*Discussion on Paper No. 6939**

Derivation of the regime equations from relationships for pressurized flow by use of the principle of minimum energy-degradation rate

by

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Dr M. B. Abbott (Reader, International Courses, Delft Technological University) had been most impressed by the Authors' verification of the regime equations from the Durand equation. It was also gratifying to see the principle of minimum energy degradation rate derived from the second law of thermodynamics. This derivation was, after all, a common procedure in books on elasticity and plasticity, and could well be used to further advantage by hydraulic engineers.

52. It seemed to be a rule of the history of science that important results were presented by two or more quite independent groups of researchers at about the same time. In the present case, Engelund and Eggert Hansen,[11] working along apparently quite other lines than the Authors, had also, in January 1967, published theoretical justifications for the regime equations. It seemed to the Writer, however, that the Engelund–Eggert Hansen approach made less appeal to empiricism and/or measurements. Moreover, their approach, based upon Engelund's 'similarity principle',[12] seemed to be more general, allowing for a specific bed profile and providing also the regime relations for meander length.

53. Would the Authors like to comment on this almost parallel work, in the light of their own researches?

Mr G. Lacey (Consultant, Sir M. MacDonald and Partners) wrote that the Paper was of great interest to all workers in the regime field of hydraulics. The application of the principle of least work, if applied to field observations as well as those in the laboratory, should prove very fruitful. The results obtained by the Authors were very encouraging, the links between the laboratory observations and those made on canals were, however, still somewhat tenuous.

55. The statement that for canal design the only particle parameter required was the mean diameter was debatable despite the fact that the Writer at one time held this opinion.[13] White as an alternative employed V_s the settling velocity of the particle[14] and at a later date stated that although the variable V_s was a gross simplification a better parameter had yet to be found.[15] Inglis was one of the first to insist that both the settling velocity and the median diameter entered regime equations.[16]

56. Reference had been made to the Blasius 'law'. It was generally contended that this equation applied only to a very limited range of the Reynolds number and had been superseded by the more accurate formula of Nikuradse. As Levin had shown in 1966 observations made by Biesel on large smooth conduits suggested that the range of Reynolds numbers within which the Blasius equation applied could be greatly extended and this indicated that the Blasius equation could with propriety be termed a law.[17] This fact had a bearing on observations made on large channels transporting fine sediment to which the Stokes law was applicable.

57. The Writer had found from a study of observations made on the Rohri

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canal, West Pakistan, that two equations applied simultaneously.[18] The first, the linear resistance equation of Phillips $V=K_1RS$, and the second, the Manning equation $V=K_2R^{2/3}S^{1/2}$. But if these two equations both applied the Blasius equation applicable to smooth turbulence was implicit.

58. The fact that the Blasius equation appeared applicable to channels transporting fine sediment, and that the Manning equation also applied did not imply that either equation was appropriate when the sediment was coarser. In the Writer's first paper on this subject[13] in 1930 he produced two equations

$$V/R^{1/2} = K_0,$$

analogous to the Froude number, and

$$P = K_1Q^{0\cdot500}.$$

On elimination of V and P the equation

$$R = K_2Q^{0\cdot333}$$

was obtained.

59. In 1930, engineers in India and also in the United States relied almost exclusively on the cumbrous Kutter equation for the computation of velocities. The Writer adopted the simpler Manning equation as a more manageable expression. From the equations for P and R it followed that A (the cross-sectional area) varied as $Q^{0\cdot833}$ and on substitution in the Manning formula an equation for the slope was obtained

$$i = K_3Q^{-0\cdot111}.$$

60. It was in the reply to the discussion on his 1930 paper[13] that the Writer assumed that the exaggeration in the vertical scale of the smaller channels was none other than the increase in the ratio R/P and thus obtained the equation

$$i = K_4Q^{-0\cdot167}.$$

61. It should be noted that the powers assigned to Q were identical with the Blench equations quoted in the Paper. It should also be noted that King[19] improved the equations by substituting for P (the water surface width) and for R (the mean depth equal to A/W_s). Thus in all large channels P and W_s and R and D_m approximated very closely to one another. Values of W_s and D_m could be obtained with accuracy but the Blench system of a mean width and a mean depth on that portion of the bed deemed to be approximately horizontal was somewhat indeterminate.

62. It was of interest to note that as early as 1937 Blench, by the application of the principle of least work, succeeded in finding an energy equation from which two of the 'Lacey relations' followed.[20] Of these, however, one (understood to be a general equation) was of limited range. The general equation it would now appear took the form

$$V/R^{1/2} = K(R^{1/2}S)^n$$

in which the power n was a function not only of the particle size but also of the settling velocity of the particle. Thus when the sediment was fine (when the Stokes equation applied) the power of n was unity, when the sediment was medium sand the power was 0·50, and for coarse sediment the power was 0·25. The so-called general equation

$$V = KR^{2/3}S^{1/3}$$

was an approximation due to scatter in the observations and was quite inapplicable to the finer sediment range.

63. The equivalent proportionalities in § 25 of the Paper were of great interest. When the Manning 'law' was employed the power of the slope equation was $-0\cdot118$ in very fair agreement with the value of $-0\cdot111$ previously obtained by the Writer. When the Blasius equation was employed the powers of Q assigned to B and R respectively appeared to differ negligibly whereas the power of Q in the slope relationship had risen from $-0\cdot118$ to $-0\cdot130$. The power of R in the Manning equation

was 0·667 and that of Blasius 0·714. The Writer had found that with regime canals with beds of medium sand the equation

$$V = KR^{0\cdot75}S^{0\cdot50}$$

applied, the correlation being high. This equation was as valid as that of Manning or Blasius. The indications were that if it were employed the power of the slope would approximate much more closely to the power of $-0\cdot167$ which the Writer assigned. The Writer would be grateful if the Authors could examine this possibility.

Dr D. I. H. Barr (Senior Lecturer) and **Mr J. G. Herbertson** (Research Assistant, Department of Civil Engineering, University of Strathclyde) wrote that the Authors had described a most valuable and pertinent study. Two basic conditions of sediment transport under pressurized conditions had been described, viz. with and without deposition to form a bed. It seemed worthwhile to complete the Authors' correlation criteria for the first of these cases.

65. Adopting the concept of dynamic velocities[21-25] the complete velocity matrix for the quasi-uniform flow system could be drawn up as follows. There were three linear variables, d the equivalent diameter of the particles, and B and T the breadth and total depth of the horizontal rectangular duct. There were two boundary actions V_s and V, the average velocity of sediment and of the fluid through the whole cross section. It was here assumed that the ratio V_s/V, the charge, could be imposed, as could V. The three actions or partial actions of active forces were those of the driving force, of gravity and of viscosity, specified by sg, g and μ respectively, where s was the pregometric gradient in terms of the fluid. This gave the matrix

	V	V_s	Sg	g	μ
d	V	V_s	$\sqrt{Sgd}$	$\sqrt{gd}$	ν/d
B	V	V_s	$\sqrt{SgB}$	$\sqrt{gB}$	ν/B
T	V	V_s	$\sqrt{SgT}$	$\sqrt{gT}$	ν/T

66. Seven terms had to be selected to allow completion of the matrix in the absence of the other terms, for example:

$$\phi(V, V_s, \sqrt{gd}, \sqrt{gB}, \sqrt{Sgd}, \nu/d, \sqrt{gT}) = 0. \quad . \quad . \quad . \quad . \quad (9)$$

Hence

$$\phi\left(\frac{V_s}{V}, \frac{V}{\sqrt{gd}}, \frac{\sqrt{Sgd}}{\sqrt{gd}}, \frac{\sqrt{gd}}{\sqrt{gB}}, \frac{\sqrt{gd}}{\nu/d}, \frac{\sqrt{gd}}{\sqrt{gT}}\right) = 0. \quad . \quad . \quad . \quad . \quad (10)$$

67. Equation (10) was an example of a complete non-dimensional criteria, on which Figs 3 and 4 could be considered to be based. The first term was the charge, C, the second term was proportional to discharge Q in the circumstances of the tests, and the third term gave S (the Authors' j). Terms four and six reduced to linear ratios, kept constant, in each of the test series. It was also necessary that the fifth term, $g^{1/2}d^{3/2}/\nu$, was kept more or less constant—as it presumably was by maintaining the water temperature reasonably constant.

68. The Authors' correlation was thus completely secure, regardless of whether the flow of liquid was virtually all above the bed surface or otherwise. This point should be elucidated by the Authors—presumably dye injections could be viewed at the transparent panel. More important, details of the clear waterway depth, H, should be given; these data could have been incorporated on Figs 3 and 4 because on these diagrams $V/\sqrt{gd}$ was the essential basis of the variable chosen as dependent, and the insertion of a second dependent length into the matrix could give $\sqrt{gH}/\sqrt{gd}$ and hence H/d as a second dependent group. If there were little or no sub-bed flow, the value of the term d/T of equation (10) would be immaterial in correlation, given that stable conditions were obtained in each test.

69. The Writers considered that a slightly modified equation (10) had a wider significance. If free surface flow were to be considered, whether in a laboratory flume or in a straight canal, the inclusion in the matrix of allowance for gravitational active forces signified that equation (10) with the now irrelevant final term d/T, omitted, allowed correlation of circumstances where surface waves were present, as well as covering sediment movement. It was emphasized that it was not implied that the same data would be obtained, with and without a free surface, but that both circumstances were individually covered by the basis of correlation.

70. The importance of approaching laboratory test series with a similarity criteria which is both complete and also compatible with feasible testing procedures could not, in the opinion of the Writers, be over-emphasized. If testing would be restricted to depth to width ratios greater than about 5, the term d/B could be omitted and the object would be to obtain *parts* of diagrams based on

$$\phi\left(\frac{V_s}{V}, \frac{V}{\sqrt{gd}}, S\right) = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

71. Each diagram would be applicable to *one* value of $g^{1/2}d^{3/2}/\nu$, and the set would apply to one value of the ratio ρ_s/ρ, i.e. initially that for quartz sand and water. In addition, H/d parameters could be superimposed, as could details of the bed formation. Equation (11) could equally be obtained as

$$\phi\left(\frac{q_s}{q}, \frac{q}{g^{1/2}d^{3/2}}, S\right) = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where q was flow per unit width. The point about equations (11) and (12) was that a large recirculating flume could be filled with a particular sand and quasi-uniform flow obtained for some condition. It was then feasible to hold one of the groups, chosen as a dependent group for the plot, constant, and obtain a second point on the same line. It should be noted that lines of constant value of the second dependent group H/d would only be obtained by interpolation. It was a remarkable fact that most of the established correlations were neither rigorously complete, nor were they compatible with this apparently simple requirement of allowing consecutive tests to provide points of the *same* value of dependent parameter.

72. There was an alternative approach—that the length H be considered as a primary variable in place of the partial action specified by Sg. Then, working as before

$$\phi\left(\frac{V_s}{V}, \frac{V}{\sqrt{gd}}, \frac{d}{H}\right) = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

again for one value of $g^{1/2}d^{3/2}/\nu$ and of ρ_s/ρ. In this case the most natural dependent group for plotting was d/H, and this would be held constant while the whole range of V commensurate with the tilt and the discharge capacities of the apparatus would be explored—again at a single value of $g^{1/2}d^{3/2}/\nu$ and of ρ_s/ρ. Again, values of the second dependent group (in this case s) may be superimposed but lines of constant s will only be obtained by interpolation.

73. Neither of these plots would bridge the gap between laboratory and field observations. However, with the laboratory range rigorously covered, it should be possible to use well authenticated field measurements to fill in the diagrams.

74. The Writers realized that the production of either of these sets of diagrams would take considerable effort. Yet, compared with the present worldwide effort on sediment transport correlation, the task would be small. If either set of diagrams were available as a design tool, the need for the regime relationships would disappear.

75. The Writers wondered whether the Authors were justified in using Q in their fit equations for Figs 3 and 4. Was not the variable q discharge per unit width? Was not q the true variable of interest in the whole of regime studies, rather than Q since surely a natural channel in establishing its regime is merely adjusting its width and depth until an acceptable discharge intensity (q) is obtained?

Dr T. M. Prus-Chacinski (Partner, C. H. Dobbie and Partners) wrote that the Authors should be congratulated on their most interesting and valuable Paper. They had demonstrated in an elegant manner the basic unity of hydraulics and the deep underlying physical principles which governed the multiphase motions of liquids. By applying the principle of least work or the minimum production of entropy the Authors were able to derive three basic 'regime theory' relationships from the considerations of two-phase flow in pressure conduits. Their results offered an independent and possibly a decisive support to the almost identical results based on similar concepts which had been obtained by the recent research in the USA. Following the long series of publications on the morphology of natural streams by the US Geological Survey, and utilizing the concept of stream power[26] formulated by Bagnold, Leopold *et al.*[27–29] have demonstrated that the basic equations of canal and river 'geometry' (or 'regime') can be derived by applying the principle of minimum production of entropy in an open system. It was also demonstrated that the conditions of equilibrium or 'regime' were the most probable conditions and that therefore any given stream would tend to oscillate around such conditions or would tend to approach them. The above concept was also extended to tidal estuaries.[30] The discussion and the theoretical results were further expanded and compared with the field data by Leopold *et al.* in their recent book.[31] In 1966 a similar concept was applied by Leopold *et al.* in a most convincing manner to meandering[32] and it was demonstrated that meanders were the most probable form of a natural river and that they tended to minimize production of entropy. The Authors' derivation seemed to be a special case of the more general treatment applied in the above mentioned American publications. The Authors stated the energy degradation rate as being equal to vi or power per unit weight of liquid and their 'regime' equations were derived according to the above statement.

77. Leopold *et al.* used three not quite equivalent statements: (*a*) power per unit of mass which in practice was equivalent to the Authors' statement; (*b*) power per unit area $= VHi$; and (*c*) power per unit lengths $= Qi$. They consider that the applicability of any of the above statements will depend on the shape of the longitudinal profile of a stream. For irrigation canals statement (*a*) was applied but for natural rivers was applied a compromise between statements (*b*) and (*c*). It has to be noted that whereas for irrigation canals the longitudinal profile would tend to be either almost straight or even concave upwards, profiles of most natural rivers were convex but less convex than if statement (*c*) was applied. The references quoted above treated this most important aspect in great detail and there was no need to enlarge on this here. It would be useful, however, to compare the results obtained by the Authors with the American results. The three 'regime' equations obtained by the Authors were: $B \propto Q^x$; $R \propto Q^{x_1}$; $i \propto Q^{x_2}$. Table 1 compared the values of x, x_1 and x_2. Table 1 demonstrated a striking agreement regarding x and x_1 but a significant divergence in x_2 in relation to the Authors' results and the field data for canals as opposed to the same for rivers.

78. The Writer considered that the above was of great importance. The longitudinal profile of the hydraulic gradient or energy gradient in a pressure conduit must be, of course, straight and therefore the Authors' results were practically the same as Langbein's for the canals and they did not differ very much from Lacey equations. The case for natural rivers was different but again the agreement between Langbein's theory and the field data was very reasonable. In the Writer's opinion this should clarify once and for all the question of the status of the slope in the 'regime theory'. It was quite obvious that slope was the dependent variable but, paraphrasing the famous George Orwell saying, it was 'less dependent' than the other dependent variables. On the other hand, the power index of the width emerged as the most conservative quantity. Indeed it was well known that both natural and artificial channels adjusted their width quite rapidly whereas adjustment of slope took a much longer time and in the case of some purely natural streams the time might take a geological

TABLE 1

	x	x_1	x_2
Authors (theory)	0·471 (or 0·478)	0·353 (or 0·348)	−0·118 (or −0·130)
Langbein canals (theory) . . .	0·47	0·36	−0·12
Lacey canals (field data) . . .	0·50	0·33	−0·17 (also −0·11)
Langbein rivers (theory). . . .	0·53	0·37	−0·73
Leopold rivers (field data) . . .	0·50	0·40	−0·75 (from −0·49 to −0·95)

aspect. The Writer had commented some years ago upon the fundamental aspects of width in respect to any engineering computations.[33] In the Writer's opinion the Paper and the recent American publications had provided a firm physical basis for the Lacey's regime theory extended now to the natural streams. The basis of this theory was the same as for common hydraulics or classical hydrodynamics. Professor White had said[34] more than 30 years previously, that Lacey's theory at least provided some sort of framework on which to base the field data. It seemed that it was much more than that. It was quite likely that it was also a framework on which the bed-movement theories would have to be based in the future. It was probably not an exaggeration to say that to date (1967) the success of the bed-movement theories was at best qualified. On the other hand there was a great need for a team or an individual with a Benedictine capacity for hard work to clear the regime theory from the colossal amount of misinterpreted or simply wrong information and comments which had accumulated over so many years. Now, when the physical basis was firm, the 'regime theory' which was already a very useful tool in skilled engineers' hands could be made very much better. Probably the regional approach advocated by the Writer[33] could prove the easiest and most promising. However, the amount of necessary research and field work was still very great.

The Authors, replying to the discussion, were pleased that the Paper had occasioned so much discussion and thanked all those contributing for their interest.

80. Dr M. B. Abbott had referred to the work by Engelund and Hansen. The Paper derived the regime formulae from the similarity principle combined with an empirical equation for the effective shear stress. Letting τ represent the total shear stress at the bed level, τ' the reduced shear stress corrected for the effect of the bed features, and τ_c the critical tractive stress, then the effective shear stress was defined as $\tau' - \tau_c$ and the empirical relationship used by Engelund and Hansen took the form:

$$\tau' - \tau_c = \frac{0{\cdot}4\tau^2}{\rho g(S-1)d} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

It might be remarked that this equation was not of general validity and specifically that it did not apply to the flow with plane beds encountered in the Authors' experimentation, although the latter did display regime behaviour.

81. It was the Authors' opinion that, in seeking a derivation for the regime formulae, there was a risk in using as a basis other empirical relationships applicable only to flow with a movable bed. Since the minimization process which occurred under this condition tended to obscure the causal relationship between the variables, the result might succeed in showing the compatibility of the regime formulae with the empirical equations used without demonstrating the underlying physical principles involved. In the Authors' Paper the only empirical relationship employed was the

Durand-type equation which was widely used for two-phase flow in pipe-lines. The regime formulae resulted from an application of the principle of minimum energy-degradation rate to this equation, and the experimental work reported by the Authors served merely to verify the general form of the Durand-type equation and the resulting regime relationships. Thus the Authors were unable to see any justification for Dr Abbott's view that the Engelund–Hansen approach makes less appeal to empiricizm.

82. The Authors were gratified to receive the comments of Mr G. Lacey, who had made such great contributions to the early correlation of field data into the regime equations. With regard to Mr Lacey's question, the friction equation $V=KR^{0\cdot75}$ $S^{0\cdot50}$ was, indeed, equivalent to the use of the powers 0·500, 0·333 and −0·167 employed by Mr Lacey in the breadth, depth and slope equations respectively.

83. Dr Barr and Mr Herbertson had given a verification of the suitability of the Authors' correlation parameters, and had raised the question of the amount of flow below the bed surface. During the experimental work it was found that the seepage flow through the sand bed was completely negligible, while that through the nylon beds was very small in comparison with the flow through the waterway itself. Thus the value of the term d/T was indeed immaterial. It was not feasible to include all of the experimental observations in the present publication but tabulations of the waterway depth H referred to by Messrs Barr and Herbertson formed part of the complete experimental results which were set forth in Queen's University Civil Engineering Report No. 51 by K. C. Wilson, copies of which were available on request from the Department of Civil Engineering, Queen's University, Kingston, Ontario.

84. With regard to the final point raised by Messrs Barr and Herbertson, for the constant-breadth conduit used in the experimental work the difference between a plot of total discharge Q or of the unit discharge q was, of course, merely a difference in scale on the discharge axis. However, in the general case, the total discharge was one of the basic independent variables of the system, and it was the thesis of the Authors' work that for given Q, sediment properties and sediment concentration, the system would adjust the width and depth until a minimum value of the energy-degradation rate was attained. Thus the 'acceptable discharge intensity (q)' would be a dependent quantity, the value of which was determined by this process.

85. Dr Prus-Chacinski had referred to recent work of Leopold *et al.* on stream geometry and meandering, and had suggested that the Authors' derivation might represent a special case of this type. The Authors were not aware of this work by Leopold at the time the Paper under discussion was submitted for publication, but after reviewing this work they wished to note that although the results were very similar the methods of derivation differed significantly. The Authors had used the standard thermodynamic definition of entropy which, as noted by Dr Prus-Chacinski, was in practice equivalent to statement (*a*) of § 77. The Authors failed to see how the alternate formulations (*b*) and (*c*) could also be taken to have validity for cases in which they did not coincide with (*a*). In other words, a more precise definition of the entropy concept than that adopted by Leopold *et al.* could not result in a series of alternative possible relationships. In order to derive the regime equations, it was necessary to use some further relationship in conjunction with that supplied by the minimum energy-degradation rate. In the case of the work of Leopold *et al.* this additional relationship was apparently supplied by an assumption of uniform distribution of internal energy (the theoretical justifications of which were not clear), and a statistical approach had then been used to determine the regime exponents. On the other hand, the Authors took the required additional relationship from the form of an experimentally-determined equation for flow in conduits without deposition, and derived the regime-type equation directly. Thus the two methods of derivation were quite distinct.

86. The work reported by the Authors was confined to straight-line flow, and thus at present no comparison could be made for meandering rivers. In the case

of straight-line flow through canals for which comparison was possible, the Authors agreed that the near-equivalence of the results of the two methods was indeed striking.

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