

**A proposed 'exact' theory for analysing shells, and its solution
with an analogue computer**

by

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Mr P. G. Lowe (Civil Engineer, London Computer Centre, Ferranti Ltd) wrote that he was interested to read the Author's views on some aspects of shell theory and his suggestions for solving the resulting equations. But it was regrettable that Dr Wilby should claim to have produced a more 'exact' theory than that given by Jenkins.¹

30. The Paper claimed that this latest theory made 'even fewer assumptions than Jenkins' theory'. What then were the assumptions in the present theory? No indication was given in the Paper, nor were there any references to a prior discussion of these assumptions.

31. In fact the basic equations (1-16) of the Paper were a simplified set derivable from Jenkins' results, bearing in mind differences in sign convention, with the sole exception of the term involving N_2 in equation (4).

32. It was clear that Jenkins, too, could have retained such a term, along with numerous others, if he had been inconsistent in the examining and discarding of his primary, secondary, and higher order terms.

33. On the question of the present theory's relation to other possible theories it was sufficient to note that Novozhilov,⁵ Goldenveizer,⁶ and Flügge,⁷ of the contributions in book form, developed 'general' theories of greater complexity than did either Jenkins or Dr Wilby. (By 'general' in this context was meant a theory applicable to a shell of arbitrary shape, not necessarily cylindrical as with Jenkins' and the present theory.)

34. Further, Novozhilov⁵ in the section beginning on p. 84 systematically simplified his preceding discussion of the theory to obtain a theory which, when specialized to the cylinder, was *identical* with the K-D-J theory.† This suggested that the effect of addition of the N_2 term referred to above was to produce a hybrid theory and not a revision of the K-D-J theory.

35. If Dr Wilby intended his claim for 'exactness' to be considered alongside the claims of other theories, he must surely be prepared to state his reasons for wishing to add this further term in order that these can be verified.

36. Despite the above criticism of the Author's claim for 'exactness', it was not Mr Lowe's intention to detract from the main interest in the Paper, which was the Author's contribution to solving the sets of resulting equations.

37. Very many points arose from other aspects of the Paper. First, in fairness, surely the word 'cylindrical' should have been included in the title. Special-purpose devices were described and the equations presented applied only to the cylindrical case.

38. A point of considerable interest was that the Author used the analogue circuit to solve sets of ordinary differential equations, the usual separation of variables by use of a single, length-wise Fourier expansion being adopted.

39. It was this device which permitted progress whether by hand or digital machine

* *Proc. Instn civ. Engrs*, vol. 22, July, 1962, pp. 291-308.

† Karman-Donnell-Jenkins, referred to here as 'Jenkins' Theory'.

and also it would seem when an analogue was used. The present approach, therefore, would not allow an extension of horizon to problems which could not be conveniently handled by hand or digital machine. (It was appreciated that this was not one of the expressed purposes of the Author.) But in addition to attempting solution of problems which could be handled by other means, would the Author care to forecast where the analogue computer might show intrinsic advantages over the digital machine?

40. The description in the Paper was quite naturally limited to the inclusion of a single Fourier co-efficient in the solution. In cases where a number of terms were required, would this necessitate alteration to the circuit connexions?

41. When a series of structural elements was suitably joined together, but the idealization to 'end' and 'interior' spans was not satisfactory, would it be true that a circuit representing the entire structure need be assembled?

42. In the light of the recent advances made in analogue computer technology, could the Author give some idea of the likely difficulties, if any, which might be encountered as the circuit representing progressively larger multi-span structures increased in size?

43. Tottenham⁸ recently suggested a method for dealing with certain shell problems with digital computers where the solution was obtained by Jenkins' theory with much the same effort as Schorer's or other theories. Thus the statement in the Paper, 'normal methods of design with . . . electronic digital computers are far more arduous using Jenkins' than Schorer's theory,' could be challenged. But, perhaps of greater importance, Tottenham's proposed methods allowed programmes of very useful specification to be written for the smallest digital computers available.

44. The Author's desire to make available in a sufficiently inexpensive form an apparatus for solving what was, manually, a difficult problem, was to be commended. The merits of the analogue (and the digital) machine are only imperfectly appreciated and much remains to be documented. It was therefore useful to have the present Paper for reference.

45. Undoubtedly analogue and digital techniques each had something to contribute to the engineer's ability to understand and satisfactorily design real constructions. Despite the above criticism, it was in the spirit of 'not one, not the other, but both' that Mr Lowe submitted this contribution to the discussion.

The Author, in reply, pointed out that to have made his Paper very explicit to a reader not conversant with shell analysis and also analogue computers would have involved a lengthy treatise on both these subjects prior to describing the particular contribution that he considered his work made to this dual subject. Consequently the Paper was written as briefly as possible to make it sufficiently short for publication in the *Proceedings* but yet for it to describe adequately the contribution to this particular field of knowledge. A more explicit account of the method developed for analysing shells by electronic analogy with shell theories, other than the one expounded in the Paper, was given in a recently published monograph,⁹ and this might be useful to Mr Lowe.

47. The foregoing explanation accounted for the omission of a list of the assumptions made in developing the mathematical equations in the Paper. Jenkins' theory was referred to in the Paper and anyone familiar with that theory would realize that the difference between Jenkins' theory and the theory expounded in the Paper was that the latter included the term N_2/R . Mr Lowe seemed to have realized this in § 31, in which he answered his query of § 30. The Author wondered how Mr Lowe could sensibly regard it as 'regrettable' that the Author should have taken this extra term into account—especially when one bore in mind that, with the analogue computer method of solution developed, the analysis with the extra term, as the Paper explained, caused little extra work in designing the analogue circuits and no extra work in the subsequent numerical solution of the equations.

48. It would be realized that higher order terms such as mentioned by Mr Lowe in § 32, and indeed further terms of Fourier's series, could be included in the analysis

when using an analogue computer as outlined. Obviously this meant extra circuits, more complicated (and demanding a larger analogue computer) than the simple extra circuits shown in the Paper and which give a theory slightly more 'exact' than Jenkins' theory. That, therefore, was the reason for including the term in N_2 rather than other terms.

49. With regard to § 33, the Paper dealt only with theories for cylindrical shells, not for shells of any arbitrary shape. It was therefore not a good comparison. With regard to §§ 34 and 35, all shell theories differed according to how many assumptions were made and the assumption which the Author had managed without (compared with the K-D-J theory) was one which could easily be made because of the method devised for solution with the electronic analogy.

50. Mr Lowe should know that editors usually liked short titles and with this in mind the word 'cylindrical' had been omitted. The Author considered that this method could be extended to deal with shapes other than cylindrical, so that perhaps it was a good omission.

51. In the Author's opinion, digital computers could not solve problems which could not in theory be solved 'by hand'. However, in practice the digital computer could solve problems which would never be attempted or solved 'by hand', because of the enormous effort required. The analogue computer might have a comparable advantage over the digital computer in that it required much less programming effort. This might mean that the analogue computer would be a reasonable apparatus for use in the solution of some complicated analysis which would be possibly too formidable a task for programming for a digital computer. The intrinsic advantages of the analogue computer over the digital computer were as described in the Synopsis and Conclusion to the Paper. In this respect, it was sometimes quite surprising which variables affected other variables the most, and sometimes the most effective dimension to alter to reduce, say, a certain stress, was the one which would be least expected.

52. To enlarge on the other advantage of relative inexpensiveness, an analogue apparatus specifically for designing cylindrical shells could be sold for £2000, which was very inexpensive compared with the price of a digital computer or with the salaries of shell designers. A circuit pattern could even be very inexpensively constructed for use in conjunction with a standard type of electronic analogue computer, costing about £2400.

53. As mentioned in the Paper, the Author saw no particular difficulty in using further terms of Fourier's series. In dealing with shells adjacent to one another where one does not wish to make the simplifications into 'end' and 'interior' spans, he considered that this problem could be solved on the electronic analogue computer but it would to some extent increase the number of integrators etc. required. In studying this problem some of the ideas given by Tottenham⁸ might be useful. Mr Lowe had apparently misunderstood this paper by Tottenham; it dealt with the treatment of boundary conditions for continuous (side by side) shells and hence the work applied equally to any theory of shell analysis. The methods of Jenkins and Shorer were dealt with in the same way in this respect but this did not mean that the work used in the solution by these two theories was the same for a digital computer or for conventional 'hand' computation.

54. The Author agreed that analogue computers should be used as well as digital computers, and, of course, some of the very large electronic computers were both analogue and digital, the analogue portion being used to obtain an initial 'guess' so as to reduce the time required for the digital portion of the computer.

REFERENCES

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7. W. FLÜGGE. 'Stresses in shells.' Springer Verlag, Berlin, 1960.
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9. C. B. WILBY *and* N. W. BELLAMY. 'Elastic analysis of shells by electronic analogy.' Arnold, London, 1962.

CORRIGENDA

In the original Paper, R should be deleted from equations (46) and (47) on p. 297.

Page 292, line 16: for ' I_z and I_y ' read ' I_y and I_z '.

Page 303, 22, line 5: for ' $\theta_2 = 20^\circ 29 \text{ min}$ ' read ' $\theta_2 = 20^\circ 49 \text{ min}$ '.

Page 303, equation 72: for ' $\dots - 31 \cdot 343$ ' read ' $\dots - 28 \cdot 742$ '.
