*Discussion on Paper No. 6693**

Analysis of data from non-equilibrium pumping tests allowing for delayed yield from storage

by

N. S. Boulton, M.Sc., M.I.C.E.

Mr R. W. Stallman (Hydraulic Engineer, U.S. Geological Survey, Denver, Colorado) considered that the Author's contribution to the understanding of response to pumping in unconfined aquifers is a milestone in groundwater hydraulics. The magnitude of effects on drawdown caused by delayed yield had in the past been estimated only by intuition or wishful thinking, and enforced with intestinal fortitude rather than with academic reason. A host of semi-plausible explanations, mostly unpublished, had been advanced to account for the shape of observed response curves such as presented in Fig. 2. The Author's result (Fig. 1) would eliminate many of the speculative aspects of analyzing unconfined flow. However, reduction of speculation had given rise to another difficulty. When only the Thiem or Theis equations were available for analyzing response to pumping, choice was limited to questioning the analytical results. Now, armed with an array of type curves for analyzing response, the hydrologist was faced with choosing the curve most likely to yield reliable quantitative results. Thus, uncertainty still remained having been shifted from one phase to another. Needless to say, present uncertainty was becoming less acute because of the greater understanding of flow generated through advances such as Professor Boulton's.

37. Two questions came to mind in studying the Paper, both outside the sphere of assumed conditions but well within the sphere of study of unconfined flow.

38. Drainage of porous media was dependent upon the relations between $\partial s/\partial t$, t and s resulting from pumping. If $\partial s/\partial t$ was always large $S+S'$ would be nearly equal to zero. If $\partial s/\partial t$ was of the order of 10^{-4} times the vertical conductivity of the aquifer, S would be approximately 90% of $S+S'$. The latter estimate followed from study of the nature of flow through undersaturated media. Therefore, one might expect a pronounced change in S with distance from the pumped well. This possibility was mentioned in the Paper, and quantitative estimates or measurements of this relation were needed to assess the utility of Fig. 1 for general application.

39. The Author had postulated that flow to the well was radial. Near the pumping well, vertical flow components were much greater than the radial flow components for some time after pumping began.[12] Significance levels of these times and radii were dependent on the values of specific yield, transmissibility, and anisotropy. In another recent study[13] of transient flow to wells in unconfined aquifers, the effects of vertical flow were investigated. It was found that response curves were similar to the Theis curve where Tt/r^2S_y was greater than 1, and $(r/m)(P_z/P_r)^{0\cdot5}$ was greater than about 3. S_y was the specific yield, assumed to be a constant, P_z and P_r were the vertical and horizontal permeabilities, m was the thickness of the aquifer, and other terms in the nondimensional groups were as defined by the Author. For most applications, the Author's Fig. 1 should be applied only to drawdowns and

* *Proc. Instn civ. Engrs*, vol. 26, November 1963, pp. 469–482.

aquifer situations satisfying the limits given above, so as to comply with his assumption of radial flow. Some shift in these limits would be likely to occur owing to the effects of delayed yield upon the flow system, but the magnitude of such a shift was presently unknown.

40. Analysis of field data was complicated because the response curves near the pumping well and at short times for a constant specific yield had shapes comparable with the Author's $W = -Ei(-1/\theta')$ where the effects of vertical flow are considered. Such response data had been observed on an electric analogue. Appendages like those marked r/B might instead be functions of $(r/m)(P_z/P_r)^{0 \cdot 5}$, which further were functions of the depth at which the observation well tapped the aquifer having a constant and uniform specific yield. Thus, at best, it was difficult to separate the effects of delayed yield from the effects of vertical flow. Such a separation would be possible only if drawdowns were observed at both short and long times, small and large radii, at the water table and at depth in the aquifer. Required magnitudes of both time and distance of observation had to be judged only after consideration of the hydraulic characteristics of the test site.

Mr W. C. Walton (Engineer in charge of groundwater research, Illinois State Water Survey, Urbana, Illinois) observed that the non-equilibrium formula[14] had been used by many hydrologists for nearly 30 years with pumping test data for determining the hydraulic properties of an aquifer. Basic assumptions of the formula were not given due regard; attempts had been made to force the application of the formula to aquifer situations differing greatly from those assumed in the derivation of the formula.

42. It had been recognized that under water-table conditions the non-equilibrium formula did not describe completely the drawdown in wells, especially during short periods of pumping (references 14, p. 521; 15, p. 110; 16, p. 384; 17, p. 565; 18, p. 131; and 19). Under water-table conditions, water was derived from storage by the slow gravity drainage of interstices. The non-equilibrium formula assumed that water was released from storage instantaneously with a decline in artesian head. The important effects of gravity drainage were not considered in the non-equilibrium formula; therefore, the application of that formula to the results of pumping tests made in unconfined aquifers was not justified except under certain limiting conditions.

43. Three distinct segments of the time-drawdown curve might be recognized under water-table conditions. Unconfined stratified sediments often reacted to pumping for a short time after discharge had started, as would an artesian aquifer. Gravity drainage was not immediate but water was released instantaneously from storage by the compaction of the aquifer and its associated beds and by the expansion of the water itself. The second segment of the time/drawdown curve represented the intermediate stage in the decline of water levels when the cone of depression slowed in its rate of expansion as it was replenished by gravity drainage of sediments. The slope of the time/drawdown curve decreased as it reflected the presence of 'recharge' in the form of interstitial storage in the vicinity of the pumped well. The third segment, which might start from several minutes to several days after pumping started, represented the period during which the effects of gravity drainage were small and drawdown was described closely by the non-equilibrium formula.

44. In his Paper, the Author had developed a theory and a type-curve method for analyzing pumping test data under water-table conditions taking into consideration the slow gravity drainage of interstices. His theory and type curves described very closely the three distinct segments of time-drawdown curves and were easily applied to pumping test data to determine the hydraulic properties of an unconfined aquifer.

45. Mr Walton had recently applied the Author's theory and type curves to the results of a pumping test made by the Illinois State Water Survey near Lawrenceville in south-eastern Illinois, U.S.A. Fairly coarse sand and gravel of glacial origin underlay the well field to a depth of about 100 ft. No distinct clay lenses were believed

to be interbedded in the glacial deposits. The consolidated rocks beneath the glacial deposits were relatively impermeable interbedded shales and sandstones of Pennsylvanian age. The effects of pumping a well (diameter 16 in., depth 105 ft) at a rate of 1000 (U.S.) gal/min for a period of 24 h were measured in 13 observation wells.

46. The time/drawdown graphs for several observation wells with type-curve traces were shown in Fig. 4; and the distance/drawdown graph for a pumping period of 24 h was shown in Fig. 5. The results of the analysis of test data were summarized in Table 3.

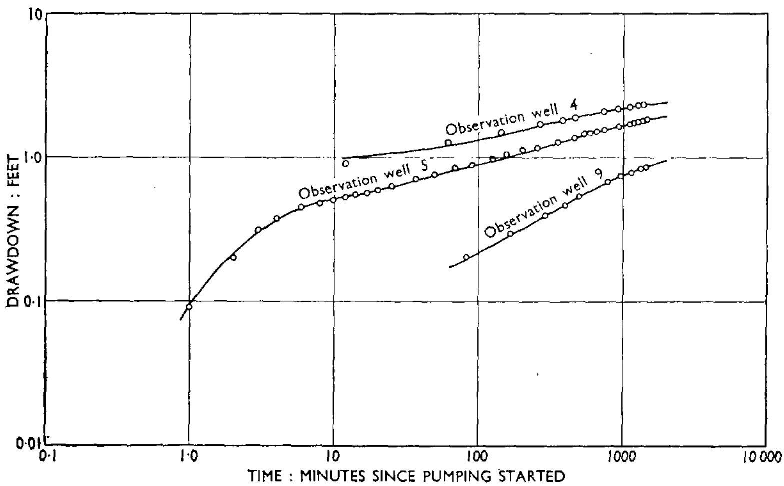

FIG. 4: TIME/DRAWDOWN GRAPH FOR SELECTED OBSERVATION WELLS

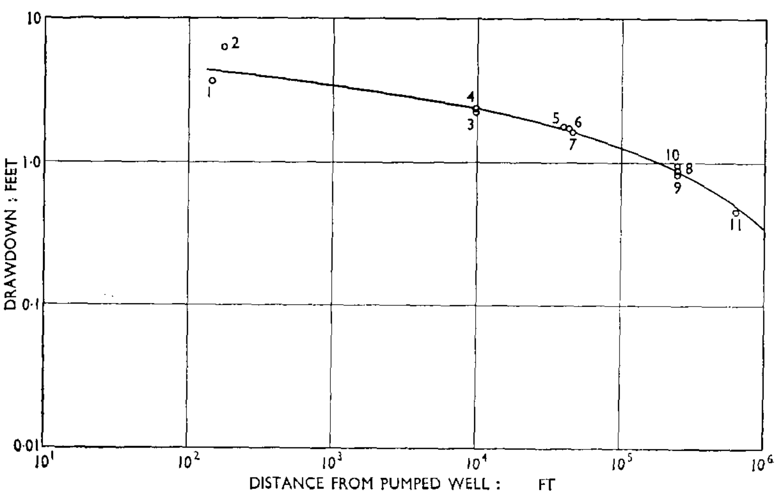

FIG. 5: DISTANCE/DRAWDOWN GRAPH FOR PUMPING PERIOD OF ONE DAY

TABLE 3: SUMMARY OF RESULTS OF ANALYSIS OF PUMPING TEST

Observation well No.	Distance from pumped well, *ft*	Coefficient of trans-missibility (early data), *gpd/ft*	Coefficient of trans-missibility (late data), *gpd/ft*	Coefficient of storage (early data)	Coefficient of storage (late data)	r/B	Delay index, *min*
1	12·1		Data greatly affected by partial penetration of pumped well				
2	13·2		Data greatly affected by partial penetration of pumped well				
3	100	—	280 000	—	0·031	0·316	116
4	100	—	286 000	—	0·035	0·4	147
5	200	290 000	274 000	0·0024	0·026	0·6	111
6	210	—	244 000	—	0·034	0·6	181
7	214	—	267 000	—	0·039	0·7	147
8	500	—	300 000	—	0·032	2·0	72
9	500	—	280 000	—	0·025	1·5	106
10	500	—	330 000	—	0·030	2·0	62
11	800	—	300 000	—	0·043	—	—
Distance/Draw-down	—	—	250 000	—	0·039	—	—

47. For the pumping test considered, the Author's equation, and the type curves based upon it, correctly represented the time/drawdown graphs, including the time-interval during which gravity drainage was an important factor. Values of the hydraulic properties computed from individual time/drawdown graphs and the distance/drawdown graph were consistent. There was a variation in the late-time coefficient of storage with distance from the pumped well.

48. It was believed that the Author's equation and the type curves based upon it would greatly aid the analysis of pumping tests made under water-table conditions in Illinois. Numerical values for plotting delayed-yield type curves for the general case when η was finite were needed and should be published in the near future.

Dr J. Ineson (Water Department, Geological Survey) wrote to congratulate the Author on a further Paper contributing to the problem of drainage effects within unconfined aquifers during discharge from wells. This analysis of groundwater flow was particularly appropriate, not only in the context of the study of observations derived during pumping tests, but in attempting to predict the effects of future abstraction. Misleading conclusions might be drawn from observations during the initial period of testing, particularly when curves appropriate to values of r/B greater than about 1 were being considered. The curves in Fig. 1 merited study in relationship to the development of groundwater resources and the significance of prolonged abstraction. Studies of groundwater flow in the Chalk in this country had shown that where unconfined conditions occurred, there might be some suggestion of 'a leaky artesian aquifer'. However, values of r/B appeared to be small, so that errors in prediction of long-term interference effects were correspondingly reduced. It should be noted that the effect of drainage from storage would be delayed considerably.

50. One important feature of practical importance was the necessity to maintain frequent measurements of water levels in observation wells during the early stages of pumping, a fact not infrequently overlooked. Although this comment had been made previously on several occasions, the present analysis and the observations plotted in Fig. 2 confirmed this. Similarly, maintenance of a constant rate of discharge during this initial period was essential. Unless greater care was exercised in deriving observations during pumping tests, critical analysis of groundwater flow around discharging and recharging wells would be of little value.

51. In § 24, attention had been drawn to variations in values of S', as derived for observation wells located at increasing distances from the discharging well, namely, a tendency for values of S' to decrease systematically. Similar results had been obtained by Dr Ineson from non-steady state analyses for observations in Gt Britain. In the described pumping test (§§ 17–24), both S and S' decreased with distance. Although the Author had suggested that these might be related to small values for drawdown, could this discrepancy be associated with smaller pressure relief as distance to the point of observation increased, thus producing variable elastic responses within an aquifer? An interesting comment was that S for the nearer well was somewhat higher than might have been expected for an elastic confined aquifer less than 90 ft in thickness.

52. The Author had indicated that the type curves in Fig. 1 had been compiled for large values of η. It was to be hoped that similar curves for finite values of η would soon be made available. Such curves might have some bearing on unconfined flow in chalk, where 'leakage' from the bulk of the chalk into the fissure system might be appropriately analyzed by taking delayed yield from storage into account.

The Author, in reply, said that Mr Stallman had pointed out in § 39 the effect of vertical flow components under water-table conditions in limiting the applicability of Fig. 1. In order to avoid further complication, the treatment of delayed yield in the Paper was deliberately confined to cases for which, in the absence of delayed yield, the Theis equation was appropriate.

54. The Author believed that the limits given by Mr Stallman were unnecessarily restrictive. In a previous paper (reference 10, pp. 570, 571), in which vertical motion was fully taken into account, it was shown that the Theis equation gave the drawdown of the water-table with sufficient accuracy under certain limiting conditions. Recent consideration of these conditions suggested that the Theis equation would give the drawdown in a shallow well to about 3% if the following conditions (here given for the isotropic case $P_z = P_r$) were satisfied:

$$\frac{Tt}{r^2 S_y} > 0{\cdot}5, \quad \frac{Tt}{m^2 S_y} > 5, \quad \frac{r}{m} > 0{\cdot}8.$$

55. While these conditions were unlikely to limit the use of the Type B curves in Fig. 1, they might restrict the use of the Type A curves. As pointed out in § 20, only S need be determined from the Type A curves. Values of T, S', and α might be found directly from the Type B curves. If the above conditions were violated in using the Type A curves it had to be accepted that the value of S obtained not only represented the immediate yield from storage, due to compression of the aquifer and expansion of the water, but also included an allowance for the effect of the vertical component of the seepage flow.

56. The Author thought that Mr Stallman's suggestion in § 40 (that, owing to the existence of vertical flow, r/B might instead be $(r/m)(P_z/P_r)^{0{\cdot}5}$) was hardly tenable, since the curves would then be independent of S' (total coefficient of storage). Moreover the type curves obtainable from reference 10, which allowed for vertical motion but not delayed yield, did not closely resemble a typical double-curvature delayed yield curve (Fig. 1).

57. All three contributors to the discussion referred to the variation of S or S' with distance from the pumped well.

58. The Author found Mr Stallman's argument in § 38 difficult to follow, particularly since the basis of the 90% figure was not given. The value of $\partial s/\partial t$ was not always large, as postulated by Mr Stallman, even close to the pumped well, and it did not follow therefore that the delayed yield S' would be nearly equal to zero. There seemed to be insufficient theoretical grounds for a pronounced change in S with distance from the pumped well, though the Author agreed with Mr Stallman that further data were needed on the variation of S with distance.

59. It was interesting to note from Table 3 for Mr Walton's recent test (§ 46) that the variation in the late-time coefficient of storage with distance was a small increase as compared with the large decrease in the earlier test quoted by the Author (§ 24).

60. Dr Ineson had commented in § 51 that S for the nearer well was somewhat higher than might have been expected for an elastic confined aquifer less than 90 ft in thickness. Under the water-table conditions which existed in the described pumping test, however, a fraction of the pore-water above the water-table would be instantaneously released as the water-table fell. This release increased the effective coefficient of storage above that for an elastic confined aquifer.

61. The Author agreed with Dr Ineson's opinion expressed in § 51 that the decrease in both S and S' with distance might be associated with the smaller pore-pressure relief during drawdown as the distance to the point of observation increased. The lowering of the water-table caused an increase in the vertical effective pressure on the aquifer above and below the water-table. For uncemented sand and gravel, provided that the compressibility of the aquifer below the water-table were constant, and did not increase with effective pressure, the value of S would be constant. The increase in effective pressure in the dewatered zone above the water-table would however increase S and S' near the pumped well where the drawdown was greatest. The vertical seepage as the water-table fell would further compact the aquifer and lead to an increase in the effective values of S and S'. These considerations might explain the observed decrease in S and S' with increase in r noted in §§ 19–24.

62. Dr Ineson and Mr Walton had expressed the need for delayed yield type-

curves when η was finite. Values of W had now been computed by Miss N. B. George from equation (14) for assumed values of η, αt, and r/B. The results, in a form suitable for plotting the type curves, were given in Table 4. The curves might be extended to the right and left of these points by using appropriate values from Table 1 and Table 2, respectively.

63. Comparing the curves plotted from Table 4 with those obtained by the approximate tangent method described in § 16, it was found that the latter method was sufficiently accurate in the few cases which the Author had considered. This conclusion was confirmed by Mr W. C. Walton* who had examined the 10 curves obtainable from Table 4. He found that the maximum difference between the tangent method and the curves based on Table 4 occurred when $\eta = 6{\cdot}5$ and $r/B = 2$, but this difference was remarkably small.

TABLE 4: NUMERICAL VALUES FOR PLOTTING DELAYED YIELD TYPE CURVES FOR FINITE η

(Values for η – Infinite are given in Appendix 2 of the original Paper.)

η	$r/B = 0{\cdot}1$					$r/B = 0{\cdot}4$					$r/B = 2{\cdot}0$				
	$\theta = N \times 10^n$		W	$\theta' = N \times 10^n$		$\theta = N \times 10^n$		W	$\theta' = N \times 10^n$		$\theta = N \times 10^n$		W	$\theta' = N \times 10^n$	
	N	n		N	n	N	n		N	n	N	n		N	n
6·5	1	2	3·82	1·82	1	1	1	1·57	1·82	0	1	0	0·118	1·82	−1
	1	3	5·08	1·82	2	2	1	1·96	3·64	0	2	0	0·221	3·64	−1
	2	3	5·42	3·64	2	1	2	2·61	1·82	1	5	0	0·372	9·09	−1
	5	3	6·10	9·09	2	5	2	3·79	9·09	1	1	1	0·576	1·82	0
											2	1	0·920	3·64	0
											5	1	1·602	9·09	0
10	5	2	4·76	5·56	1	2	1	1·95	2·22	0	1	0	0·116	1·11	−1
	1	3	4·99	1·11	2	5	1	2·28	5·56	0	2	0	0·211	2·22	−1
	2	3	5·23	2·22	2	1	2	2·48	1·11	1	5	0	0·319	5·56	−1
	5	3	5·76	5·56	2	2	2	2·77	2·22	1	1	1	0·456	1·11	0
						5	2	3·41	5·56	1	2	1	0·701	2·22	0
											5	1	1·26	5·56	0
20						1	1	1·57	5·26	−1	1	0	0·115	5·26	−2
						2	1	1·94	1·05	0	2	0	0·202	1·05	−1
						5	1	2·23	2·63	0	5	0	0·272	2·63	−1
						1	2	2·35	5·26	0	1	1	0·342	5·26	−1
						2	2	2·51	1·05	1	2	1	0·474	1·05	0
						5	2	2·92	2·63	1	5	1	0·822	2·63	0
34						2	1	1·93	6·06	−1	2	0	0·199	6·06	−2
						5	1	2·21	1·51	0	5	0	0·253	1·51	−1
						1	2	2·30	3·03	0	2	1	0·375	6·06	−1
						2	2	2·40	6·06	0	5	1	0·598	1·51	0
						5	2	2·66	1·51	1	2	2	1·389	6·06	0
						1	3	3·03	3·03	1					

* Private communication to the Author.

References

12. R. W. STALLMAN. Electric analog of three-dimensional flow to wells and its application to unconfined aquifers. *U.S. Geol. Survey Water-Supply Paper 1536-H*, 1963, pp. 205–242.
13. R. W. STALLMAN. Effects of water-table conditions on water-level changes near pumping wells (abstr.). *Trans. Amer. Geophys. Un.*, vol. 44, no. 1, April 1963, p. 46.
14. C. V. THEIS. Relation between the lowering of the Piezometric surface and the rate and duration of discharge of a well using ground-water storage. *Trans. Amer. Geophys. Un.*, 1935, pt. 2.
15. L. K. WENZEL. Methods for determining permeability of water-bearing materials. *U.S. Geol. Survey Water-Supply Paper 887*, 1942.
16. C. E. JACOB (ed. Hunter Rouse). 'Engineering hydraulics', chapt. V, Flow of groundwater. John Wiley, New York, 1950.
17. N. S. BOULTON. Unsteady radial flow to a pumped well allowing for delayed yield from storage. International Association of Scientific Hydrology, Assemblee Générale de Rome, Tome II. *Publication No. 37*, 1955.
18. K. UBELL. Unsteady flow of ground water caused by well-drawdown. International Association of Scientific Hydrology Symposia Darcy, Tome II, 1956.
19. W. C. WALTON. Application and limitations of methods used to analyze pumping test data. *Wat. Well J.*, Feb.-Mar., 1960.