires would have been reached, or even slightly exceeded at failure, as might be predicted from many tests, both in Great Britain and abroad, since the beams were under-reinforced. A formula recently published by Guyon,[1] to which Dr Hajnal-Kónyi would refer again later, assumed a considerable excess of the ultimate strength of the wires. With pre-tensioned wires the depth of the rectangular stress block would have been $\frac{1{\cdot}13 \times 224{,}000}{16{\cdot}5 \times 4{,}800}$ $= 3{\cdot}2$ inches; the lever arm, $10{\cdot}375 - 1{\cdot}6 = 8{\cdot}775$ inches; and the moment at failure, $1{\cdot}13 \times 224{,}000 \times 8{\cdot}775 = 2{,}222{,}000$ lb.-inches.

The time lag between pre-stressing and testing to failure was not stated in the Paper. It might be inferred from *Fig. 25* that it had been between 3 and 4 weeks. If the test had been carried out at a later date (say, a year after pre-stressing) the losses resulting from the relaxation of the steel and from further shrinkage and creep of the concrete would have been greater and both the cracking and the ultimate moment would have been smaller, reducing the overall factor of safety to less than 1·8.

With pre-tensioning, the effective pre-stress in the wires after release would have hardly exceeded 65 per cent of the ultimate strength of the wires as compared with 82 per cent in the beams described by the Author, so that the loss of pre-stress by relaxation would have been less. In that case, an increased time lag between manufacture and testing would have had much less influence on the cracking moment and none on the ultimate.

Whether or not an overall factor of safety of less than 1·8 was satisfactory in pre-stressed concrete bridges was a matter of opinion, but when different systems of pre-stressing were compared from the point of view of economy, the same factor of safety against failure had to be assumed. Dr Hajnal-Kónyi wondered if the Author had taken into account the difference in ultimate load-bearing capacity between pre-tensioned and post-tensioned beams when stating, on p. 467, " that for spans from 25 feet to at least 35 feet the post-tensioned pre-cast slab section is more economical than in-situ slabs, I-beams, or composite construction."

In order to have a fair assessment of the factor of safety, it was essential to make a close approximation of the load-bearing capacity of a structure and to distinguish between pre-tensioned and post-tensioned systems. In the case of pre-tensioned under-reinforced beams the ultimate moment was practically independent of the degree of pre-stress, whilst in post-tensioned systems the ultimate moment depended primarily on the pre-stress and on the efficiency of the grout. The Author's test had shown that although " the grout had completely filled round the wires " (see *Figs 30*),

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[1] " A Study of Continuous Beams and of some Statically Redundant Systems in Prestressed Concrete." *Inst. Tech. Bât. Trav. Pub.*, Circular J No. 8 (20 Sept., 1945). See C.A.C.A. Library Translation No. 33, p. 10.

its efficiency in establishing bond between the wires and the pre-cast concrete was negligible, since an increase above the initial pre-stress as shown in the preceding calculations (that was, 5 per cent of the ultimate strength) was to be expected even with non-bonded wires.

With an assumed ultimate strength of 100 tons per square inch of the wires, the formula recently published by Guyon would give the following ultimate moment : $36 \times 0{\cdot}0314 \times 224{,}000 \times 10{\cdot}375 = 2{,}628{,}000$ lb.-inches as compared with an actual moment of 1,978,000 lb.-inches, that was to say, only about 75 per cent of the value obtained by Guyon's formula. With a smaller initial pre-stress or with the test carried out much later, the discrepancy would have been even greater.[1]

The Author's test was therefore a confirmation of Dr Hajnal-Kónyi's view, expressed elsewhere, that Guyon's formula was not applicable to the type of construction adopted by the Author.

Dr P. W. Abeles observed that the Author had mentioned two distinct approaches to pre-stressed concrete design based on (1) the "service" (working) load, and (2) the ultimate load giving a certain factor of safety. However, the design had to comply with both conditions, as was pointed out in the "First Report on Prestressed Concrete," published in 1951,[2] and as Dr Abeles had advocated for the past 10 years. That view could not be better expressed than it had been by Dr F. G. Thomas at the Conference on Pre-stressed Concrete in 1949, when he had said "(i) the load factor against ultimate failure shall be sufficient, and (ii) the deformation at working load shall be within tolerable limits."[3] The two extreme views which the Author had mentioned in the Introduction were closely associated with the two approaches. Those engineers who had been so impressed by the "novel qualities" of pre-stressed concrete "that they have failed to recognize its relationship to traditional methods and so treat it with excessive caution" most likely considered only working-load conditions and ensured a high factor of safety against cracking, assuming that to be the most important condition. A design in which ultimate-failure conditions were disregarded might result, in spite of all caution, in a brittle structure like cast iron or glass in which cracking and failure would occur simultaneously. On the other hand, engineers who considered pre-stressed concrete "merely as a further development of ordinary reinforced concrete" and "fail to make full use of its unique properties" most likely disregarded the deformation at working load.

As mentioned above, the provision of a large factor of safety against cracking might result in a brittle failure. In fact, a high factor of safety against cracking could not be combined with great resilience ; those were

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[1] In his book "*Béton Précontraint*" (Paris, 1951), on p. 599, Mr Guyon attached a coefficient of 0·9 to his formula. Even so, the test load represented only about 83 per cent of Mr Guyon's value.

[2] "First Report on Pre-stressed Concrete." Instn Struct. Engrs, 1951.

[3] "Conference on Pre-stressed Concrete." Instn Civ. Engrs, 1949.

contradictory conditions. Resilience, however, was the "unique" property of pre-stressed concrete. As could be seen also from the Paper, cracks closed and became invisible on removal of loads approaching even failure conditions. Thus, a very small factor of safety against cracking would be sufficient for repeatedly occurring loads in bridge construction. Even if cracks developed owing to overloading, they would do no harm, so long as heavy impact did not occur and provided that they closed up entirely under ordinary load. That statement was based on fatigue tests [1] carried out at Liège in 1951, when cracks which had developed under special loading closed up entirely after millions of repeated loadings. On the other hand, if there was a possibility of heavy impact, as with railway underline bridges, it might be advisable to have a higher factor of safety against cracking so as to avoid cracking under an unforeseen overload (if such a case were really probable) and the possibility of gradual destruction of the bond near the cracks owing to the impact.

It would appear from the Paper that the safety factor was related to the live load only and not to the entire service (working) load, as was required for other materials and also for pre-stressed concrete.[2] Similarly, the permissible stresses appeared to relate to live load only instead of to the entire working load.

Turning to the question of the ultimate resistance of steel, Dr Abeles observed that it was interesting to note from the Paper that a good bond resistance was ensured by grouting-in the cables. However, from the test it would appear that the bond resistance was overcome just at failure. In under-reinforced sections (and the slab tested was under-reinforced), the ultimate force taken up by the steel, $T_{ult} = \frac{M_{ult}}{a}$, where a denoted the lever arm, could be taken as $K_u \, . \, A_t \, . \, t_{ult}$, where t_{ult} denoted the tensile strength of the steel and A_t its section area, whilst K_u represented a factor indicating to what extent the steel was used at failure. With pre-tensioned wire, K_u could be taken as unity, whilst it would appear from the test that K_u was less than unity in spite of the good bond of grout.

Dr Abeles considered that it would be interesting to hear from the Author particulars of the efficiency of transverse pre-stressing in ensuring an equivalence to a homogeneous slab structure, particularly if the transverse cables were provided at mid-depth. The case was different if separate cables were provided at the top and bottom, as shown in *Figs 19*, where transverse resistance was ensured by the steel.

Composite constructions on general lines had been excluded by the Author in his Paper except for the block solution indicated in *Figs 13*. In that case, tensioned cables were entirely embedded in the additional concrete placed between adjacent blocks. Such a solution had, in Dr

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[1] P. W. Abeles, "Some New Developments in Pre-stressed Concrete." *Structural Engineer*, Oct. 1951.

[2] See ref. 2, p. 213.

Abeles's opinion, great possibilities, particularly if the pre-cast part was reduced and an in-situ topping was provided in addition to the in-situ ribs between the individual blocks. If sufficient topping was available, transverse pre-stressing was obviated, which would be a great advantage.

The use of curved members was a pleasing solution and seemed to be particularly suitable for pre-tensioning, since it was possible to provide straight wires which allowed handling from the ends with the self-weight always counteracting. However, the solution presented by the Author in which the lower cables were curved downwards seemed, in Dr Abeles's opinion, to be somewhat unsatisfactory from the point of view of ultimate load condition, when in a cracked state the steel tended to act downwards and should be anchored to the concrete as in ordinary reinforced concrete.

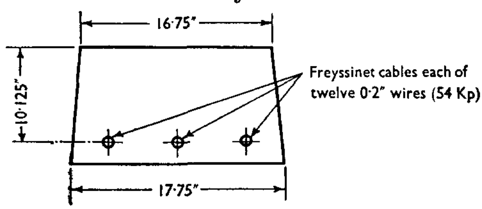
The Author's statement that beams of lengths up to 40 feet were cheaper when pre-cast under factory conditions had to be queried in the light of the experience gained with about twenty bridges, of spans between 20 and 50 feet, which had been built as composite constructions with a pre-cast pre-stressed component of minimum quantity, and which had proved to be very economical. Similarly, Dr Abeles could not understand the basis for the statement that, for spans of 25 feet to at least 35 feet, pre-cast slabs with post-tensioned cables were most economical. To his knowledge an alternative solution, consisting of a pre-tensioned girder of a weight of 6 tons and 40 feet in length, had recently been tendered at a lower price than the alternative solution with post-tensioned cables made near the site, although in the first case transport from the factory to the site and double profit of concrete works and contractor had had to be taken into account. However, it seemed to be doubtful that it was permissible to make such general statements with regard to economy, since each case had to be considered on its own merits. In any case, a composite slab, comprising a pre-stressed pre-cast component with pre-tensioned wires, costing £2 per cubic foot complete, placed in position, together with in-situ concrete costing 5*s.* per cubic foot complete, would cost only from 14*s.* 3*d.* to 16*s.* 8*d.* per cubic foot of composite slab, depending on whether the pre-cast component represented one-quarter or one-third of the entire quantity. That seemed to be less than the price per cubic foot of a pre-cast concrete slab with post-tensioned cables.

The Author, in reply, observed that the main point raised by all three contributors had been the ultimate moment of resistance. He agreed that the value obtained in the test had been slightly less than the theoretical value. He considered that the bond in the cables had been good, but agreed with Professor Ross that the system of loading might have been a possible cause of the slightly premature failure. The Author had found similar effects with two-point loadings in other tests of his own, and considered that it was not the best method of testing the resistance of a beam to bending only.

In order to clarify that point, he had decided to repeat the test on a similar beam, using a four-point loading, which would greatly reduce the chances of a shear failure. That had been done and the results for the ultimate load were given below.

It would be seen from *Fig. 31* that the beam tested had not been a precise duplicate of the previous one, the main difference being the slightly smaller lever-arm. That, however, did not affect the point in question—the efficiency of the bonds between grout and cable and between grout

Fig. 31



MID-SPAN CROSS-SECTION
Span: 34′–0″. Design live load 700,000 lb.-in.

and duct. The measure of the bond was the ultimate stress developed in the pre-stressing steel.

The applied bending moment at failure had been 1,898,000 lb.-inches.

Total bending moment was then	1,898,000
plus dead-load bending moment of beam . .	389,000
	2,289,000 lb.-inches.

The steel used in the cables had had a tested ultimate tensile strength of 111 tons per square inch, and the concrete cube strength had been about 8,000 lb. per square inch. The theoretical ultimate moment of resistance was then given by :—

Ultimate strength of three Freyssinet cables:

$$= 3 \times 12 \times 0{\cdot}0314 \times 111 \times 2{,}240$$
$$= 282{,}000 \text{ lb.}$$

Ultimate moment of resistance:

$$= 282{,}000 \times 10{\cdot}125\left(1 - \frac{282{,}000}{2 \times 16{\cdot}75 \times 10{\cdot}125 \times 4{,}800}\right)$$
$$= 2{,}860{,}000\ (1 - 0{\cdot}172)$$
$$= 2{,}340{,}000 \text{ lb.-inches}$$

The actual value obtained in the test was therefore $\frac{2{\cdot}28}{2{\cdot}34} \times 100 = 97{\cdot}5$

per cent of the maximum theoretical value, which showed that the steel was stressed to a value approximating closely to its ultimate, and that therefore the bond had been very effective.*

The factor of safety expressed as a ratio of total bending moment to design bending moment was $\frac{2,289,000}{389,000 + 700,000} = 2{\cdot}1$ which was a value comparable with that obtained in other forms of construction; that answered the point raised by Dr Hajnal-Kónyi and Dr Abeles, namely, that it was not proper to compare the costs of two types of construction that had different ultimate strengths.

The Author agreed that composite construction could be very economical, and was in fact developing a system of construction employing that principle with post-tensioned members. However, the costs of pre-tensioned beams did depend on the amount of repetition obtainable. In the case of the railway overbridges constructed to Dr Abeles' designs, there was doubtless a great deal of repetition and it was possible that the factory overheads of manufacture had been shared by other products. The Author had since prepared designs employing the solid post-tensioned beams described in the Paper in competition with pre-tensioned composite schemes, and had found the solid beams to be cheaper.

He agreed with Dr Abeles, however, that accurate comparisons were difficult to make, since the size of the scheme and the amount of repetition had a great influence.

The method of transverse pre-stressing appeared to be effective in practice and was certainly better than depending merely on the provision of an in-situ filling on top of the beams. The Author agreed that it would be better to place the cables near to the top and bottom of the beams, and that would be done in beams of greater depth. It had not been possible in the shallow beams described in the Paper.

The amount of curvature in the beams shown had not been sufficient to require stirrups on the cables, since at ultimate load the beam was straightened out and the downward component of the cables was reduced.

* The ultimate moment of resistance obtained by Mr Guyon's formula was $282{,}000 \times 10{\cdot}125 \times 0{\cdot}9 = 2{,}570{,}000$ lb.-inches, which was a greater value than that found in the test.

It was possible that Mr Guyon had had T-sections in mind when producing that formula, since it would be much more accurate when applied to them.