Analysis of curved bridge decks

A. COULL & P. C. DAS

Mr F. Sawko, University of Leeds

The Authors are to be congratulated on extending the analysis of Coull and Ergin for circular slabs under uniform loading to cover the concentrated point load condition. An accurate analysis of radially supported curved isotropic slab bridges is now a practical possibility.

34. The Authors were faced with the difficult task of obtaining confirmation of accuracy for their theoretical approach, and had to resort to model testing on small scale slabs. The agreement between theory and experiment is very satisfactory considering the size of test specimens. It is a pity, however, that the two models were not made geometrically similar. This could easily have been achieved by making the inner and outer radii of the cement asbestos sheet 14 in. and 26 in. respectively, double that of the perspex specimen. The comparison thus obtained would have enabled the Authors to investigate the effect of the different Poisson's ratio of perspex and asbestos cement. Perspex, with its high $\nu = 0{\cdot}375$, might give misleading results for concrete bridge decks, but asbestos cement with $\nu = 0{\cdot}2$ would approximate to concrete very well. Figs 3–6 do show differences in both bending moment and deflexion profiles for the two models.

35. In a former discussion[5] I used an equivalent grillage composed of circular and straight members to analyse the perspex slab under uniform loading. The same approach has been used here to provide further verification of the accuracy of the Authors' approach.

36. The asbestos cement model was analysed as an equivalent grid shown in Fig. 7 using an electronic computer program which I developed for grillages curved in plan.[6] The perspex model was analysed using a similar grillage[5] but with fewer curved and radial members. The Authors have not given material constants for either model or actual (as against nominal) dimensions. The deflexion profiles obtained from the grillage analysis are similar to the ones obtained by the Authors, the ratios of outer edge to inner edge deflexions being shown in Table 1. Bending moments could, however, be compared explicitly, and the distribution under central point loading is shown in Figs 8 (a) and (b). This independent verification does confirm the accuracy of the Authors' method.

37. I feel that the title of the Paper might mislead the readers as to its contents. A more descriptive title of 'Analysis of Bridge Slabs Curved in Plan'[4] used before seems more appropriate, and even this could be modified with the addition of 'radially supported'. I have been in the past and am at present engaged on the analysis of several bridges curved in plan. The only bridges with radial supports were the beam and slab decks of Lofthouse Interchange forming a roundabout above two intersecting motorways. All the others have their supports parallel to the centre line of the highway they cross. A number of these are continuous over the central reservation, with supports parallel to the abutments.

38. As a practical solution to bridge decks, a reinforced or prestressed slab is sometimes employed for continuous bridges with randomly spaced discrete supports, since the supports do not lead to any geometric arrangement of main and transverse girders. For curved bridges a similar situation could arise, requiring an isotropic or

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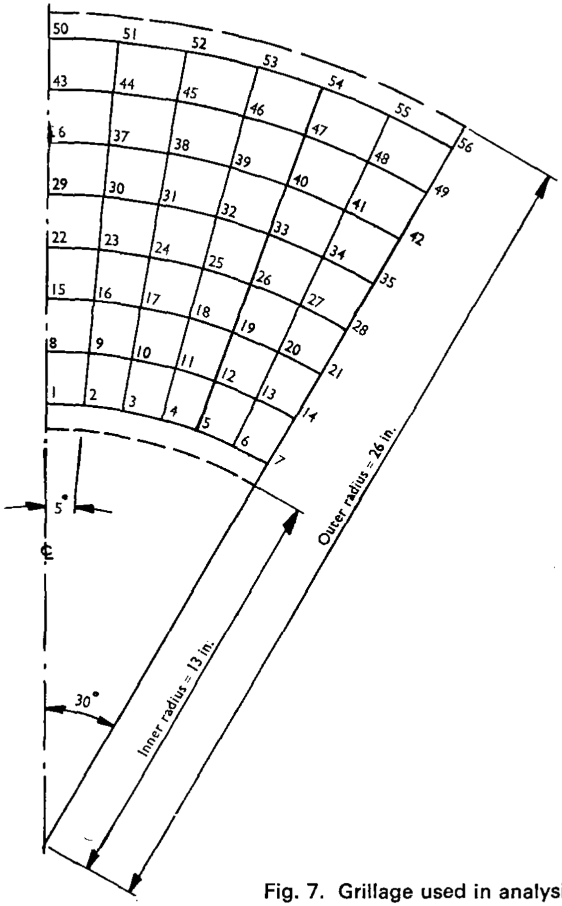
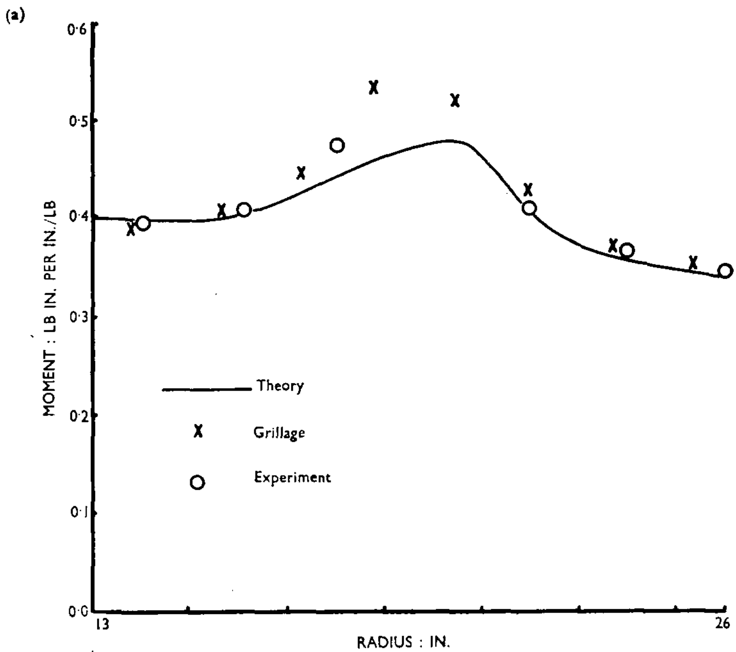


Fig. 7. Grillage used in analysis of asbestos cement model

Table 1. Ratio of outer edge to inner edge deflexion

	Load at:	Theory	Experiment	Grillage
Asbestos cement model	inner radius	1·11	1·03	1·07
	central	2·70	2·85	3·03
	outer radius	4·10	4·66	4·55
Perspex model	inner radius	1·07	1·25	1·12
	central	2·66	3·00	2·60
	outer radius	4·00	4·66	4·10



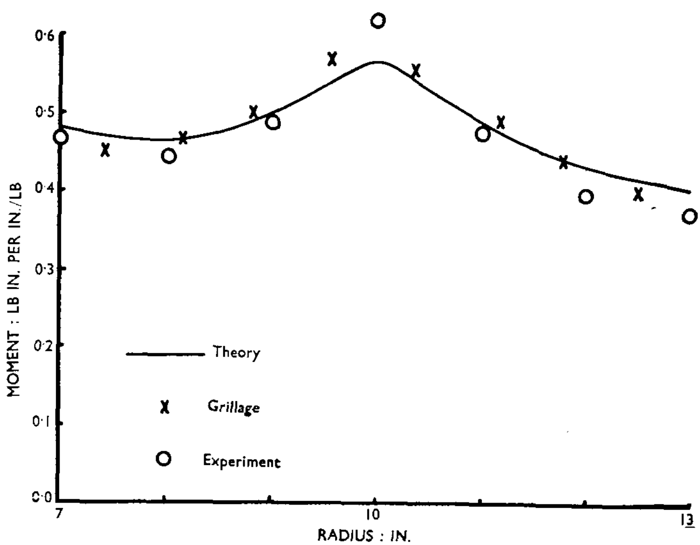


Fig. 8. Longitudinal bending moment distribution under central loading
(a) asbestos cement model
(b) perspex model

orthotropic slab, most probably stiffened at the edges. The type of structure analysed by the Authors would be used only infrequently. Reference to Figs 5 and 6 will further indicate the large difference between maximum longitudinal and transverse moments (3:1) indicating that an orthotropic slab would be more economic than an isotropic one. Could the Authors indicate whether a solution for orthotropic slabs is possible, and if so, whether it would not be too complicated?

39. Finally, a point made in § 32 on the advantage of the present method over numerical solution is only true when it refers to the analysis of 'radially supported isotropic bridge slabs curved in plan', not to curved bridge decks. Grid analogy and finite element methods can automatically include edge stiffening, non-parallel and discrete supports and discrete members, and it is structures incorporating some or all of these that constitute the majority of curved bridge decks.

Dr W. M. Jenkins and Mr J. M. Siddall, University of Bradford

We note the good agreement between the results of the Authors' analysis and the experimental results obtained from perspex and asbestos cement models and have investigated the problem of analysis of skew and curved bridge decks using a stiffness matrix approach and representing the deck slab with finite elements. Parallelogram elements are used for skew bridges and annular segments for curved bridges. Both models described by the Authors have been analysed using our program and the results are shown in Figs 9 and 10. A fairly coarse division into elements was used, the slab being represented by six elements in the circumferential direction and four in the radial direction.

41. The Authors did not quote values for the elastic constants and these were assumed to be: Young's modulus $4{\cdot}3 \times 10^5$ lb/sq. in. and 3×10^6 lb/sq. in. and Poisson's ratio 0·35 and 0·33, for perspex and asbestos cement respectively.

42. It appears that the finite element method gives rather better agreement with the experimental values in the case of the asbestos cement model but there does seem to be a fairly serious disagreement in the case of the perspex model with the applied load at mid-radius. Perhaps the Authors would like to comment on this, particularly in regard to the values of the elastic constants used.

43. It should be pointed out that in the program developed at Bradford an increased fineness of representation by elements does not increase computational difficulties since the program is devised in such a way that this requires only a simple alteration to the basic data of the problem. Difficulties may arise however in regard to the size of the computer store. The storage required for the models described in the Paper was 13 k.

44. A finite element treatment has certain advantages when considering the general problem of bridge analysis. The treatment of a bridge deck curved in plan is the same as that for a right or skew bridge except for the type of element used, the remainder of the structural analysis program involving the assembly and solution of the equilibrium equations and the computation of forces remains the same. An element approach permits the treatment of either isotropic or orthotropic conditions and the bridge programs prepared at Bradford can accept a composite beam-slab structure with the beams lying in both longitudinal and transverse directions. Variations in slab thickness can be easily accommodated as can continuity in support conditions.

The Authors

The Paper presents an exact solution of the biharmonic equation of plate theory applied to the problem of radially supported bridge slabs curved in plan. Consequently, the work may be used as a yardstick to test the accuracy of such numerical approaches as the grillage or finite element methods, which replace the continuous slab by a series of one or two dimensional elements connected only at a discrete series of nodes.

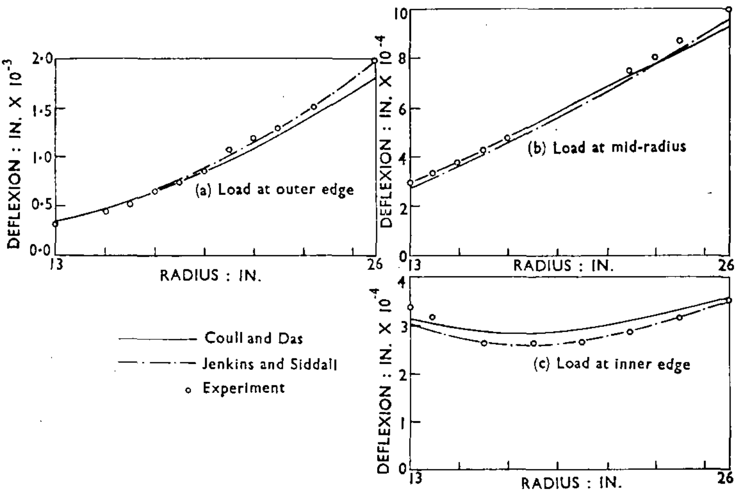


Fig. 9. Radial distributions of mid-span deflexions due to unit load (asbestos cement model)

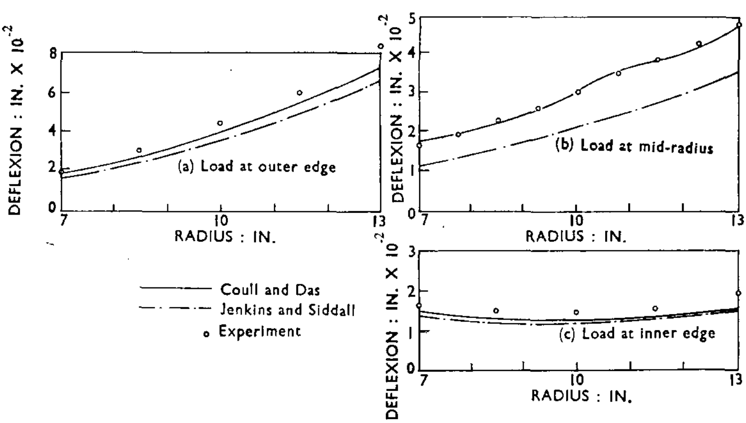


Fig. 10. Radial distributions of mid-span deflexions due to unit load (perspex model)

46. The solution corresponds to Massonnet's work[7] on right bridge slabs, which has been publicized by Morice, Little and Rowe[6] in their well-known distribution coefficient method of analysis of bridge decks. There is little difficulty in extending the analysis to deal with orthotropic decks (with principal planes of stiffness in the radial and circumferential directions), the only difference being that the roots of the equation are no longer fixed as in equation (8), but depend on the ratios of the bending and torsional stiffnesses (as in the case of the right bridge deck). The analysis has been extended to deal with bridge slabs stiffened by edge beams, with good agreement having been reached between theory and experiment, and has been used also for the analysis of two-span curved bridge slabs.

47. Mr Sawko, Dr Jenkins and Mr Siddall have commented on the statement made in § 32 concerning the computational effort involved. With the grillage and finite element techniques, the accuracy of the solution is increased by increasing the number of elements and nodes, and hence the order of the stiffness matrix which has to be inverted. Problems of storage in a machine will eventually occur, and the accuracy of the computation may be affected by round-off errors. In any event, the amount of computation increases according to some power of the order of the matrix. This is not the case in the present solution, which is built up to any desired degree of accuracy by the addition of a number of terms, each of which is calculated quite independently of the preceding ones. The amount of computation thus increases linearly, and the rate of convergence may be easily examined as the computation proceeds.

48. In reply to Dr Jenkins' and Mr Siddall's question, the elastic constants for the models were

perspex: $E=4{\cdot}6\times10^5$ lb/sq. in., $\nu=0{\cdot}35$
asbestos cement: $E=3{\cdot}0\times10^6$ lb/sq. in., $\nu=0{\cdot}18$

49. In the case of the perspex model, the variations in the nominally $\frac{3}{16}$ in. thick sheet were so great that the model was ground on one side, producing a final thickness of 0·168 in. These figures explain the discrepancies between Dr Jenkins' and Mr Siddall's results and our measured and computed deflexions. However, it is felt that a better test of the accuracy of the finite element method would have been afforded by a comparison of stress-resultants rather than the directly computed deflexions.

References

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