*Discussion on Paper No. 6659**

Measurement of stress in concrete beam reinforcement

by

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Mr M. Sas-Skowronski (Civil Engineer, East African Railways & Harbours) wrote that the Author's Paper was an interesting one. The purpose of his contribution was to add a few supplementary remarks to the Paper.

48. Advancement in the field of structural design and analysis must, of necessity, proceed with extreme caution and deliberation.

49. A pronounced interest in the ultimate strength of structural members dated back nearly three decades, but its origin and the use of a design theory based on the ultimate strength of sections was, in effect, a return to the concept of design further back than the concepts of elasticity and working stresses. In 1638, 325 years ago, Galileo's work regarding flexure of beams was devoted to the ultimate strength, but the first fundamental theorems of the theory of elasticity were developed by Navier in 1821.

50. The first theories of ultimate-strength design were published in 1897 by Professor Thullie of Lwow University, and in 1899 by Ritter in Zurich (see Fig. 9).

51. The straight-line theory of Coignet and Tedesco was published in 1894 and became generally accepted about 1900. There were two reasons why this theory was accepted:

1. The straight-line theory was mathematically very simple.
2. The resulting safety factors, with respect to ultimate loads observed in tests, were sufficiently controlled to satisfy the requirements of that time.

52. The elastic theory became so widely used that there was a tendency to overlook the approximation involved in its assumptions, and applications beyond the range of validity of the theory resulted.

53. In 1931 Emperger wrote a 'revolutionary' critical study of the allowable stresses, modular ratio, and the elastic theory as used in reinforced-concrete design. The paper initiated intense theoretical and experimental studies of the ultimate strengths of reinforced-concrete elements of structures.

54. In the writer's own country, the impetus was given by R. H. Evans in his excellent paper 'The plastic theories for the ultimate strength of reinforced-concrete beams'.[4]

55. Ultimate strength design had been adopted by Brazil and Russia before World War II and by several countries in Europe and America after the war.

56. Knowledge of the entire field of reinforced-concrete design had advanced so far that a transition to ultimate-strength design should be very short indeed.

57. Studies and practical experience showed that the ultimate load capacity of a reinforced-concrete section could be predicted with an accuracy within design requirements. For simple beams, the ultimate capacity equalled the computed capacity; for indeterminate structures, the maximum moments at various sections were due to different load arrangements and therefore the maximum load capacity of

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* *Proc. Instn civ. Engrs*, vol. 25, June 1963, pp. 127–146.

a structure might be considerably greater than that indicated by the capacity at one section because of redistribution.

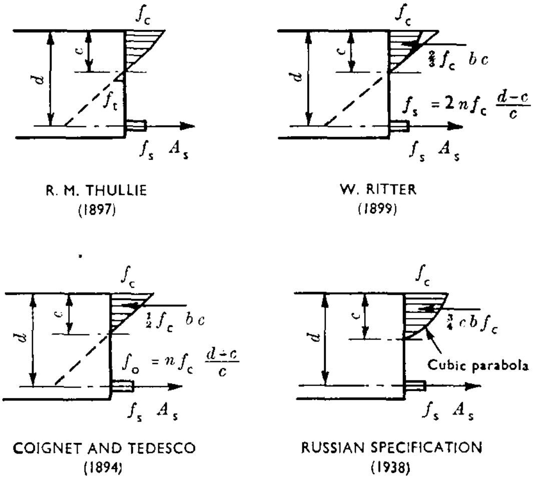
58. The stress and strain were not proportional beyond the elastic limit, therefore the elastic theory could not give a reliable prediction of the ultimate strength of a section, which might be about 50% greater than that computed by the elastic theory. That showed that the actual factor of safety could not be determined by the elastic theory.

59. It was also unreasonable to apply the same load factors to dead and to live loads. Ultimate-strength design conveniently permitted the use of different factors which resulted in more economical design.

60. Numerous papers had been published in recent years to demonstrate that ultimate-strength design procedure was no more complicated than present-day working stress methods, and even simpler. However, the tendency of avoiding it still existed in many designing offices. The writer believed that the main obstacle was lack of suitably qualified staff in the design offices, and convenient design tables and curves based on ultimate-strength design. The Author in his paper referred to CP114:1957. It should be borne in mind that the recommendations of each Code of Practice were conservative and too safe. When shear reinforcement was required and was designed to CP114, such designs were generally excessively conservative, as the Code did not utilize percentage of vertical stirrups of less than 0·5%. This was indeed a fallacy as smaller percentages of web reinforcement were definitely effective for resisting shear.

61. The recommendation as it stood was illogical, as it needed either no shear reinforcement at all or an excessively large quantity of such reinforcement. The writer believed that CP114 was only a temporary stage in further progress, and it was to be hoped that a standard form of practice would be outlined in the near future.

62. In the design of reinforced-concrete structures, the aim was to attain economy consistent with adequate safety against all possible manner of failures. Bending and shear failures were the most common types of failure encountered in reinforced-concrete beams. Although the safety of beams with respect to bending failures could



R. M. THULLIE (1897)

W. RITTER (1899)

COIGNET AND TEDESCO (1894)

RUSSIAN SPECIFICATION (1938)

FIG. 9

be calculated to a fair degree of certainty, the same could not be said in regard to shear failure. The mechanism of shear failures was not yet clearly understood, due to lack of rationality in the approach to the problem of shear. The phenomenon of diagonal tension cracking was one which involved a combination of flexural and shear stresses.

63. Recent research work regarding shear and diagonal tension had been devoted primarily to members without web reinforcement. Although several investigations had included members with stirrups, the variables affecting shear strength had not been thoroughly and systematically studied.

64. The addition of stirrups added one more variable to the complex problem of understanding shear and diagonal tension. One thing was certain, that neither laboratory tests, nor performance of members of the existing structures designed in accordance with CP114, indicated a lack of safety for members with the web reinforcement.

65. The Author's Paper was a valuable contribution to knowledge of reinforced-concrete design, and the writer wished to congratulate him on his results.

Dr H. Robinson (Assistant Professor of Civil Engineering and Engineering Mechanics, McMaster University, Hamilton, Ontario) wrote that the general form of the distribution of strain along the tensile reinforcement, particularly for beams Nos 1 to 7 was similar to that which would be obtained if the beams were treated as composite beams with incomplete interaction between the steel and the concrete, and analysed in accordance with the solution by Newmark *et al.*[5] for composite T-beams.

67. The notation used in this contribution was essentially that of the Appendix of reference 5.

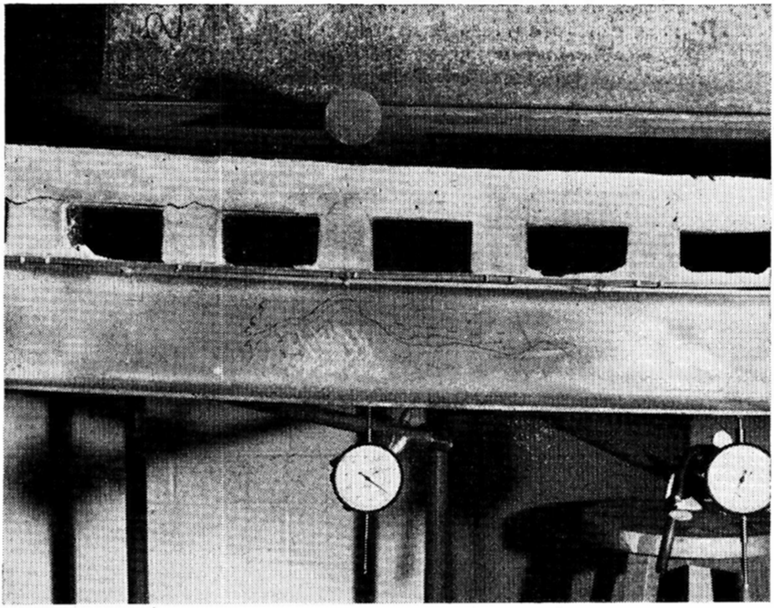
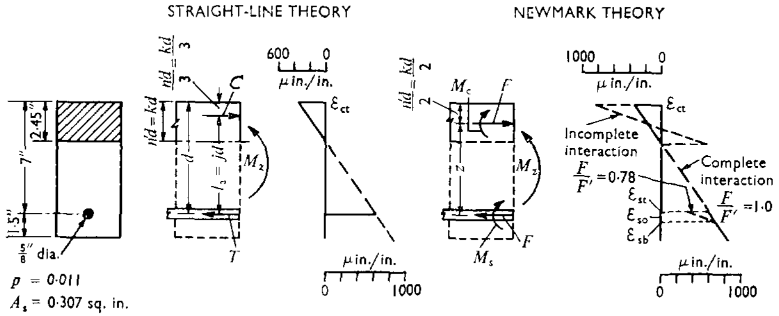


Fig. 10: Showing influence of breakdown of interaction, between the steel and concrete, on the variation of tensile strain along a composite T-beam; and the occurrence of a diagonal tensile crack near the load point

68. The ratio F/F' was a convenient measure of the degree of interaction at any section and it could be shown that for $F/F' < 1{\cdot}0$ the mid-height strain in the steel reinforcing rod was less than that occurring with complete interaction, while the lower steel fibre strain was increased slightly. It was assumed that strain distribution across any section was linear.

69. The writer had carried out tests on composite concrete and steel T-beams which incorporated a cellular zone between the concrete slab and the steel I-beam. Interaction was achieved by means of shear connectors. In that case there was no distinct interfacial plane between the two materials; however, strain distribution at any section had been observed to be linear in the elastic range. The total slip between the concrete slab and the steel beam could be considered to consist of an interfacial slip between the lower part of the cellular zone and the steel beam and a larger slip, particularly after cracking, due to the rotation of the concrete ribs formed by the cells (Fig. 10). That might be considered to be analogous to the rotation of the 'cantilevers' or 'teeth' referred to by Moe[7] and Kani.[8] Fig. 10 showed, for various loads, the extremities of cracking in 'stress coat' sprayed on to the steel beam in the region of the load point. It was perhaps of interest too to notice the diminutive diagonal tensile crack in the concrete under the load point and that cracking in the concrete did not occur until the steel had yielded.

70. Considering a reinforced-concrete beam with dimensions and properties similar to those chosen by Dr Plowman, it could be shown, for example, that at his design load, identical results were obtained by using the straight-line theory and the Newmark theory for the case of complete interaction between the steel and concrete (Fig. 11).



STRAIGHT-LINE THEORY

$E_s = 30 \times 10^6$ lb/sq. in.
$f'_{cu} = 3500$ lb/sq. in.
$E_c = 3{\cdot}5 \times 10^6$ lb/sq. in. (see reference 9)
$M = 36\,200$ lb in.

Strain
$M = Tl_a = Tjd$
$n' = k = 0{\cdot}35$
$a' = j = 0{\cdot}874$

NEWMARK THEORY

Strain
$M = M_s + M_c + Fz$
$\epsilon_{sb} = 10^{-7}[0{\cdot}182 - 0{\cdot}162(F/F')(1{\cdot}05 - 1{\cdot}085)]M$
$\epsilon_{st} = 10^{-7}[-0{\cdot}182 + 0{\cdot}345(F/F')]M$
$\epsilon_{so} = 0{\cdot}176(F/F')M \times 10^{-7}$

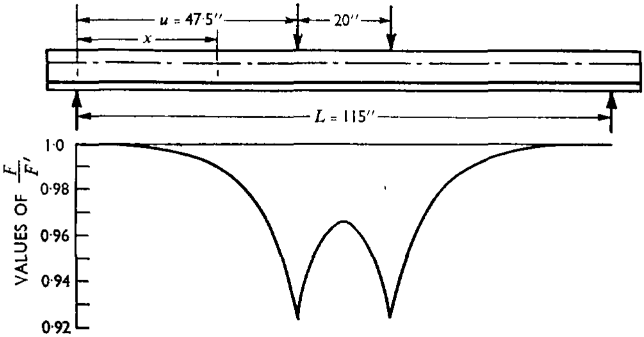
FIG. 11: ESTIMATED STRAINS IN CRACKED SECTION AT DESIGN MOMENT

71. If, however, there was a loss of interaction represented by $F/F' < 1$, the Newmark theory indicated that the mid-height strain in the reinforcing rod was reduced while the lower fibre strain could increase slightly above that for complete interaction.

72. Fig. 12(a) showed the variation of the ratio F/F' along the length of a beam with a two-point load system. A comparison between strain variation in the reinforcing rod for complete interaction and incomplete interaction was shown in Fig. 12(b) together with the fibre strains for incomplete interaction.

73. For simplicity, the strains had been estimated on the basis of a constant cracked section throughout the beam, and it was perhaps of interest to note that the general form of the variation of strain was similar to that observed by Dr Plowman.

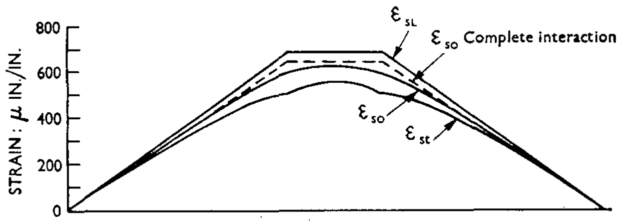
74. Whereas it was not suggested that such an analysis would necessarily account for the extremities of strain observed by the Author, it was probable that any small



$$\frac{F_L}{F_L'} = 1 - \frac{L\sqrt{c}\cosh \pi/\sqrt{c}(\frac{1}{2}-u/L)\sinh \pi x/L\sqrt{c}}{\pi x \cosh \frac{1}{2}\pi/\sqrt{c}} \quad x < u$$

$$\frac{F}{F'} = 1 - \frac{L\sqrt{c}\sinh \pi u/L\sqrt{c}\cosh \pi/\sqrt{c}(\frac{1}{2}-x/L)}{\pi u \cosh \frac{1}{2}\pi/\sqrt{c}} \quad x > u < L/2$$

(*a*) VARIATION OF F/F' ALONG LENGTH OF BEAM FOR $1/c = 28$ DESIGN LOAD



(*b*) VARIATION OF UPPER, MID-HEIGHT, AND LOWER STRAINS ALONG REINFORCEMENT OF REINFORCED-CONCRETE BEAM WITH CONSTANT CRACKED SECTION FOR $1/c = 28$ DESIGN LOAD

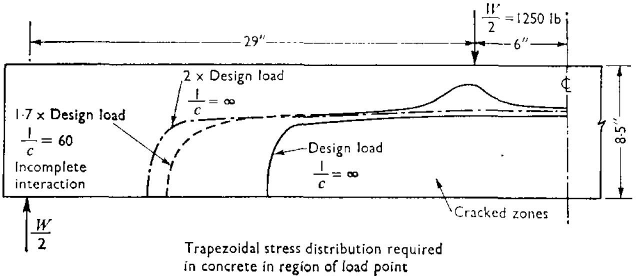
FIG. 12: INFLUENCE OF BREAKDOWN OF INTERACTION IN REINFORCED-CONCRETE BEAM

displacements of the gauge points from the mid-height of the rod would result in deviations of measured strain. Also it might be that the lower steel fibre strain in the steel was as large or larger in magnitude than that predicted by the straight-line theory.

75. A better description of the strain distribution could be made by estimating on the basis of the uncracked section at any position along the beam for incomplete interaction. That would result in lower strain values in the steel towards the ends of the beam, as observed by the Author. That would not, however, necessarily result in higher average strains in the steel in the regions of the load points, since the lever arm increased together with the development of a 'stress block' in the concrete.

76. Fig. 13 showed the approximate extremities of the cracked zones due to bending stresses for complete and incomplete interaction at various loads, for a beam of type No. 1.

77. For the value of $1/c = 60$ and a load of 1·7 × design load the Newmark analysis indicated that the lower steel fibre strain under the load point was at the yield strain. Perhaps it might be speculated that the cracked zone constituted a region of visible cracks and tensile micro-cracking and that increased deformation due to local yielding of the steel reinforcing triggered off the diagonal tension crack. The load at which this occurred would depend on the degree of interaction or bond modulus between the steel and concrete. This could be estimated from measured recordings of slip between the steel and concrete.[5,6,10]



Estimated according to theory for composite beams with complete and incomplete interaction

Limiting tensile strain in concrete assumed to be 100 μin./in.

FIG. 13: SKETCH OF REINFORCED-CONCRETE BEAM SHOWING APPROXIMATE EXTREMITIES OF FLEXURAL CRACKING FOR BEAM TYPE No. 1

78. Even if bond strength were high, however, and there was no perceptible slip between the steel and the concrete at the ends of the beam (hooks), slip and associated loss of interaction could occur locally in the region of the load points.[6,10]

79. Dr Plowman's results were of particular interest in that they seemed to indicate loss of interaction, between the steel and concrete, similar in nature to that observed with composite T-beams. Measurement of bond slip as well as strain variation and consideration of the beams as composite beams with incomplete interaction might lead to a further understanding of the development and nature of the diagonal tension cracks.

Dr J. T. Manning (Messrs A. J. and J. D. Harris) wrote that the measurement of steel strain along the reinforcement of a concrete beam was of little importance practically as it was well known that the maximum steel stress in flexure occurred at a crack. An investigation of the distribution of steel stress and hence bond stress was, however, of high academic importance. In this field the measurement of steel stress gave little information unless bond stresses were derived by differentiation and related to the factors influencing their distribution, viz. crack widths and spacing, reinforcement cover, diameter and surface conditions.

81. It was therefore probable that a gauge length of 2 in. was too large to enable steel strains to be accurately differentiated to obtain the distribution of bond around a crack.

82. The method of investigation of bond distribution used by Dr Plowman was an extension of the many methods measuring steel strains directly at the steel–concrete interface by baring the steel.[1, 11, 12] The obvious disadvantages of this method were that bond was destroyed where concrete had been removed and cracks were initiated at those places, which influenced the distribution of steel strain and crack widths, and there was inability to measure differential movement between the steel and the concrete associated with steel stress.

83. The X-ray method obviated these disadvantages. Radiographs of beams reinforced with bars containing minute platinum markers which protruded into the concrete could be obtained at various stages of loading. Measurements with a travelling microscope on the images of the markers gave steel strain, concrete strain, and relative slip.

84. The disadvantages were, as pointed out by Dr Plowman, expensive equipment, time taken to obtain results, a high degree of control, and limitation of specimen sizes. Control was required to standardize or eliminate penumbra effects, temperature and humidity changes, shrinkage due to processing of films, grain size, and sensitivity of film, and human inaccuracies in measurement of the films, which was a

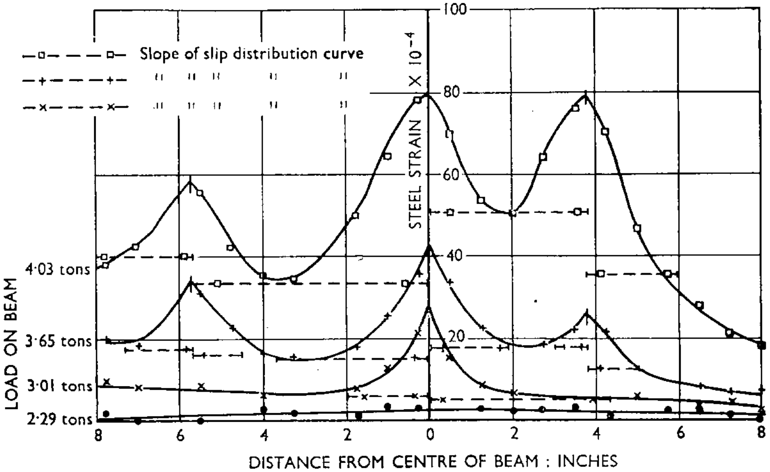
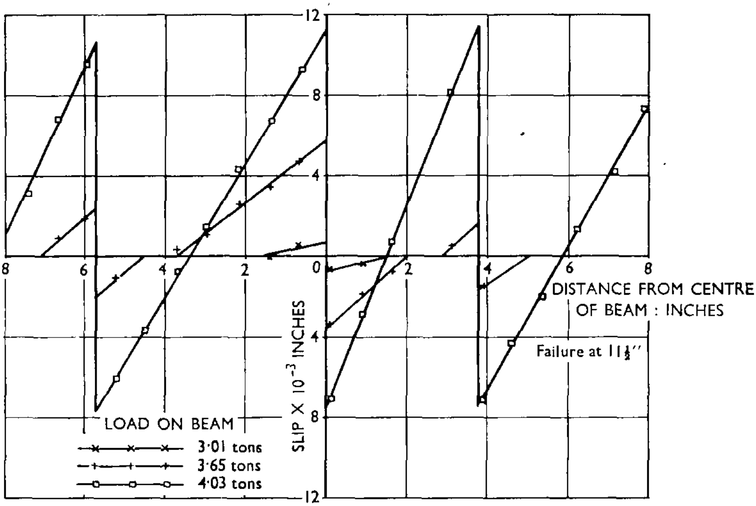


FIG. 14: STEEL STRAIN DISTRIBUTION FOR BEAM X2



CRACK WIDTHS AT LEVEL OF REINFORCEMENT

0·0050 0·0220	6·3"	0·0022 0·0172	5·7"	0·0010 0·0088 0·0208	3·8"	0·0028 0·0134	7·7"	0·012 0·0465

FIG. 15: SLIP DISTRIBUTION FOR BEAM X2

long and tedious operation. The size of the specimen which could be X-rayed was limited by two factors, (i) the thickness of concrete which could be penetrated by the X-rays and retain a clear image, and (ii) the maximum differential steel strain which could be accurately measured by this technique. With the most recent X-ray apparatus used at Leeds University to investigate bond, specimens 4 in. thick could be investigated and differential movement of $0{\cdot}5 \times 10^{-4}$ over a ¾-in. gauge length could be accurately measured. (This was equivalent to bond stresses of 40 lb/sq. in. with 0·08-in. wires, 140 lb/sq. in. with 0·276-in. wires, or ¼-in.-dia. bars, and 250 lb/sq. in. with ½-in.-dia. bars.) Therefore the maximum specimen which could be successfully investigated was one 4 in. thick reinforced with bars or wires about ¼ in. in diameter.

85. Tests had been conducted at Leeds University over the past three years in an attempt to find the influence of bond stresses on crack patterns in prestressed beams using the X-ray technique and the results were being prepared for publication. Prestressed beams were used to obtain high strains and still limit deflexions.

86. A typical graph of distribution of steel strain was shown in Fig. 14. The dotted horizontal lines were derived from measuring the slope of the slip distribution curve; a typical curve was shown in Fig. 15. That could be compared with steel strain as, considering strains between markers, it could be shown that at any point,

(steel strain) = (concrete strain) + (slope of the slip distribution curve).

The difference of those two curves gave the concrete strain distribution. Bond stresses had been obtained by graphical differentiation and were shown plotted in

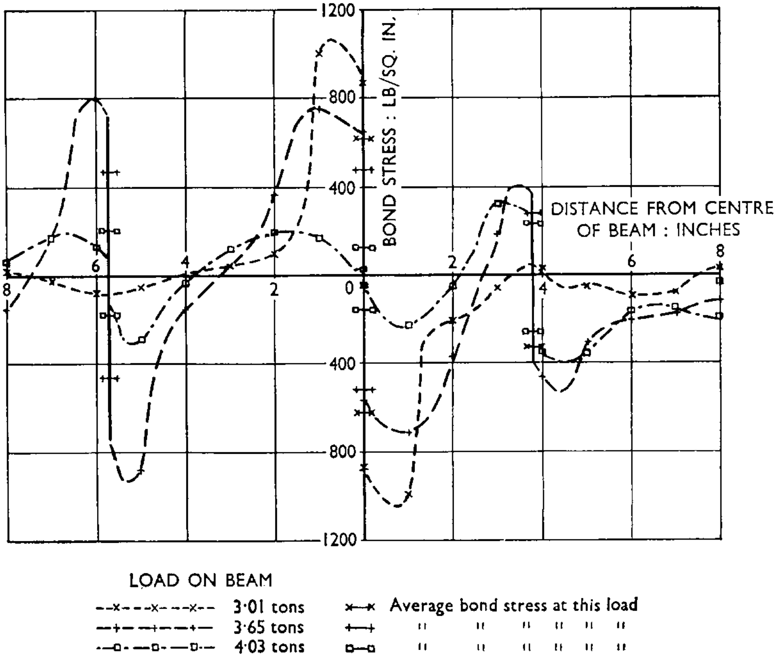


FIG. 16: BOND STRESS DISTRIBUTION FOR BEAM X2

TABLE 2

Beam No.	Reinforcement		Steel %	t_m, Steel stress at failure, *lb/sq. in.*	t_m/t_u
	Diameter, *in.*	Surface condition			
X1	0·08 H.T. wires	Polished smooth	0·396	320 000	1·02
X2	0·08	As delivered	0·396	333 800	1·069
X3	0·08	Roughened with emery paper grade 0·2	0·396	338 900	1·086
X4	0·2 H.T. wires	Polished smooth	0·745	215 400	0·92
X5	0·2	Rusted by placing in curing room	0·745	227 600	0·974
X6	0·2	Polished smooth	0·745	202 800	0·862
X7	¼ in. square twist bar	As delivered	1·86	69 100	1·038

t_u = ultimate steel stress in lb/sq. in.

Fig. 16. The technique adopted for graphical differentiation was that described by Mains,[13] aided by a mirror device. This was a long and tedious process, as small errors in steel strain were greatly magnified when slope was considered.

87. The effect of surface condition of the reinforcement on bond was investigated by determining the steel stress at failure of beams. These were obtained by extrapolation of the X-ray results, and were summarized in Table 2.

88. Crack spacing and width were also recorded and compared as a qualitative measure of bond.

89. From these results it could be seen that although the X-ray technique was tedious and slow to execute, and restricted the size of specimens, the results obtained were very detailed and could be extended to investigate many factors influencing bond.

The Author wrote in reply that there were two ways in which a building could be said to 'fail' from the point of view of its user. It could collapse, which would be a short-term structural failure, or it could develop over a period of time deflexions or settlements which interfered with or prevented its use. For the first a knowledge of the ultimate strength was required, for the second was needed a knowledge of the properties of creep, shrinkage, and elasticity. Mr Sas-Skowronski set out the history of the ultimate load theories but these were not adequate to deal with the second problem since the properties of concrete are dependent upon the stresses within the materials and these can only be determined by experiment.

91. The Author's method was developed to provide an easy and cheap way of obtaining such information on stresses. It had the merit that it could be used in actual buildings as well as in the laboratory on members of any size. Although the Author carried out a lot of research on shear in composite action some years before the work described, the possibility of a relationship between this and bond in normal beams had not occurred to him. Dr Robinson's comments were therefore very welcome. The purpose of measuring stress in steel included not only the determination of the points of maximum stress but also points where the stress was low enough for the area of steel to be reduced.

92. Undoubtedly the X-ray method had certain advantages over the stud method in avoiding openings in the concrete. These had been listed by Dr Manning, but he had omitted the possibility of the platinum inserts acting as stress raisers. It was always a good thing when two methods were used in solving a problem, since this reduced the chances of unknown or unsuspected errors influencing the results. The Author had a great respect for the Evans school on the subject and regarded it as complementary to his own work. Further work had now been carried out on long-term loading.[14]

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