

CORRESPONDENCE
on two Papers published in
Proceedings Part III, April 1954

Hydraulics Paper No. 2

**“ Measurement and Utilization of the Water Resources
of the Nile Basin ” †**

by

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Correspondence

Dr Serge Leliavsky, of Cairo, thought that **Dr Hurst** had underestimated his splendid work, for a casual reader studying his valuable Paper might possibly be misled into believing that the practical importance of the widespread net of **Dr Hurst's** measurements related chiefly to future works to be carried out on the Upper Nile. **Dr Leliavsky** wished, therefore, to place on record, and to emphasize, that in the two decades he had been in charge of the Designing Service of the Reservoirs and Nile Barrages Department of the Egyptian Ministry of Public Works, he had had frequent opportunities to use with advantage the precisely recorded data on the Nile regimen, supplied by the Physical Department, organized and directed by **Dr Hurst**. That information had been of inestimable help to **Dr Leliavsky** in preparing the basic outlines of several major works on the Nile; they were, the remodelling of Assiut Barrage, the construction of the Mohammad Aly Barrages, the remodelling of Esna Barrage, but especially, the second heightening of the Aswan Dam.

The latter work called for particular mention, for, in that case, **Dr Hurst's** discharge observations had been employed by **Dr Leliavsky** to solve one of the most important problems of modern river-control technique, that was, abstracting from the natural river the maximum possible volume of water for filling a storage reservoir without causing the latter to be obliterated with silt. That problem was of acute importance at the present time, for troubles with sediment deposits had lately been reported in connexion with several of the American storage schemes, which had recently been completed. Since the earliest time when the Aswan Dam had first been conceived, the importance of that problem had been fully realized, and, when the second heightening scheme had been put forward

† Proc. Instn Civ. Engrs, Part III, vol. 3, p. 1 (April 1954).

in the middle nineteen-twenties, much apprehension had existed as to whether the larger reservoir could indeed be filled without dangerous effects on its capacity by deposited silt. Dr Leliavsky's solution of that problem, in collaboration with the Director General of Reservoirs, had subsequently been published by the Egyptian Government,²⁶ and since it had been vindicated by two decades of practical application, its original points might possibly deserve quotation as enhancing the value of the information supplied by the Physical Department.

The first question was, whether a "statistical" year might be substituted, in the study of the filling problem, for a number of actual years, which would have rendered the solution materially simpler. In fact, from Dr Hurst's accurate discharge observations the percentage probability of the 10-day mean discharges for the complete water-year had been known (the water-year in Egypt started and ended on July 15 when the reservoir was empty). A reasonably severe set of equal-probability discharges for the entire cycle of operation of the storage scheme could, therefore, have been easily calculated, and conclusions deduced therefrom.

A 95-per-cent-dry year had actually been considered, but not used on further examination, chiefly on account of the criterion for the date on which the filling operation should start. That operation, of course, could not begin until the Nile water became clear of silt after flooding; and therefore, the rule, which had been followed in the period previous to the investigations, had been that filling began when the water level on a certain gauge, 15 kilometres ($9\frac{1}{2}$ miles) downstream of the dam, fell to the level R.L. 88.00.*

The effect of the rule had been twofold. In the first place, it prevented the use of statistical years, in general, because it made the amount of water available for storage a function of the rate of fall of the discharge, the probability of which was beyond the usual statistical analysis; and secondly, it stood in the way of a second heightening scheme, for, with so late a date for its beginning, the filling operation of the larger reservoir could not be completed in time.

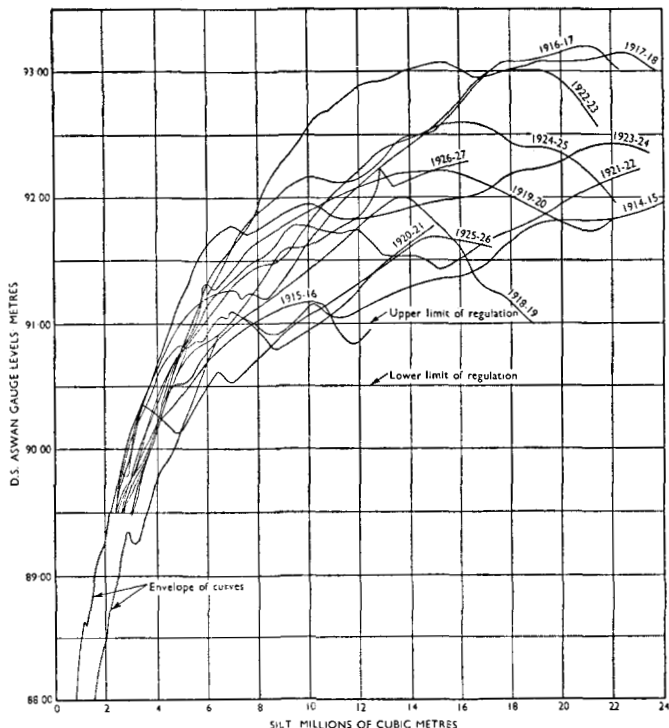
The R.L. 88.00 rule had been, therefore, the main objective of the investigation. No one knew, when, and for what reason, it had first been put forward, but since it had become part of the established routine and since anything even remotely connected with the Aswan Dam was taboo, it was not easy to find arguments against it which were convincing in the right quarters. One of these arguments is reproduced in *Fig. 13*, which appeared in Dr Leliavsky's publication²⁶ and was reproduced, again, in the printed report of the resident engineer. Unless its principle was fully explained its argument might be difficult to grasp. Although the ordinates

* The reduced levels were supposed to refer to a conventional datum which at one time was believed to be the mean sea level at Alexandria.

²⁶ References 26 to 41 are given on pp. 593 and 594.

were levels, they should be interpreted as criteria for the dates of the beginning of the yearly storage operations, and the corresponding abscissae were then the volumes of silt carried by the river, after the dates in question, in each individual year investigated. The characteristic point of the diagram was, that up to a certain level, all the curves fell close to each other and rose evenly (the "envelope" region of the diagram), but above that level they dispersed. *Fig. 13* was, therefore, interpreted to mean

Fig. 13



CUMULATIVE AMOUNT OF SILT REFERRED TO WATER LEVELS

that beneath a certain critical limit the river level was symptomatic of the safety (or otherwise) of the filling operation, and could, consequently, be used as a criterion for the date on which the filling should start. Since the limit in question was R.L. 90.50, or even R.L. 91.00, but not R.L. 88.00, the antiquated rule had been modified accordingly, and Dr Hurst's reference to the absence of deposits confirmed the correctness in judgement in taking that step.

Dr Leliavsky concluded that the described incident was believed to substantiate the following points :—

(1) The operational programmes of proposed storage schemes should not be calculated from statistical years (whether mean years or otherwise), but from a representative series of actual past years.

(2) To ensure safety against sedimentation, storage programmes should be based on hydraulic criteria, and not on fixed dates of the year.

(3) The significance of river levels, as such criteria, could best be ascertained by diagrams such as *Fig. 13*.

Mr R. P. Black, of Cairo, stated that a measurement of discharge consisted of sounding and velocity observations at a number of equally spaced points across the stream. That was at best a long and tedious process especially under flood conditions when, to the other difficulties was added the management of a launch in a strong-flowing stream.

In order to reduce the time required to reasonable limits, it was necessary to reduce the number of observations to a minimum. The computation of a discharge reduced basically to the determination of an area bounded by a curve, its end ordinates, and the axis of co-ordinates, hence the accuracy depended on the number of observations whatever the width of the stream might be and it was highly important that the end points should be so close to the banks that the unmeasured portions at the banks were negligible. On the Nile, under ideal conditions, it had been found by trial that twenty-five equally spaced observation points, the first and last being as close as possible to the banks, were sufficient. Current-meter observations were made only at half depth, and a factor of 0.96 was used to reduce the half-depth velocity to the mean velocity in the same vertical. The meter was assumed to measure the velocity at a point in the section, and that was sensibly true for a stream of any size, where the velocity was practically constant over the area of the rotor. That was the condition under which the meter was rated and set a limit to the smallness of the channel in which it could be used.

The 0.96 coefficient had been determined at Khannaq, which was the discharge site some 40 kilometres (25 miles) below the Aswan Dam, where the depth of water varied from 4 to 12 metres at the different stages of the river. Some 500 vertical-velocity curves had been drawn and although individual curves might depart considerably from the parabolic form, the mean of a fair number of them was very approximately parabolic and no systematic departure from that form was detectable at depths of less than 4 metres. The curves also showed that the mean of the velocities at 0.2, 0.6, and 0.8 depth was 0.99 multiplied by mean velocity, but as the three-point method took so much longer, the half depth was standard on the Nile.

The ratio mean velocity half depth velocity had been carefully determined at a number of points and nowhere had a departure of as much as 5 per cent from the 0.96 value been found.

Mr Black quoted the following examples of departures :—

- (1) In the Blue Nile, 6 kilometres (approx. 4 miles) from Lake Tana where the mean depth was $3\frac{1}{2}$ metres and the bed was of very rough basalt boulders, the factor was 0.92.
- (2) In a new canal, of pitched side slopes and smooth concrete bed, at Atf in Lower Egypt, where the depth was 2 metres, the factors were 0.955 over the smooth bed, and 0.915 over the pitched sides.
- (3) In a flume, 4 metres wide with vertical sides, and a depth of 2.70 metres the factor was 0.935.

To anyone who had had experience of flood-discharge measurement on the Nile, where every source of error tended to exaggerate the discharge, Dr Hurst's Table 2, showing the close agreement of sluices and current meters was a remarkable tribute to the reliability of the Gurley current-meter when used at a well-chosen discharge site.

As a further vindication of the reliability of the current meter, Mr Black thought it might be of interest to quote a comparison with a Venturi-meter survey which he had carried out at the Delta Barrage. The sites, just downstream of a regulator, had been far from ideal, but had been the only ones available.

TABLE 4.—COMPARISON OF CURRENT METER AND VENTURI-METER AT THE DELTA BARRAGE

Site	Mean of four current discharges : cubic metres per sec.	Venturi-meter discharge : cubic metres per sec.
No. 1	3.68	3.80
No. 2	3.82	3.80
No. 3	3.66	3.80
No. 4	3.98	3.80
Mean	3.78	

From an experience of hundreds of current meters and many thousands of discharges, Mr Black could list only two objections to the Gurley meter : (a) the rotor turned with any up-and-down motion of the meter, and (b) the meter was not sensitive enough below 25 centimetres per second. The results of those objections were that, (a) caused the flood discharges to be exaggerated, and (b) demanded a site not easily found on the Nile, where the low-stage velocity did not fall below 25 centimetres per second. His S-rotor meter eliminated both those faults and would give reliable results at velocities as low as 10 centimetres per second.

With reference to current meter rating Dr Hurst had described the

Delta Barrage Rating Station, and it was only necessary to add that the rating was entirely automatic and quite independent of the skill of an observer. The chronograph sheets, which contained the whole of the observations and computations, were filed and thus a record was kept of every instrument throughout its whole period of service. Anyone interested could check a rating Table against the original observations at any time.

In the past, objections had been raised against the rating of a meter by towing it in still water, the contention being that the instrument should be rated in flowing water. He had never heard, however, of a meter rated in flowing water having any claim to the accuracy obtained by a meter rated in still water. If the flume used was small, it was not possible to get in the required number of measuring points, nor was it possible to get rid of the boundary effect, nor was the rotor in any sense a point in the water-section. Not one of the three conditions for the accurate measurement of a discharge mentioned above could be satisfied. If the flume was large enough for those conditions to be satisfied, then the volume of water to be supplied was prohibitive. If the volume of water to be measured did not constitute a difficulty as at Aswan, to get the required range of velocities would require half a flood season and the method was quite impracticable.

A few years previously Mr Black had experimented with a small flume 30 centimetres wide and 15 centimetres deep, and had secured what had looked like a steady uniform flow with a velocity of about 10 centimetres per second, the discharge being measured directly into a tank. When a small Gurley meter had been moved about in the flume, variations of velocity of 10 per cent had been recorded, and later when the flume had been explored with his miniature S-rotor meter, velocity variations of 50 per cent had been recorded, or just about the same as were found in a section of the Main Nile at low stage. As a matter of interest, the miniature meter had a rotor diameter of 12 millimetres, gave 47 revolutions per minute for a velocity of 10 centimetres per second, and could be carried in the waistcoat pocket. It was very useful for measuring low velocities in a model where it would record satisfactorily in a depth of water of only 2 centimetres.

Fortunately the problem of rating current meters in flowing water was no longer of practical importance. Dr Hurst's Table 2 where current meter measurements at Aswan were compared with what amounted to absolute measurements into a tank, should convince anyone that the method of rating by towing in still water, which was in general use to-day, and which could be simply and expeditiously carried out with great accuracy, was sound.

Mr R. B. Bulman thought that one significant aspect had been neglected in the Paper and also appeared to have been neglected in more detailed works, such as "The Nile Basin."⁵ That aspect was what

happened to the water after it had been delivered into the field water-courses ?

On several occasions recently attention had been drawn to the much greater dividends which would result from a small increase in agricultural productivity rather than bringing large areas of additional land into cultivation.

By analogy, would not a small increase in the efficiency of use of irrigation water, at present available, pay better dividends than the expensive schemes of storage and control now being proposed ?

If efficiency of use of irrigation water was defined as the proportion of the whole actually transpired by useful crops, what efficiency was experienced in Egypt ? In other words, could the Author say how much of the water available at the field water-course was transpired by crops and how much was wasted in direct evaporation, etc. ?

In the traditionally irrigated countries there appeared to have been a gap in research and a lack of co-operation between the irrigation engineer and the agriculturist. Each had been afraid of trespassing on the domains of the other. The irrigation engineer had considered his work finished when an adequate supply of water arrived at the field water-courses. The agriculturist had taken for granted the soil moisture made available for his crops.

An interesting field of research existed in the gap between the provinces of the irrigation engineer and the agriculturist. It could make a valuable contribution to world food problems.

Mr William Allard in connexion with half-depth velocity measurements, drew attention to a Paper published by the Institution.²⁷ In the discharge measurement examined, incidentally, the coefficient was 0.91, not 0.96, whilst R. F. Wileman (p. 293, in reference 4) thought 0.92 to be a possible value for a certain British river.

He wondered why, at present, whenever a single-point method was used, 0.6 depth was (except in Egypt) largely preferred to mid-depth. Was it because some arithmetical labour was obviously avoided in subsequent computations by the use of a coefficient of unity rather than one of 0.96. W. A. Liddell,²⁸ some years previously, had quoted, without himself supporting, a claim that a greater constancy existed for mid-depth velocities than for those at 0.6 depth. Whichever of the two single-point methods was used, it seemed probable that there would be, from gauging station to gauging station some variation in the value of the standard coefficient, whether that was 0.96 or unity.

What exercised the minds of some river gaugers more was the relationship between results obtained arithmetically by the full-scale method, that was, multiple points in each vertical, and those secured by the short-cut methods, that was, one, two, or three points in the vertical. Mr Allard's impression was that in the past those who favoured the full-scale method, including most gaugers on the Continent of Europe, were inclined to dismiss

the short-cut methods without actually testing their adequacy, as several of the short-cut school had done. The reason usually given was that the validity of all short-cut methods depended upon a parabolic distribution of the velocities in a vertical; that form of distribution was, in practice, rarely found in vertical velocity curves, "owing to pulsations and surges"²⁹ and other causes. The possibility of such irregularities cancelling one another largely or entirely did not seem to have been examined by such critics.

During the last few years, however, the matter had been discussed helpfully by British river gaugers such as R. F. Wileman,^{4, 30} R. H. MacDonald³¹ and J. J. O'Kelly.³¹ A notable investigation of the matter had now been reported from the Antipodes, where modern hydrometric practice had already been described in Papers by Australian³² and New Zealand³³ engineers. One of these engineers, K. D. Green, had said³⁴ that the 0.2/0.8 method had not been adopted in the State of Victoria as general practice but added—"However, data accumulated from plotting the results of many gaugings indicates some cases in which the method may be used, but velocity pulsations must then be considered. . . ."

The investigation reported recently was made by A. C. Hopkins, of the staff of the Soil Conservation and Rivers Control Council of New Zealand.³⁵ He compared with the discharge calculated by graphical means, from what he called the plotted mass curve, the discharges obtained purely arithmetically by the following methods:—

- (1) One-point, one measurement at 0.6 depth.
- (2) Two-point, the mean of measurements at 0.2 and 0.8 depth.
- (3) Three-point, the arithmetic mean of measurements at 0.2, 0.6, and 0.8 depth.
- (4) Two-and-One (2/1), the mean of the two values obtained by (1) and (2) above, respectively.
- (5) Multiple point, the arithmetic mean of measurements at four or more points in each vertical.

From 52 gaugings related to (1), (2) and (4); 59 to (3); and 45 to (5) he found that the differences from the discharges calculated from the plotted mass curve were as indicated in Table 5.

Mr Hopkins had stated further that the greatest percentage differences related to the smaller flows, particularly in wide shallow streams. He considered that in general for best results, when time was limited for a particular gauging, velocities at 0.2, 0.6, and 0.8 depth should be observed and the two-and-one method used for computation of discharge. Elsewhere in the report he remarked: "Unless a particular gauging site has been the subject of an analysis of several gaugings which decrees otherwise, the two-point and one-point methods are best avoided if at all possible."

Mr Allard recommended that the combinations of measuring points

TABLE 5

Method	Range of differences : per cent	Number of deviations		Overall average of differences, neglecting plus and minus signs : per cent
		Neg.	Pos.	
(1)	- 7.4 to + 9.3	33	19	3.0
(2)	- 6.6 + 10.2	33	19	2.6
(3)	- 6.6 + 6.3	38	21	2.4
(4)	- 6.8 + 5.1	35	17	2.3
(5)	- 4.2 + 5.5	16	29	1.4

shown in Table 6, should be adopted, when any current meter of the type and size of a "Price" meter was being used.

Mr Allard suggested that Mr Hopkins's thorough quantitative study and other Australasian information had done much to help define the situation more clearly and establish a balance between the views of the opposed schools.

Mr Allard asked whether Dr Hurst felt that seasonal rates of loss and gain in transit could be calculated for lengthy reaches of a river like the

TABLE 6

Depth of vertical : feet	Points in total depth at which to measure
1-2	0.6
2-4	0.2, 0.6, and 0.8
4-5	0.1, 0.2, 0.4, 0.6, 0.8, and 0.9
5-6	0.1, 0.2, 0.3, 0.5, 0.6, 0.7, 0.8, and 0.9
6-10	Nine points at 0.1 intervals
Over 10	0.04, 0.20, 0.24, 0.42, 0.50, 0.58, 0.76, 0.80, and 0.96

Nile, or whether the possible error in any individual discharge measurement rendered it questionable to set an upstream station's results against those of another downstream? Dr Hurst would doubtless be acquainted with what was said on the subject in the Handbook of Egyptian Irrigation.³⁶

He asked also whether Dr Hurst felt able to express any further views upon A. B. Buckley's Paper³⁷ on the influence of silt on the velocity of water flowing in open channels, which was based upon data obtained in Egypt. It was thought that there had been some recent work in the U.S.A. on the subject.

He assumed that, had space sufficed, Dr Hurst would under "Other Measurements" have included a reference to the notable use made in Egypt of sluice openings in river barrages and canal regulators for the

measuring of discharges, the openings being calibrated by current-meter. Details were, however, obtainable elsewhere.^{38, 39}

The Author, in reply, stated that the question of the most efficient use of irrigation water, mentioned by Mr Bulman, was a matter of great importance. Apart from technical considerations, which were difficult enough, it involved in Egypt the whole system of life of the country, and would make a large chapter in a treatise on irrigation. It was not possible therefore to deal with it in a short Paper into which many subjects mainly of a technical nature had been compressed. Volume 9 of "The Nile Basin" was still in preparation. The work dealt principally with hydrology and its connexion with major projects, so that irrigation matters were introduced only to make clear the objects of the work.

Work had of course been done on the question of plant requirements, but Mr Bulman was right in emphasizing the need for the closest collaboration between the agriculturist and the irrigation engineer. However, whatever improvements were made in the distribution and use of water on the land, the rapidly increasing population of the Nile Basin would still make necessary the major projects designed to render available the maximum amount of Nile water.

Mr Allard had asked about losses over long stretches of rivers like the Nile. It was the regular practice in Egypt to use average losses derived from many measurements made in the past of the discharges at the head and tail of the various reaches of the river and its tributaries, from Ros-eires on the Blue Nile and Lake Albert on the White Nile to the Delta Barrage. Those entered into all forecasts of the water supply reaching Egypt, as well as into the distribution of water for irrigation, where, for example, the supply sent down from Aswan must be sometimes larger and sometimes smaller than the amount which was required at the canal heads. As Mr Allard had suggested, no reliance could be placed on a loss of the order 5 or 10 per cent based on two or three pairs of discharges. However, where a good deal of information was available about the past, together with regular measurements in the time immediately preceding the regulation, very useful information could be obtained as to probable existing losses. Losses as used were dependent also on rates of travel of changes of discharge.

The influence of silt on the velocity of water in open channels had been discussed in a Paper by Vanoni⁴⁰ and in a forthcoming book on fluvial hydraulics by Dr Serge Leliavsky.

Several contributors had given valuable information on the vertical distribution of velocity in a stream, which covered a wide range of conditions, from small rapid streams with stony beds to large rivers in alluvial plains like the Nile and Indus. More information was contained in the valuable references given by Mr Allard. As he had remarked the single observation at half-depth was not much used outside the Nile Basin. That was strange considering its obvious advantages over the point at 0.6

depth, which had been previously mentioned. The very valuable data from New Zealand³⁵ did not specifically mention half-depth observations, though it was clear that information on that point could be extracted from some of the individual discharge records. It was recommended in various publications that when measuring point velocities on a vertical the readings at 0.2, 0.6, and 0.8 depth should wherever possible be included. It would add greatly to the value of such observations if 0.5 depth could also be added. It was better to have direct measurement at those points rather than deduce values from a curve.

In assessing the value of short-cut methods of measuring the mean velocity it was important to distinguish between accidental and systematic errors, since accidental errors would be made of small importance by increasing the number of observations, whilst the systematic errors remain unaffected, and in the long run were often of much more importance. In a steadily flowing river at a good discharge site, where the discharge and level remained constant over several hours, the velocity at individual points was continually varying, irregularly with the time to a greater or less extent, and also from point to point over the cross-section. At the same time the average velocity over the whole section would vary irregularly, but with a much smaller amplitude. Excluding very shallow or very turbulent streams and those of very steep slope which perhaps carried much matter in suspension, about which there was not much available information, if velocities were measured at say ten points on a vertical for periods of 2 or 3 minutes at each, the velocity-depth curves approximated to parabolae. Experience on many rivers showed that the mean velocity on a vertical occurred at about 0.6 depth below the surface; that the mean of velocities at 0.2 and 0.8 depth was approximately the mean for the vertical; and that the velocity at half-depth was approximately 1.04 times the mean. (The Author drew attention to contributions by Brigadier Hawes (p. 31), Mr Newhouse (p. 41), Mr Black (p. 583), and to references 28, 29, 31 and 35 on p. 593). It should be noted that in any particular case the average values of those relations could not be established with precision except by a number of discharges giving something of the order of 10 or 20 times as many vertical velocity curves each based on 6 to 10 individual observations. It was probable that at least 100 to 200 vertical velocity curves (5 to 10 discharges at a station) were required to establish fairly precisely the factor required to reduce the velocities for a given combination of 1, 2, or 3 observation points to the mean velocity on the vertical. With that amount of evidence the probability was that such factors would not depart by more than 1 or 2 per cent from the values given above. A point to be noted was that a reasonable length of time must be adopted for each velocity observation. The practice in Egypt, when observing at half-depth, was to take three runs of 1 minute each on about twenty verticals. That reduced the effect of the accidental irregularities which occurred. As an example of those, from a

discharge of the Nile in flood at Aswan, three runs of a minute each were taken at half-depth on twenty-two verticals. The average range of a set of three was 6 per cent, and there were five ranges of 10 per cent or more, with a maximum of 13 per cent. The range of the means of revolutions for the 1st, 2nd, and 3rd minutes was 0.5 per cent. Thus to reduce the effect of those variations to a reasonable amount three observations at each point was not excessive, and it was very probable that on any ordinary river three such observations at half-depth using a reduction factor of 0.96 gave a more accurate result and took less time than two observations occupying 1 minute each at 0.2 and 0.8 depth. The practice of observing velocity at a point over 1 minute only was common (see references 28, 29 and 35), and it seemed probable that the shortness of the total time during which velocity was under observation was largely responsible for the irregularities observed.

To illustrate some of the above remarks, Table 4 was taken from a Table in Mr Hopkins's Paper,³⁵ in which he had attributed anomalous variations to the small number of observations.

TABLE 4.—FIVE DISCHARGES MEASURED ON DIFFERENT NEW ZEALAND RIVERS.

Characteristics of site			No. of verticals	Total No. of point velocities	Percentage differences from mean of vertical velocity curves			
Width: feet	Depth: feet	Mean vel. ft/sec.			Depths			Multiple point
					0.6	0.2, 0.8	0.2, 0.6, 0.8	
105	5	3.5	14	56	− 4.1	+ 4.7	− 0.6	+ 5.4
35	8*	1.3	7	34	+ 4.9	+ 2.3	+ 3.9	+ 1.0
45	9*	1.5	8	62	+ 0.5	− 3.9	− 2.6	+ 0.8
40	3.5	3.4	7	28	+ 3.8	− 0.5	+ 0.7	+ 2.9
168	6	2.5	14	93	− 1.0	+ 0.3	− 0.2	+ 0.2
Means					+ 0.8	+ 0.6	+ 0.2	+ 2.1
Total number of point velocities					50	96	144	273
Mean number of point velocities					10	19	29	55

Duration of point velocity observation: 40 to 60 secs.

*Depth at centre of river

The Author's (Dr Hurst's) comments on Table 4 should be considered as supplementary to Mr Hopkins' Paper,³⁵ which was devoted entirely to discussion of irregular variations of point velocity observations, and not

to derivation of reduction factors for various velocity combinations. The facts which appeared in Table 4 were :—

- (a) The average number of velocities measured at a single depth in a discharge was ten, covering about 10 minutes.
Judging by experience on the Nile, to reduce the effect of irregular time-variations of velocity to the order of 1 per cent, about 6 times that length of observation time was required.
- (b) Assuming the variations in Table 4 to be mainly random, if equal numbers of observations were made for each combination there would be little to choose between 0.6 ; 0.2 and 0.8 ; and 0.2, 0.6, and 0.8 depths. The ranges of 8.7, 8.6, and 6.5 per cent for those combinations tended to confirm that.
- (c) The multiple point method with an average of $5\frac{1}{2}$ points to a vertical had a range of 8.3 per cent and differed from the result obtained by drawing vertical velocity curves through the points by more than any of the previous combinations. That was probably because of the inclusion, in 29 out of 50 verticals, of observations at 0.4 depth, where normally the velocity was about 8 per cent more than the mean, thus throwing out the balance of observations.
- (d) In determining a reduction factor for a combination of observations at various depths at any station, it was important to have a total time of velocity observations long enough to render unimportant irregular velocity fluctuations of short period, and to make the observations on a number of different occasions. The reduction factor as determined was no doubt a quantity subject to slight fluctuations about an average value, due partly to errors of observation and partly to natural variations. The Author suggested that for observations at a single point, for example, 0.5 depth, ten discharges, consisting of twenty verticals each and 3 minutes of velocity observation at each point should be measured. For comparison with that, in order to draw reasonably accurate vertical velocity curves, observations should, whenever possible, be made at ten points on a vertical with a duration of one minute at each. In shallow streams it might not be possible to put in as many as ten points to a vertical and in that case the same precision (percentage) could not be expected.

Mr C. L. Berg⁴¹ had described a measurement of the discharge of the Victoria Nile, which consisted of multiple points on nineteen verticals, including observations at half-depth. From them he had deduced a reduction factor of 0.91 to reduce the velocity at half-depth to the mean

velocity. In view of the experience already described it would seem advisable to wait for further observations before deciding to adopt that value instead of the value of 0.96 previously used.

The process of plotting velocities and drawing vertical velocity curves was costly in time, and depended a little on the judgement of the draughtsman. The work could be done much more quickly and without any personal bias by an arithmetical method, and of course could be equally quickly checked.

If $v_1 v_2 v_3 \dots v_n$ were a series of velocities on a vertical, observed at depths of $x_1 x_2 x_3 \dots x_n$, and x_{n+1} was the depth of the river, then, following the ordinary process of numerical integration :

$$\begin{aligned} \int v \cdot dx &= v_1[x_1 + \frac{1}{2}(x_2 - x_1)] + v_2[\frac{1}{2}(x_2 - x_1) + \frac{1}{2}(x_3 - x_2)] + \\ &\dots + v_n[\frac{1}{2}(x_n - x_{n-1}) + (x_{n+1} - x_n)] \\ &= \frac{1}{2}v_1(x_1 + x_2) + \frac{1}{2}v_2(x_3 - x_1) + \\ &\dots + \frac{1}{2}v_{n-1}(x_n - x_{n-2}) + v_n(x_{n+1} - \frac{1}{2}x_n - \frac{1}{2}x_{n-1}) \end{aligned}$$

If the first interval was a and the last c and all the intermediate ones were equal to b then :

$$\int v \cdot dx = av_1 + b(\frac{1}{2}v_1 + v_2 + \dots + v_{n-1} + \frac{1}{2}v_n) + cv_n$$

and the mean velocity was $\frac{1}{x_{n+1}} \int v \cdot dx$.

In conclusion the Author thanked the Institution for accepting the Paper, and the contributors to the discussion for the valuable information which they had supplied.

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²⁷ C. L. Berg, "Detailed Analysis of a Discharge Measurement on the Victoria Nile." Proc. Instn Civ. Engrs, vol. 2, Pt. III, p. 609 (Dec. 1953).

²⁸ W. A. Liddell, "Stream Gaging," McGraw-Hill Co., New York, 1927. p. 37.

²⁹ "Stream Gaging Procedure." United States Geological Survey Water-Supply Paper No. 888 (Washington, D.C., 1945).

³⁰ S. M. Dixon, G. FitzGibbon, and M. A. Hogan, "The Flow of the River Severn, 1921-36." J. Instn Civ. Engrs, vol. 6, p. 81 (June 1937). Discussion by R. F. Wileman, p. 148.

³¹ G. N. Croker, "Records of Flows in the River Wye System, as Determined by Current Meter Measurements, with a Note on Flood Warning Arrangements." J. Instn Wat. Engrs, vol. 5, p. 39 (Feb. 1951). Discussion by R. H. MacDonald and J. J. O'Kelly, pp. 84-88.

³² A. L. Galbraith, "Investigation of Potential Water Power Resources." J. Instn Engrs, Australia, vol. 21, No. 12, p. 189 (Dec. 1949).

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³⁶ J. D. Atkinson, "Handbook of Egyptian Irrigation." Pt 2, p. 40. Government Press, Cairo, 1935.

³⁷ A. B. Buckley, "The Influence of Silt on the Velocity of Water Flowing in Open Channels." Proc. Instn Civ. Engrs, Pt II, vol. 216, p. 183 (1922-23).

³⁸ Ref. 35, Pt 2, pp. 49-62.

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⁴⁰ V. A. Vanoni, "Transportation of suspended sediment by water." Trans. Amer. Soc. Civ. Engrs, vol. 3, 1946, p. 67.

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Structural Paper No. 36

"A Moment Distribution Method for the Analysis and Design of Structures by the Plastic Theory"†

by

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Correspondence

Mr A. Fruchtländer observed that the special moment distribution method for structures by the plastic theory, introduced by the Author, was very interesting but it might be noticed that it could only be applied to structural members which were actually in the collapse mode. It was obvious that, for a partial-collapse mode, the application to members which were still in a fully elastic state might lead to incorrect results as would be shown by the following example.

In *Example 1*, on pp. 53-55, the final improved design for a two-span continuous beam gave :—

Point	A	D	B	E	C
Moment : (inch-tons) .	108	108	108	54	72

The moments at points E and C were not correct and should be 63 inch-tons at E and 54 inch-tons at C. When the plastic hinge had developed at the support B under the load $P = 4$ tons, the short span BC was still in the elastic state, since at no point throughout the span had the plastic moment been reached ; the member BC acted therefore at that stage as a beam hinged at B and fixed at C. The distribution of the unbalanced

† Proc. Instn Civ. Engrs, Part III, vol. 3, p. 51 (April 1954).