*Discussion on Paper No. 6661**

Simplified rational analysis of filter behaviour

by

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Mr H. E. Hudson, Jr (Partner, Hazen and Sawyer, Engineers, New York) observed that filter designers were gladdened by the Author's complete exploration of what went on during filtration, but troubled by the mathematical treatment necessary to make the description rigorously correct. Its correctness was established by the excellent agreement of laboratory observations with computed results, and the relationships were in general accord with Mr Hudson's laboratory and plant observations.

47. The Author's analysis should now be put into application by assembling data on results obtained in operating works, and determining the basic values required. This required keeping in mind two major criteria: one dealing with water quality, the other with filter clogging rates.

48. Mr Hudson considered that values of C/C_0 in the range of 10^{-1} to 10^{-2} ought to be produced in filter effluents, and the smaller the ratio, the better for the public health.[11] Limiting values ought not to be founded on averages nor on brief laboratory trials. They should be screened from the results of years of plant-operating records and should take into account the worst cases found. Where high values were due to design or human failures, it was appropriate to ask how such failures could be prevented by better design. As Mr Hudson saw it, the most important parameter to be evaluated in this regard was σ_u, which appeared to depend on the character of the material to be removed by the filter. Other variables could be controlled by the designer, but the maximum specific deposit seemed to depend on natural phenomena and poorly-understood chemical variables that had not been quantitatively evaluated.

49. The second major unknown to be evaluated from long-term experience was the clogging tendency of the material being filtered. It, too, required the searching of records for extreme cases, and it required only appraisals of the head loss constant k. This parameter was also dominated by an array of unappraised factors involved in the process of preparation of the water for filtration.

50. These suggestions made the interpretation of results seem simple. This would not be the case until there was consolidation of the relationships, additional simplifying assumptions, and perhaps the preparation of numerical tabulations showing the relations between parameters whose functioning could not be simply expressed. This further work was needed to make the analysis useful to the practising engineer. It was a pity to propose the conversion of the elegant into the mundane, but even more simplification was needed to fit this tool to the grasp of the designers.

Mr A. K. Deb (Lecturer in Civil Engineering, Bengal Engineering College, Howrah, India), observed with great interest the simplified procedure laid down by the Author for the design of sand filters. In most of the cases it was found that the head loss versus time curve was not a linear one, especially when flocculated influent water was

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used. The suggestion of the Author in that case was to deduct the head loss across the first layer from the total head loss readings; in some cases this might not be helpful. This was especially true towards the beginning of the run.

52. Mr Deb suggested the following alternative method for calculating the value of k. Considering any layer of filtering media between depths L_1 and L_2, the head loss across the layer could be written from equation (18);

$$H_1 = \int_{L_1}^{L_2} hdl = \int_{L_1}^{L_2} h_0 dl + \int_{L_1}^{L_2} k\sigma dl = H_{0_1} + \int_{L_1}^{L_2} k\sigma dl \quad . \quad . \quad (25)$$

$$\int_{L_1}^{L_2} \sigma dl = \frac{H_1 - H_{0_1}}{k} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25a)$$

where H_1 and H_{0_1} were total and initial head losses (water column) in the layer bounded by depths L_1 and L_2. For uniform filter k was assumed to be constant at all depths.

53. The left hand side of equation (25a), $\int_{L_1}^{L_2} \sigma dl$, represented the volume of deposit in the unit area of the filter layer bounded by depths L_1 and L_2.

54. From the continuity equation, $-\delta C\, v\, \delta t = \delta\sigma\, \delta L$; $v \int_0^t \delta C dt$ represented the deposit in unit area of filter bed of any layer, where δC showed the difference of influent concentration and effluent concentration of particles in that bed layer in time δt. Therefore, the volume of deposit in the unit area of filter bed bounded by depths L_1 and L_2 could be equated as:

$$\int_{L_1}^{L_2} \sigma dl = v \int_0^t \delta C dt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

55. From equations (25a) and (26):

$$\frac{H_1 - H_{0_1}}{k} = v \int_0^t \delta C dt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

$$k = \frac{H_1 - H_{0_1}}{v \int_0^t \delta C dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

The integral $\int_0^t \sigma C dt$ represented the area between the curves of influent particle concentration and effluent particle concentration between any layer of depths L_1 and L_2, when plotted against time.

56. The equation (28) could be written in the summation form as:

$$k = \frac{H_1 - H_{0_1}}{v \sum_0^t (C_1 - C_2)\delta t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

If the initial and the total head losses and influent particle concentration, C_1, and effluent particle concentration, C_2, in any layer of uniform media could be measured at different intervals during test run, k for any layer could be calculated from equation (29). Equation (29) would be valid for any layer of filter column, even in case of stratified bed.

57. Considering full depth, for uniform bed the equation (29) becames:

$$k = \frac{H - H_0}{v \sum_0^t (C_0 - C_e)\delta t} \quad . \quad . \quad . \quad . \quad . \quad (30)$$

When C_0 was constant,

$$k = \frac{H - H_0}{vC_0 t - v\sum_0^t C_e \delta t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30a)$$

where C_0 is influent particle concentration and C_e is effluent particle concentration. Measuring initial head loss, H_0, and total head loss, H, for full filter depth and influent particle concentration, C_0, and effluent particle concentration, C_e, at different time intervals, the value of k for uniform bed could be computed from equation (30a). If C_0 varied with time, equation (30) should be used for k value.

58. Neglecting C_e, which would be negligible for full filter depth, equation (30a) could be approximated as:

$$k = \frac{H - H_0}{vC_0 t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

which could also be compared with the Author's approximate equation (20a).

59. If the concentration of particles and head losses at different depths and time intervals could be measured during test run, the values of k at different layers for stratified bed could be calculated from equation (29).

60. It seemed from various data given by the Author, that the value of k depended not only on bed characteristics, but possibly on influent particle characteristics also. The test run had therefore to be conducted with the same influent particle characteristics as might be expected in actual plant.

Dr L. E. Robinson (First Assistant Engineer, Croydon Corporation Water Undertaking) remarked that the Paper represented a methodical approach to the design of filters from the use of a simple model. However, examination of the curves of concentration ratio, C/C_0, against depth, L, showed that the calculated values of C/C_0 were less than the experimental values, particularly in the lower regions of the filter. This led to the corresponding values of storage ratio, σ, being greater than the experimental values. The reasons given by the Author for these discrepancies might have had some significance but were not considered to be the main cause.

62. The theory had been developed from an assumption that the removal of particles from the flow was proportional to their concentration in the flow, this assumption being expressed mathematically as $C = C_0 e^{-\lambda L}$. Use of this basic equation, allowing for the change in the value of λ with time, to calculate values of C and hence σ at any interval of time throughout the depth of the filter, was believed to be incorrect. From the experimental values of C at various depths in the filter, obtained during a programme of research at University College, London, it was observed that the curve of C against depth, L, could not be represented by one negative exponential curve of the form $C = C_0 e^{-\lambda L}$ but that the distribution of material with depth appeared to be a more complex function with a discontinuity occurring at a depth which, for any given grade of filter media, appeared to be dependent on the concentration of suspension used. It was found that a very accurate representation of the actual experimental results could be obtained by using a concentration/depth curve consisting of two negative exponential curves superimposed one on the other. Each had a different value of λ and a different imaginary value of C_0.

63. The values of λ and C_0 for each of these curves, for any given instant of time, could be obtained by plotting on semi-logarithmic paper, C on the logarithmic scale against L on the linear scale. These plots produced two straight lines of different slopes, and the lines produced back to zero depth gave the two imaginary values of C_0 and the slopes the two values of λ. The depth at which the two lines intersected defined the boundaries of their validities. In all cases, the value of λ obtained was greater for the top portion of the filter than the value obtained for the lower portion.

Since the calculations of the storage ratio, σ, and the concentration, C, required the use of the value of λ, any calculations or equations based on the original assumption made in the Author's paper would give values of storage ratio, σ, in the lower portion of the filter greater than they actually were, and values of C in the lower portion of the filter lower than they actually were.

64. From the experimental results obtained in this programme of research the curves of hydraulic gradient, h, plotted against specific deposit, σ, were not shallow curves that could be approximated closely by straight lines. They consisted of two parts, the first part of each being accurately represented by straight lines, but the second part of each curved upwards away from the straight lines, showing a rapidly increasing hydraulic gradient with small increases in the storage ratio σ. The values of σ at which these curves ceased to be straight lines, increased as the porosity of the media increased, the values being much greater for anthracite than for the corresponding gradings of sand. The Author's shallow curves might be due to the larger values of σ obtained from the equations than were obtained in practice, which when plotted against a given head loss per unit length would tend to make the curves more shallow.

The Author replied that he was gratified to see his Paper discussed by a designer-consultant, a teaching engineer, and a practising water engineer; a significant point was the lack of participation from filter manufacturers.

66. He agreed with Mr Hudson that some further work was necessary before design methods such as he had proposed would find their way into practice. Quite clearly the laboratory tests needed to be extended to pilot plant and full scale test operation. Also some portable 'filter coefficient apparatus' would be required to make possible the on-site testing of waters to be filtered. This could then be used in conjunction with design charts or tables as Mr Hudson had suggested.

67. It was clear from his remarks that Mr Hudson appreciated the need for considering filtrate quality changes as well as the rising head loss; many designers and operators appeared to be preoccupied with the latter. However the optimum design would be one in which the maximum allowable head loss is reached just before the filtrate quality deteriorates to an unacceptable level. Such an optimum could now be sought in a rational manner.

68. Mr Deb's proposed method of calculating the head loss constant k was logical, and a valuable adjunct to the Author's method. The derivation of equation (31), making the assumption that the effluent particle concentration (filtrate) from a deep filter would be negligible, was very interesting vis-a-vis the Author's equation (20a). This equation (20a) was derived with the assumption that certain mathematical terms predominated over others in the case of deep filters. Mr Deb appeared to have discovered the physical significance of such an assumption.

69. The Author would still like to sound a cautionary note with regard to Mr Deb's method, based as it was on equation (25). This only held for head loss due to deposition in the filter. A rapid increase in head loss was frequently associated with deposition on the filter surface; such a head loss had to be deducted from H in equations (30) (30a) or (31). The use of an intermediate layer (i.e. not the surface layer) obviated this, as appreciated by Mr Deb in equation (29).

70. Dr Robinson's criticism of the lack of agreement between the zero time concentration curves for the deeper layers was justified. This however was due to a lack of conformity between the ideal conditions specified by the mathematical model and experimental conditions in Dr Robinson's experiments.

71. The mathematical model required a homogeneous unisize suspension; the experimental suspension had a size range from less than 1 micron to over 23 microns with a peak at 4·5 microns and nearly uniform distribution by weight of other sizes. The mathematical model required a homogeneous unisize bed; the experimental filters were sieve fractions (not even adjacent size sieves in 14/18A2) backwashed with

consequent hydraulic stratification. The mathematical model required a negative exponential curve at zero time; the experimental curves are at 0·5 h (not 0·05 h as given in error on Fig. 14) when some deposition had already taken place. This was evidenced by the hydraulic gradient in the 1–3 in. layer having risen from 0·35 for clean water to 0·65 at 0·5 h, in experiment 14/18A2, and 0·57 to 0·675 in experiment 18/22A1.

72. All of these destroyed the expectation of a negative exponential concentration curve in the experiments. The depth-concentration curves at 0·5 h mentioned by Dr Robinson for experiments 14/18A2 and 18/22A1 (see pp. 148, 149 of reference 6) could equally well be interpreted as smooth curves on a semi-logarithmic plot, i.e. a continuous function, rather than the two straight lines involving a discontinuous function. It was difficult to conceive a rational basis of such a discontinuity in continuous flow in continuous media. The most convincing graph analysed by Dr Robinson in the manner which he had suggested in his thesis (p. 155 of reference 6), unfortunately referred to 2 hours after commencement of filtration when the accumulation of deposit was quite high and the negative exponential curve would not be expected anyway. Recent theoretical work from the Czechoslovak Hydrodynamic Institute had indicated that a curve approximated by two straight lines on a semi-logarithmic plot would result from a suspension with two well-defined peaks in the size distribution curve. Thus the nature of the suspension probably caused the concentration curve forms found in Dr Robinson's experiments.

73. The graphs of hydraulic gradient h, against specific deposit σ, in Dr Robinson's experiments were, as he had stated, straight lines for the lower σ values, as used by the Author. The parts of the graphs which curved above the straight lines, at higher σ values were for the upper layers of the filter where deposition was greatest. This was not consistent with Dr Robinson's argument that the differences in concentration curves in the lower layers accounted for over-estimation of σ values leading to differences in the h vs. σ curves. It was possible that the higher σ values on Dr Robinson's curves were due to assumptions that all the material removed in the top layer was all within the pores, whereas some might have been in a surface mat layer, as indicated by the Author.

Reference

11. H. E. Hudson, Jr. High quality water production and viral disease. *J. Amer. Wat. Wks Ass.*, vol. 54, No. 10, October 1962, p. 1265.