*Discussion on Paper No. 6502**

The ultimate load carrying capacity of slender stanchions bent about the major axis

by

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Professor M. R. Horne (Professor of Civil Engineering, University of Manchester) wrote that the extensive theoretical and experimental study undertaken by the Author showed how difficult it was to obtain a consistent criterion of strength in continuous I-section columns. While the exposition of the theoretical treatment was for the most part clear, it was confusing not to have stated that the last equation in the Author's § 19 applied only when $n>1$. A special equation was required when $n=1$. It would be of value to know how many terms in the Fourier analysis had to be taken into account in the calculations for the experimental stanchions, since these became tedious even when only two terms were included.

54. The Author compared (Fig. 8) the experimental failure loads of his "near-perfect" stanchions with the theoretical elastic critical loads. He stated that, in these cases, the relevant critical load was that which ignored the stub-stanchions, since the lateral deflexions were small. This was a misconception, since however small the initial imperfections might be, the deflexions became large at the critical load. Using the numerical values given in Table 3, and the approximate result of equation 12, the effect of the stub-stanchion was to decrease the critical load by about 20%. If this correction were applied to the theoretical curve in Fig. 8, all the experimental results lay significantly above the elastic critical load. This indicated that the Author might not have been entirely successful in eliminating various restraints which might have acted in the tests to increase the failure loads. Thus, one possible restraint would be a lateral force resisting the free sideways displacement of the end of the stub-stanchions.

55. In Fig. 9, the Author compared the theoretical first-yield loads, allowing and ignoring the stub-stanchions. When the initial curvature was zero, one would expect no difference to arise, since the stanchion must remain straight until the elastic critical load was reached. Some further explanation of the basis of the Author's calculations would be instructive.

56. In his calculations of the load at first yield, the Author ignored the warping of the section as it affected the maximum stress. This introduced a fourth component in § 27. In the design method given in the Author's reference 4, the fourth component was neglected, but since warping resistance was also ignored in the analysis, it might be argued that the procedure was justified. In the Author's treatment, warping resistance was included, and it would have been more consistent to have allowed also for warping stresses. Could the Author give some idea of the warping stresses at the theoretical first-yield loads of his stanchion?

Mr A. N. Procter (Consulting Structural and Civil Engineer, Nottingham) wrote that, arising out of the opening remarks in § 49 of Mr Trahair's excellent Paper, the follow-

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ing simple and practical methods of calculation were suggested. Using the Author's notation, it could be shown[9] that

$$\frac{P}{Q_Y}+\frac{M_B}{Z_Y.f_Y}+\frac{M^2}{Q_Y.C}=1 \qquad \text{(24)}$$

was a suitable basic formula for columns failing in single curvature. It was found that mild steel columns in double-curvature buckling with ends flat or the normal laboratory ends-fixed conditions (Q'_Y) would carry about three times the Euler load,[10] Q_Y. In order to achieve fixity, end-restraining couples, M'_B, applied by means of the elastic ties, would be about 25% of the moment of resistance of the stanchion[11] ($Z_Y.f_Y$).

Thus $\quad \frac{P}{Q'_Y}+\frac{1}{4}+\frac{M^2}{Q'_YC}=1$ or $\frac{P}{Q'_Y}+\frac{M^2}{Q'_Y.C}=\frac{3}{4}$.

Under the action of the bending moment, M, the eccentricity, e, of the force, P, would be M/P. Thus $M=P.e$ and the formula became

$$\frac{P}{Q'_Y}+\frac{P^2.e^2}{Q'_Y.C}=\frac{3}{4} \qquad \text{(25)}$$

The stiffness of the elastic beam was considered sufficient to give the required fixity to the column. Thus $Q'_Y=3Q_Y=3{\cdot}48$ tons and $C=6{\cdot}64$ tons-in^2. Solving,

$$P=\frac{C}{2e^2}\left(\sqrt{1+\frac{Q'_Y.e^2}{C}}-1\right)=\frac{3{\cdot}32}{e^2}(\sqrt{1+1{\cdot}58e^2}-1)\text{ tons.}$$

Calculated values of e and P were shown in Table 3, columns 2 and 3.

58. In formula (25) the strength of the straight axially loaded columns would be about $\frac{3}{4}Q'_Y=2{\cdot}61$ tons. It was clear, however, that imperfections had a greater effect on axially loaded columns or on columns with small applied moments, e.g. stanchions 5b, 6d, 6k, 8c, and 8e. When the applied moment, M, was large, the experimental load was more nearly equal to the calculated load. Then the effect of imperfections became less evident, although the recorded experimental imperfections acted about the weak axis only of the stanchions. To estimate the effect merely of the initial curvature, the second term of eqn (24) should include $P.u_0/Z_Y.f_Y$ in addition to the effect of the fixing moment, M'_B. Multiplying by Q'_Y,

$$\frac{P.u_0.Q'_Y}{Z_Y.f_Y}=6{\cdot}6Pu_0\text{ tons and }P=\frac{3{\cdot}32(1+6{\cdot}6u_0)}{e^2}\left(\sqrt{1+\frac{1{\cdot}58e^2}{(1+6{\cdot}6u_0)^2}}-1\right)\text{ tons.}$$

59. Corrected values of P were shown in column 4 and the experimental values in column 5. Apart from the stanchions already referred to (§ 37) and those noted above, the agreement of the corrected loads and the test loads was reasonably consistent. The mean deviation was $+7\frac{1}{2}$% on the high side compared with 40% for B.S. 449 : 1959, the load factors of which were shown in column 6 for an effective slenderness ratio of 140. The results appeared sufficient justification for ignoring the initial twists that might modify the early deflexions rather than the final loads.[13] When the applied moment M was zero, the corrected value of $P=\frac{2{\cdot}61}{1+6{\cdot}6u_0}$ was noted in brackets in Table 3.

60. To determine the effect of an applied torque, T, the third term of equation (24) could be modified to the form[9] $\dfrac{\left(M+\dfrac{T.l_2Q'_Y}{4\pi M}\right)^2}{Q'_Y.C}$. When T was small compared with M, e.g. when caused by small imperfections, this was $\frac{M^2}{Q'_YC}+\frac{T.l_2}{2\pi C}$. The combined action of M and P with the initial twist would produce a torque that might be substituted for T, giving another small correction. This did not by any means account for the $+7\frac{1}{2}$% mean deviation between the test results and the calculated values. There would be of

TABLE 3: COMPARISON OF CORRECTED COLLAPSE LOADS WITH TEST LOADS ON STANCHIONS

Stanchion No.	Moment/force ratio, e: *in.*	Axial load at collapse: P, *tons*			B.S.499: 1959 ends fixed. Load factor
		Calculated $u_0=0$	Corrected for curvature	Test loads	
Trial	0·47	2·36	—	2·32	3·2
1	0·45	2·38	2·37	2·23	3·1
2	—	(2·61)	(2·58)	2·02	2·3
3	5·5	0·66	0·63	0·55	2·1
4	2·0	1·41	1·39	1·49	3·2
5a	—	(2·61)	(2·56)	2·70	3·1
b	0·24	2·53	2·36	2·09	2·7
c	—	(2·61)	(2·44)	2·04	2·4
d	0·49	2·38	2·27	2·06	2·9
e	0·77	2·13	2·10	1·95	3·0
f	1·09	1·93	1·90	1·84	2·9
6a	—	(2·61)	(2·56)	2·44	2·5
b	—	(2·61)	(2·53)	2·26	2·6
c	—	(2·61)	(2·23)	2·27	2·7
d	0·38	2·41	2·29	2·09	2·8
e	0·63	2·37	2·10	1·91	2·8
f	0·91	2·06	1·91	1·78	2·8
g	1·32	1·77	1·65	1·52	2·8
h	5·23	0·71	0·67	0·64	2·8
i	97·5	0·041	0·041	0·04	2·4
j	—	(2·61)	(2·20)	2·18	2·0
k	0·42	2·42	2·14	1·92	2·6
l	0·97	2·02	1·81	1·65	2·7
m	1·87	1·47	1·38	1·26	2·6
n	5·10	0·69	0·67	0·63	2·3
o	100·5	0·041	0·041	0·04	2·0
p	—	(2·61)	(1·91)	1·85	2·2
q	0·5	2·37	1·71	1·60	2·3
r	1·44	1·70	1·36	1·12	2·1
s	4·2	0·82	0·71	0·63	2·0
t	90·2	0·046	0·044	0·04	1·8
7a	—	(2·61)	(2·48)	2·50	2·9
b	0·34	2·49	2·36	2·37	3·1
c	0·80	2·08	2·06	2·01	3·1
d	1·53	1·65	1·63	1·58	3·0
e	2·97	1·08	1·06	1·08	2·8
f	8·4	0·48	0·45	0·48	2·8
g	—	(2·61)	(2·11)	1·86	2·8
8a	—	(2·61)	(2·52)	2·26	2·6
b	—	(2·61)	(2·54)	2·23	2·6
c	0·38	2·41	2·39	2·11	2·9
d	0·78	2·17	2·14	2·16	2·3
e	0·38	2·41	2·38	2·10	2·8
f	0·85	2·12	2·04	1·91	3·0
g	1·54	1·64	1·61	1·57	3·0
h	2·72	1·15	1·13	1·18	2·9
i	9·8	0·39	0·38	0·43	2·6
j	107·5	0·038	0·038	0·04	2·1
k	—	(2·61)	(2·28)	1·95	2·3
l	0·45	2·38	2·19	1·79	2·5
m	1·06	1·92	1·79	1·52	2·6

Loads in brackets from $P = \dfrac{2{\cdot}61}{1+6{\cdot}6u_0}$ tons. Mean load factor = 2·8.

course other sources of error, as noted at the end of § 37, such as unintentional imperfections or eccentricities, etc. Since they appeared to be roughly proportional to the applied forces, they could be allowed for in a load factor.

61. In practice there were appreciable bending moments applied about the minor axis of a stanchion in addition to the restraining moments. In a design these could be dealt with quite simply by utilizing formula (24). In such cases it would probably be found that there would be a further increase in the load factor imposed by B.S. 449 : 1959, since this made no provision for the fixing moments required to restrain the ends of the column.[12] It was clear that the relative importance of column imperfections decreased as the applied bending moments increased.[13] In practice some economy could most likely be achieved by bearing this in mind, since the imperfections were deduced from tests on axially loaded columns. Moreover, it was found that, when the formulas suggested above were plotted, the diagram might be used to develop a direct method of column design. This would obviate the present trial and error methods and also increase efficiency in design.

Dr R. H. Wood (Senior Principal Scientific Officer, Building Research Station) wrote that Mr Trahair was the first to give a useful derivation for the torsional elastic critical loads of stanchions, restrained about the minor axis by unloaded elastic beams, and subjected to constant moments M_x applied about the major axis by "plastic" beams. This celebrated P_xE_y case had been much debated by Horne,[1] Baker *et al.*,[4] Heyman,[5] and Wood,[14,15] the latter from the point of view of producing an immediately available design method[15] for restrained stanchions, even when bent about both axes by beam loads, rather than an all-out attempt at predicting outright collapse loads. This important case was in fact intractable,[4] especially if recent developments concerning existing stresses were taken into account,[16] but that did not mean that reasonable approximations to the collapse state could not be built into a rapid design method.

63. The Author's Fig. 8 showed how important the knowledge of the elastic critical load was from the point of view of approximating to the collapse state. But it was important to remember that by studying only symmetrical single curvature the Author would be giving extra prominence to the elastic critical load. Secondly, the general problem of determining the *alteration* in end moments in the stanchion, depending on the slenderness, the direct load, the beam/column stiffness ratio at the top and (independently) at the bottom, the out-of-balance fixed-end moments of loads on the elastic beams, top and bottom, demanded a thorough knowledge of Berry's functions (or Merchant's[17] more up-to-date tabulation). On such a treatment hung the solution of the frame-instability problem even for the no-sway case,[14] and as yet the only rapid design method which incorporated Berry's function as a basis was that derived by the Building Research Station.[15]

64. Moreover when such beam restraint (E_y) was employed, the stanchions could be astonishingly slender, and in the B.R.S. design method, for example, it had been necessary to provide for stanchions under factored loads operating at such high levels as $(P/P_E) = 1{\cdot}5$, where

$$\frac{P}{P_{Euler}} = \frac{P}{P_E} = \frac{P}{\pi^2 EI/L^2} = \frac{PL^2}{\pi^2(13\,000)\,Ar^2} = \left(\frac{\frac{P}{A}}{13}\right)\left\{\frac{L/r}{100}\right\}^2$$

approximately, in ton-in. units.

65. This simple formula was a reminder that a stanchion with a slenderness ratio of 100 and a direct stress of 13 tons/sq.in. was operating at the Euler load. If the figures were 150 and 8 respectively, then $P/P_E = 1{\cdot}38$, and there was even then the possibility of considerable bending stress before the yield stress of 16 tons/sq.in. was reached. This emphasized that the major problem in the P_xE_y case was, to begin with, a problem with Berry's functions built into slope/deflexion equations, and the Author's treatment had only encountered this general problem in a very particular case. At a more advanced

elasto-plastic stage, Berry's functions were superseded by the "conjugate beam-line method" as outlined by Wood.[18] As regards the torsional problem with beam bending as well as beam restraint, this was partly investigated by Goodwin,[19] using a differential analyser, who showed that the use[15] of very slender stanchions could be accompanied by only small torsional stresses. Moreover, torsional instability in the presence of existing stresses was then certainly intractable, and would provide a scatter of points, and was better avoided than incorporated in a design. Of course to avoid it, it was necessary to study the increase of stress arising from it and to limit this increase of stress. Here however was a dilemma. For with restrained stanchions, once the principal problem of bending stresses induced by the beams had been solved (Berry's problem) and the stanchion designed accordingly, then any further stresses arising from torsional instability were subject to severe complications coming from two sources:

(a) The deterioration of stiffness about the minor axis and about the torsional axis as a result of existing stresses, when coupled with the bending stresses and direct stresses, causing plastic zones of unknown extent.

(b) Measured out-of-straightness.

66. Previously the tendency in Britain had been to make Perry's constant $\eta = 0{\cdot}003\, L/r$, belonging to item (b), virtually incorporate all the effects of item (a). The Author's treatise followed in this stream. The tendency in America was just the opposite,[16] and seemed to be trying to invent a system of agreed locked-up stresses (item (a)) which would also virtually incorporate the effects of item (b). This debate was becoming too elaborate. The principal reason in a design process[15] for incorporating out-of-straightness and existing stresses at all was that, in the event of the loads on an elastically restrained stanchion being absolutely symmetrically disposed, the sum of the induced bending stresses about each axis (Berry's problem) being then zero, there was nothing to stop the engineer designing the stanchion so as to be at the full "squash" load $P = Af_Y$. This would cause severe deterioration of stability,[14] and to avoid it some value of η must be agreed upon. But in the end it must be remembered that no theory of torsional instability was qualitatively any better than the choice of the value of η in the first place. In improving on Perry's treatment, therefore, to include torsion, it must not be forgotten that the main design problem was Berry's problem. This was often overlooked.

The Author, in reply to Professor M. R. Horne, stated that the last equation in § 19 of the Paper was quite general and held for all values of n. However, when $n = 1$ the equation simplified and the coefficient a_1 could be expressed as in the first equation (9). This expression could also be derived from the second equation (9) with $n = 1$.

68. In the theoretical analysis, convergence of the quantity $\sum_{n=1}^{\infty}(2n-1)a_n$ was required, and ten terms of the series had to be calculated for this. However, it was found that $\sum_{n=3}^{10}(2n-1)a_n$ remained practically constant for a given value of M for the range of values of P. This fact considerably reduced the volume of calculations required.

69. Stub stanchion action was a factor introduced experimentally, and an attempt was made to account for it theoretically. If the stub stanchion acted as theoretically assumed, then its contribution to the effective length would lower the elastic critical load of a perfect stanchion below that where there was no stub stanchion. This was shown in the analysis presented in the Paper, and might also be demonstrated by using the energy method to derive the critical loads.

70. For a theoretically perfect stanchion, the lateral deflexion would be zero when the critical load was reached and would then increase with no increase in load until catastrophic failure occurred at first yield. For near-perfect stanchions, the experimental failure loads were reached before the deflexions became large and before the

stub stanchions started to move laterally. This was the reason that the stub stanchion effect was ignored for these stanchions. Professor Horne's comment that possibly additional restraints might have been introduced was quite likely to be true, but these had not been detected experimentally.

71. In the theoretical calculations, the longitudinal stresses due to warping were ignored. This was justified when either the major axis moment was zero or there were no imperfections, since in these cases the warping stresses were either zero or infinitesimal. The extreme fibre stress at the centre of the stanchion due to warping might be derived from

$$\sigma_z = -\frac{EdB}{4}\left(\frac{d^2\theta}{dz^2}\right)_{z=l_2/2}$$

$$= \frac{EdB}{4}\frac{\pi^2 M}{Cl_2}\sum_{n=1}^{\infty}\frac{(2n-1)^2 an}{1-\alpha+(2n-1)^2\gamma}\sin\frac{(2n-1)\pi}{2}$$

Now σ_z would be greatest for large moments and imperfections. The stress σ_z for $P=0{\cdot}5$ ton, $M=2{\cdot}0$ tons, and $l_2(Pa_0+K_1at)=0{\cdot}2$ ton-in., was calculated as 1·40 tons/sq. in. or 8% of the yield stress. This stress was additive in effect to the stresses previously calculated, thus reducing the yield load combinations.

72. When taken into account, this effect emphasized the increase of the experimental collapse loads over the theoretical first yield loads for stanchions with high major-axis moments and imperfections. It did not alter the conclusion that the first yield failure theory was applicable if the major-axis moments or the imperfections were small.

73. Mr A. N. Procter had given a design method based on empirical formulae and certain approximations, which was a mixture of elastic analysis and the results of experimental ultimate tests. In applying them to the experimental results given here, he found that his mean prediction was 7½% higher than the experimental collapse loads. The use of design methods such as this should be treated with extreme caution until it could be shown either theoretically or by extensive experimental tests that the method was both safe and accurate and what its limits of applicability were.

74. In § 58 Mr Procter said "It was clear, however, that imperfections had a greater effect on axially loaded columns or on columns with small applied moments". It should be pointed out that Mr Procter was referring to a comparison between his calculated loads and the experimental collapse loads and not to the actual behaviour of the stanchions tested. Further, his statement that initial twists modified early deflexions rather than final loads was not justified by the experimental evidence which showed that for high major-axis moments, the initial twists played an important part in determining the collapse loads.

75. Dr R. H. Wood had drawn attention to the fact that the work presented in the Paper formed only a part of the general problem of $P_x.Ey$ stanchions. This general problem might be attacked in two ways, the first being to determine the collapse loads, the second being to produce by reasonable approximations a workable design method which would give safe answers. This Paper represented an investigation by the first method of attack of a clearly defined part of the general problem, but it was hoped that the experimental results would be of some use to those who were using the second method of attack.

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