*Discussion on Paper No. 6868**

Twisting of thin-walled box girders

by

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Dr J. E. Spindel (Assistant (Development), British Railways Board) wished to draw attention to the problem of transverse bending stresses in the type of box girders considered by the Author.

50. He had had occasion to make an analysis of torsion stresses in the design of a multi-cell box girder for a rather complex concrete bridge.[5] Because the wall thicknesses, something of the order of one fifth of the depth of the box, were obviously too great to be neglected, the analysis was based on considering both longitudinal forces and transverse moments in the walls of the box. The forces acting on any side of the box were thus as indicated in Fig. 11.

51. Shear deformations were included in the analysis, which finally produced some 142 simultaneous equations to be solved by computer. Diaphragms were not involved. Simple cases, such as the square box used by the Author for the experimental work, could, of course, be solved quite easily.

52. In the case of the concrete box it was found that the transverse bending stiffness, to a very considerable extent, took the place of a continuous rigid diaphragm, and the same, as pointed out by the Author, was true of boxes with very much thinner sides. It was, in the writer's view, important to consider these transverse bending effects, firstly because the effectiveness of a 'rigid' diaphragm depended on the transverse bending stiffness of the box, and secondly, because the transverse bending stresses were far from negligible.

53. If the deflexion due to torsion was calculated as in equation (5), the longitudinal bending stress σ was given by the following expression:

$$\sigma = E \cdot \frac{\pi^2}{L^2} \cdot \frac{h}{2} v_1$$

For a square box with sides of equal thickness t and length h, the transverse bending stress under the torsion loading indicated in Fig. 11 was given by:

$$\sigma_T = E \cdot \frac{6t}{h^2} \cdot v_1$$

which expressed in terms of longitudinal stresses gave

$$\sigma_T = 1{\cdot}21\sigma \left[\frac{L}{h}\right]^2 \times \frac{t}{h}$$

54. This followed from the deflected form of the box shown in Fig. 12. For the two boxes tested by the Author, the ratio of transverse to longitudinal stress was 1·53 for the thinner box, and 3·05 for the thicker one. While it was appreciated that these were somewhat artificial examples, they drew attention to the importance of transverse stresses.

55. If the problem was made more realistic by assuming that torsion was due to a line load acting on one web only, the transverse bending stresses would still be 75 and 83% of the total longitudinal stresses respectively. This neglected the local stiffening of the webs due to web stiffeners, which might locally double these stresses.

* *Proc. Instn civ. Engrs*, 1966, 33 (April) 659–675.

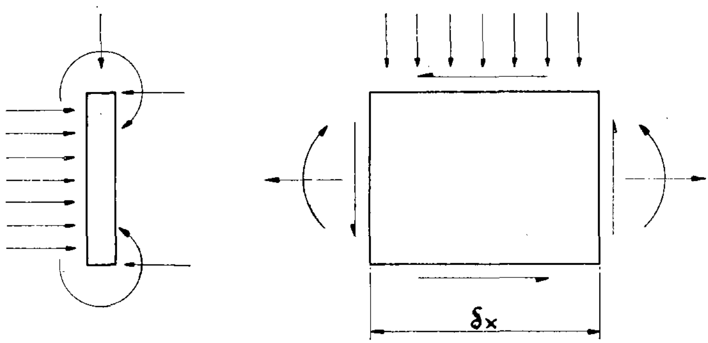

FIG. 11

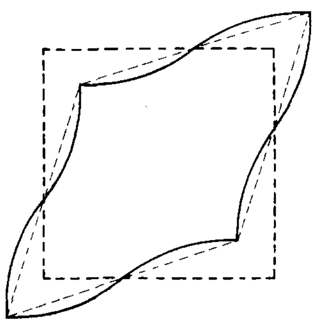

FIG. 12

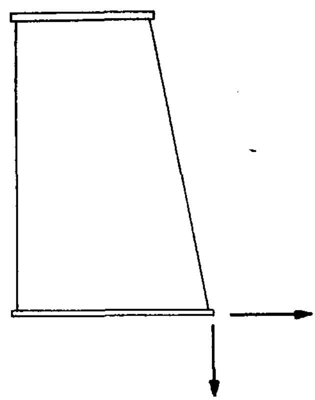

FIG. 13

56. A similar analysis had been applied to various steel boxes of the type shown in Fig. 13, where again very considerable transverse moments had been found. In this analysis, diaphragms were treated as concentrated forces applied at diagonally opposite corners, which prevented the extension of the diagonal, either absolutely or elastically. Several terms in the Fourier series were necessary to deal properly with close spaced diaphragms.

57. The main function of diaphragms was, in fact, considered to be that of reducing the transverse stresses, rather than modifying the longitudinal warping ones. This was particularly important in the case of welded steel boxes, because the joints at the corners of the boxes were usually very much weaker in transverse bending than in the sides of the box itself, even when fatigue was not a criterion. Failure, or even premature yield of these welds under the combined stresses to which they were subjected would, of course, destroy any 'diaphragm' action of the box by transverse bending stiffness, with a resulting increase of longitudinal stresses.

58. It seemed possible, as a very rough approximation, to calculate transverse bending stresses directly from the torsional deflexions of the side webs. These could

be determined as shown in Part I of the Paper. Where several diaphragms were involved, each diaphragm could be treated as supporting the web with its part of the flanges, so that the latter would become a continuous beam.

59. Some similar work had been published in Germany[6] which quoted some very interesting results by way of illustration.

60. The exact solution given in Part II of the Paper seemed vastly superior for the warping problem but it was not clear how it could be modified to deal with the transverse bending effects arising from loads applied between diaphragms. Perhaps a combination of all methods could be developed to deal with this problem.

The Author, in reply, thanked Dr Spindel for his comments on transverse bending which he agreed could be very important. The points referring to steel boxes applied more to those with smaller cross sections. The diaphragm requirements of larger boxes depended mainly upon longitudinal stresses.

62. It was suggested in § 58 that the transverse bending stresses could be found directly from the torsional deflexions of the side webs. Presumably distortional deflexions were meant here, since the deflexion caused by pure torsion represented a pure rotation of the cross section which would not induce transverse bending. This being so, the Author was sure that they could be used to obtain accurate values of transverse bending stresses for single-cell (double-web) and double-cell (triple-web) boxes and acceptable values for many multi-cell systems.

63. With regard to loads between diaphragms in the solution of Part II, these were considered, but the fixing moments and reactions shown in Fig. 10 were calculated as if the cross section had no distortional stiffness in the diaphragm sense. The resulting distortional deflexion at the mid-point of bay 1–2, say, was,

$$Ev = \frac{1}{384} \cdot \frac{wa^4}{I} = 6 \text{ ton/cm}$$

Applying the reactions of the fixing forces as loads to the actual structure gave a further distortional deflexion at the mid-point of bay 1–2 of

$$Ev = E(\theta_1 - \theta_2)\frac{a}{4} + E\frac{1}{2}(v_1 + v_2) = 197 \text{ ton/cm}$$

This gives a total value for v of

$$203/E = 0{\cdot}1 \text{ cm}$$

64. It was clear that any likely distortional stiffness of the cross section could not materially affect the solution with such small distortional displacement, but the 0·1 cm deflexion could be used to calculate any transverse bending stresses. In a box girder where this treatment was not justified, modifications could be made to the equations, but the Author would prefer to introduce fictitious equivalent diaphragms between the actual diaphragms. It was sometimes necessary to introduce extra diaphragms anyway if there was a significant variation in cross section. These could be of zero stiffness or could represent the stiffness of the cross section.

Correction

The equation given on line 3 of § 47 should read

$$\beta = -I/a + \xi$$

References

5. Turton F. Three prestressed concrete railway bridges: Discussion. *Proc. Instn civ. Engrs*, 1962, **22** (July) 321–322.

6. Lacher G. Zur Berechnung des Einflusses der Querschnittsverformung auf die Spannungsverteilung bei durch elastische oder starre Querschotte versteiften Tragwerken mit prismatischem, offenem oder geschlossenem biegesteifem Querlast. *Stahlbau*, 1962 (10), 299–308; (11), 325–335.