*Discussion on Paper No. 6667**

A simplified approach to edge stiffening of bridge decks

by

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and

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Messrs R. E. Rowe, B. C. Best, and **G. Somerville** (Cement and Concrete Association) wrote that certain points made in the Paper on the load distribution analysis should be amplified for the sake of clarity.

65. When dealing with edge stiffening (§ 6), the edge moments could be ignored when the ratio of the flexural stiffness to the torsional stiffness of the edge beam was greater than 5 and when the flexural stiffness per inch width of the edge beam was less than about 15 times that of the unstiffened deck.[5] The Authors' use of the phrase 'of the same order' as that of the unstiffened decks could imply that no edge stiffening was present and it was necessary to give an indication of the practical range of edge stiffening, which was done by the above two statements.

66. In addition, it should have been emphasized in the paragraph that, when considering edge stiffening, it was necessary that the neutral axis of the edge stiffening beam and the deck should not be at greatly different levels, otherwise local stresses were set up, and also that the edge stiffening, to be effective, must be structurally connected to the main deck structure.

67. The factor 1·1 mentioned in § 10 with regard to longitudinal moments arose primarily not from the neglect of Poisson's ratio but from the use of distribution coefficients for deflexion for the first term only in the Fourier series for the load, although the 'mean' moment was derived from simple beam theory and hence included all the terms in the Fourier series for the 'mean' moment.

68. Although no mention was made in § 37 of the actual distribution of edge shearing forces, since the value of the distribution coefficient was that corresponding to the first term in the series, the Authors had made a *basic assumption* that the distribution of shearing forces was the same as that of the applied loads.

69. With regard to § 39, it should be pointed out that the Cement and Concrete Association has for some time advocated the use of the same Fourier series representation for the edge forces as for the applied loading and has recommended that more than one term in the series should be considered when dealing with edge stiffening problems.

70. 'The method of analysis presented by the Association is in series form and therefore a rigorous solution may only be obtained by considering the various terms in the series for the applied load, edge shearing forces and, if considered, edge moments. By considering the centre of the span, only the odd terms in the Fourier series need be summed. While the first, third, and fifth terms of the series will lead to a reasonably accurate solution, this implies solving three sets of compatibility equations. This procedure may, in special circumstances, be warranted but for most practical problems it is sufficiently accurate to consider only the first term of the various series

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and apply certain correcting factors to the solution so found. Let us consider these correcting factors in more detail.

71. If only the first term is used for the applied loads, edge shears and edge moments and the compatibility equations for deflexion and slope are solved, then the deflexion of both the edge beams and the bridge structure can be predicted with an accuracy of within about 3%. This follows from the consideration of Fourier representations of various types of loading.

72. For the longitudinal moments, the Fourier representation does not give the same accuracy as for the deflexion owing to the double differentiation involved. However, the relation between the first term of the Fourier series for the "mean" moment and the exact "mean" moment derived from simple beam theory can be determined. Hence a factor can be introduced to correct for the consideration of the first term only in the "mean" moment. The same factor will be applicable to the edge shear forces F since the Fourier series for these are of the same form as that for the applied load. The second correcting factor arises because only the first term is considered for the distribution coefficient K; this factor of 1·1 has already been discussed, but it must be noted that it only applies to the region of maximum moment under or near the applied loads.'

73. The above suggested treatment for the problem had been given elsewhere[6] and had been used in the lecture notes issued by the Association to its various Training Courses.

74. It was a pity that, although the Paper was based upon load distribution theory, the Authors had departed from the accepted notation associated with that theory; this could lead to some confusion in the minds of readers. Apart from this, there were some printing errors, namely: § 38 and 40 $\mu_n\theta$ should read $\mu_{n\theta}$ and § 45 $M\theta$ should read μ_θ.

75. The basic premise in the paper was that the deck was designed for HA loading, checked for HB loading and, if the deck was over-stressed, provided with edge stiffening to make the design satisfactory. On this premise the approach suggested by the Authors was very useful in the design office when used by a designer already familiar with the load distribution process but, when used by an inexperienced designer, it could lead to the design of a bridge which was the wrong type for the given loading. This could be seen in relation to § 13 where, depending on the width of the deck and the size of the edge beams, the deck structure might conceivably span transversely as opposed to longitudinally, in which case the load position to cause, and the position of, the maximum moments would be considerably different from that normally associated with the load distribution theory.

76. The programming of the Authors' approach was a logical conclusion for design office purposes but it would have been helpful to users of the Authors' method if details had been given of the in-put data for the computer programme. If this had been done it would have been apparent that a complete distribution analysis of the unstiffened deck was necessary to obtain the data for use in the computer programme. Since that was so, it would seem more logical, in the opinion of the contributors, to use a simplified approach for the unstiffened deck to obtain an assessment of the maximum moment due to eccentric loading, then to assess the required edge stiffness to decrease the calculated moment to the appropriate value based upon information given elsewhere,[6] and then to perform a detailed distribution analysis on the deck and edge stiffening. This procedure could be very quick and could be readily understood in physical terms by the designer.

77. Despite some of the above comments, the contributors wished to congratulate the Authors on a useful contribution to load distribution theory which certainly would be an asset in design offices provided that it was used by competent bridge designers.

Dr J. E. Spindel (Assistant, Bridge Design and Assessment, British Railways Board) wrote that he was interested to see that the Authors had turned a simplified method of analysis into a method of direct design. This should prove useful for most cases. A simple approximation to the result could be obtained if it were assumed that the edge beam on one side of the slab did not affect stresses at the other. This assumption was true for 'wide' slabs.

79. It was surprising that the Authors found it necessary to consider the complications of compatibility of self weight and stiffness, as well as the use of concordant cables and transformation profiles, in order to match holes for transverse prestress. It seemed simpler to calculate the deflected form of the edge beam and then to adjust the level of the holes if necessary.

80. The Authors rightly appreciated the value of edge stiffening beams in reducing depth of construction and relieving heavy edge bending moments. This could be carried much further if the level of the neutral axis of the edge beam lay above that of the deck slab. Edge beams of the full depth of the parapet could be used in extreme cases. The deck slab then became virtually a slab supported on four sides with a corresponding reduction in longitudinal bending moments. This had to be paid for by increased transverse bending.

81. With such an edge beam, an additional condition of compatibility had to be satisfied, namely that of longitudinal shear. This was done by considering the edge beam as an L beam acting with a width of deck slab which approximated to about 1/6 of the span of the slab. In calculating the stiffness of this L beam, the second moment of area for the slab about its own axis must be ignored. Alternatively, it seemed that the computer programme prepared by Severn[7] might be used to solve the slab and plane stress problem simultaneously.

82. In the later treatment of the problem, the Authors had made certain assumptions which appeared a little doubtful. Paragraphs 39 and 40 suggested that the longitudinal distribution of load in the edge beam was such that the bending moment diagram was similar to that of the applied loads. This could not be strictly correct, since, for a load away from the edge beam, the proportion of first harmonic transmitted to the edge would be greater than that of the higher harmonics, and conversely, the higher harmonics in the edge beam loading would not be transmitted back into the slab to the same extent as the first harmonic. This could be established quite easily by repeating the calculation for higher harmonics. Fortunately, the differences due to this were unlikely to be serious when a number of loads was involved.

83. Similar considerations applied to the distribution of longitudinal shears, mentioned in § 36, which was generally less favourable than that of bending moments. However, as shears were greatest when loads were near the end of beams where the distribution was only local, this question should not affect the design.

84. The suggestion for treating continuous structures with torsional stiffness as being simply supported at points of contraflexure, made in § 21, was attractive. Presumably, it was assumed that the support moments were distributed in the same way as the mid-span moments. The contributor had found this to be some 15% in error for a load applied to an infinitely wide slab. It would be interesting to know whether the Authors had been able to confirm this statement by comparison with any known 'exact' solutions.

The Authors, in reply, thanked the contributors for their observations and generally accepted the various points made. They pointed out, however, that the object of the Paper was to present a method of edge stiffening analysis which, whilst being both quick and accurate, was relatively simple to use in a design office. For these reasons some of the refinements discussed by the contributors were deliberately omitted from the Paper.

86. Generally, in highway bridge design, the edge beams were chosen to be of the same form as the deck beams. This produced an approximate coincidence of the neutral axes and also led to a flexural stiffness per unit width ratio of much less than 15. In the example given in the Paper, this ratio could be shown to be 2·8. In

order to obtain a ratio approximating to 15 and neglecting any consideration of lateral stability, it would have been necessary to have adopted an edge beam 6 in. wide by 12 ft 9 in. deep. As it did not seem economically desirable to increase the depth of an edge beam below the general deck soffit, such an edge beam would have had its neutral axis far removed from that of the deck beams. However, in highway bridge design, it was not usual to incorporate a parapet as a structural member because of aesthetic considerations and also because of the possibility of damage from vehicular impact.

87. The effect of the neutral axes of the edge and deck beams being at different levels was, of course, to produce an eccentric longitudinal shear force between the members and thus induce longitudinal moments in them. In the case of deck beams, the moment would be distributed transversely according to the distribution factors. The evaluation and distribution of the longitudinal shear, however, would depend on an assessment of various 'unknowns'. Dr Spindel's observations and recommendations were therefore of particular interest in that an assumption for the distribution of the longitudinal shear would enable a solution to be obtained by consideration of compatibility of stress between the edge beam and extreme main beam.

88. If it was considered essential to consider the effects of edge moments, the method could be made to cater for them by using a further compatibility equation for edge slope as given by Little and Rowe.[5]

89. With regard to the distribution of vertical edge shearing forces, the Authors accepted that the Cement and Concrete Association advocated the same Fourier series representation for those forces as for the applied loading. However, when the method was first compiled in 1960, this was not apparent in the Association's publications of the period.

90. The Authors noted that the contributors from the Association would recommend a different approach to the design of edge stiffened decks from that given in the Paper. If, as it would appear from § 76, an estimate of the moment due to eccentric loading was to be made in the first instance without considering the full distribution properties of the deck, then that could possibly lead to the less experienced bridge designer having to perform the final distribution analysis several times before an acceptable load distribution was obtained.

91. The factor 1·1 was well known to devotees of the load distribution analysis and the Authors' omission to give the complete reason for and the limits of its use was regretted. At sections removed from centre span, the estimation of moments by the proposed method appeared to be sufficiently accurate for design purposes when compared with those obtained from the computer analysis based on the grillage method. Such comparison would also appear to justify the premise that continuous structures with torsional stiffness could be treated as simply supported between the points of contraflexure for the purposes of distribution and that the support moments for the load within the span would be distributed in the same way. It was assumed, however, that the moments occurring on the next span and thereafter were evenly distributed. In the Authors' experience, insufficient examples had been cross-checked by grillage analysis to justify a positive statement. In prestressed beams the zones were generally wide enough between centre span and support to tolerate some inaccuracy, but greater accuracy would be of benefit at the centre and support where the zones would be narrow. It was assumed, although Dr Spindel had found as much as 15% error in the support moments when checking against 'exact' solutions, that the live load moments were in fact better distributed than at centre span and therefore the maximum calculated moment range would lead to a conservative but safe design. That conclusion had been confirmed in the case of a symmetrical three-span grillage bridge.[8]

92. The compatibility of self weight and stiffness were included in the Paper to show that it was not difficult to achieve the necessary concordancy in the edge beam resulting in cable profile and deflected shape similar to those of the deck beams. That

would be almost a natural outcome of design. There were obvious aesthetic advantages in maintaining the same deflected shape in both deck and edge beams and there was also the practical advantage in that the ducts for transverse stressing cables could be located more easily in both beams. The location of these transverse holes can be difficult where the beam was heavily cabled longitudinally, especially where those cables had tightly defined limits as in continuous deck construction.

93. Since 1960 the use of computer methods had led to the extensive application of the grillage analysis. Those techniques, to which Mr F. Sawko of Leeds University had made notable contributions, had shown that the results from the grillage analysis fit actual observed data more accurately than the load distribution analysis. The latter analysis was also limited in its application to such problems as heavy skew decks, decks supported in unconventional ways, and evaluation of the torsions arising in the deck. It might be of interest that decks with shear distribution only had been investigated in the West Riding using the grillage analysis and found to be in excellent agreement with the more mathematical work of both Dr Spindel and Mr Best.

94. However, despite the foregoing discussion and as far as design methods were concerned, the refinements mentioned were unlikely to affect design in any radical way and the Authors considered that the method of edge stiffening analysis based on the load distribution theory did provide a direct design office technique suitable for most standard cases.

References

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