Automatic analysis of strain hardening structures (with particular reference to steel to B.S. 968)

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We welcome the Paper as a significant contribution to methods of analysis of structures in the elastic-plastic state. The Paper describes an automatic method of analysis suitable for digital computer and includes the effects of strain hardening and instability. The strain–hardening effect is allowed for by adopting a strain–hardening plastic hinge and this involves the assumption that the length of a member subjected to a moment greater than M_p (dimension z in Fig. 2) is small enough to be neglected. Clearly this will be an acceptable approximation in regions of steep moment gradient but with flatter bending moment diagrams it would seem important to include the length of the plastic zone.

42. There would appear to be some doubt as to the validity of the assumption made in § 19 that $k \simeq K_s$. If $K_s = 9$ and cot a is taken as 50 then for $M/M_p = 1{\cdot}0$, $K/K_s = 0{\cdot}86$ and for $M/M_p = 1{\cdot}3$, $K/K_s = 1{\cdot}20$ representing a not inconsiderable error.

43. The Writers are engaged in preparing digital computer methods for the same problems and have come to the conclusion that if a digital computer is to be used, and this appears absolutely necessary in problems of this complexity, then the finite length of plastic zone can be included at very little extra cost. In the computer programs being developed at Bradford the Writers have adopted a stiffness matrix method in which the stiffness coefficients of the structural members are computed from a moment curvature diagram of the form shown in Fig. 9. The length of the plastic zone is automatically included as a function of the end moments on a member segment and the discontinuities produced by plastic hinges are therefore not encountered. The equilibrium equations are treated as linear over small increments of load and an iterative process used to determine the stiffnesses at each load increment. Experience has shown that no difficulties arise over convergence and if the load increments are kept small at each stage convergence is very rapid, usually being achieved in about three iterations. A process of successive, automatic reduction of the load increments is used as the structure progresses into the plastic state.

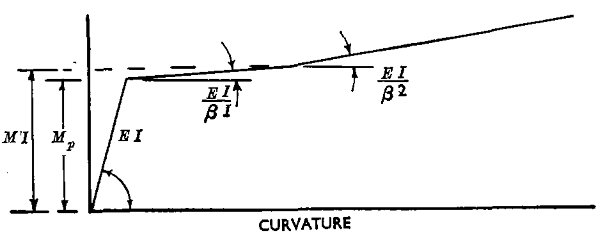

Fig. 9. Moment curvature diagram

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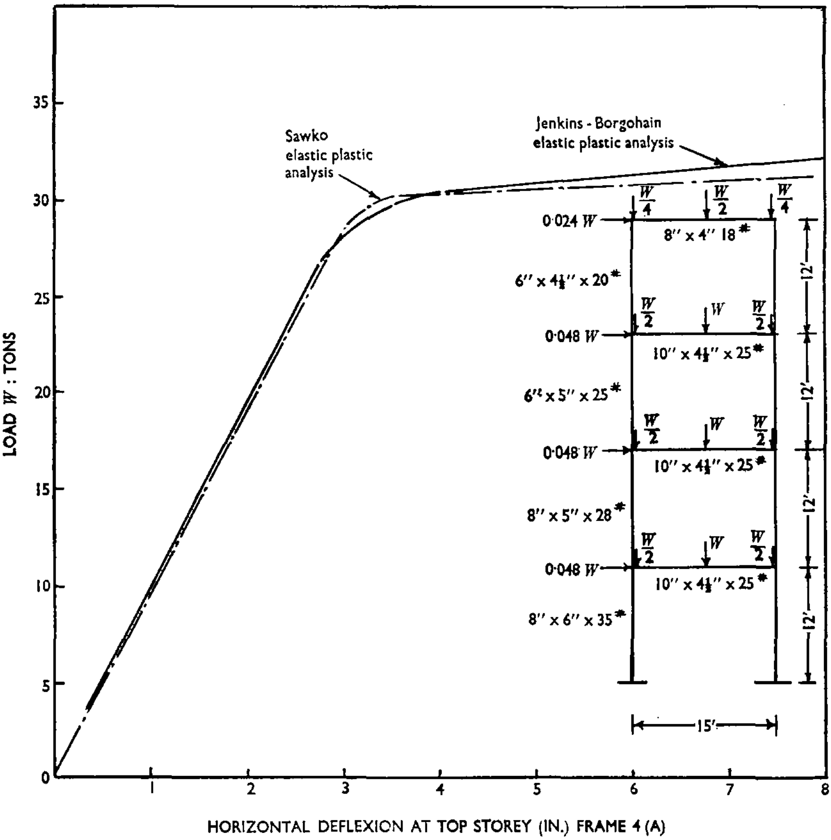

Fig. 10 and Fig. 11 (opposite) Results of computer analysis of structures shown in Fig 4(a) and Fig. 7

44. In order to obtain a comparison with the Authors' method the Writers have analysed the structures shown in Fig 4(a) and Fig. 7 with their program and the results are shown in Figs 10 and 11. These results do not include the effect of reduction in stiffness due to axial forces and this effect is at present being written in to the computer programs. The calculations were carried out on an Atlas computer.

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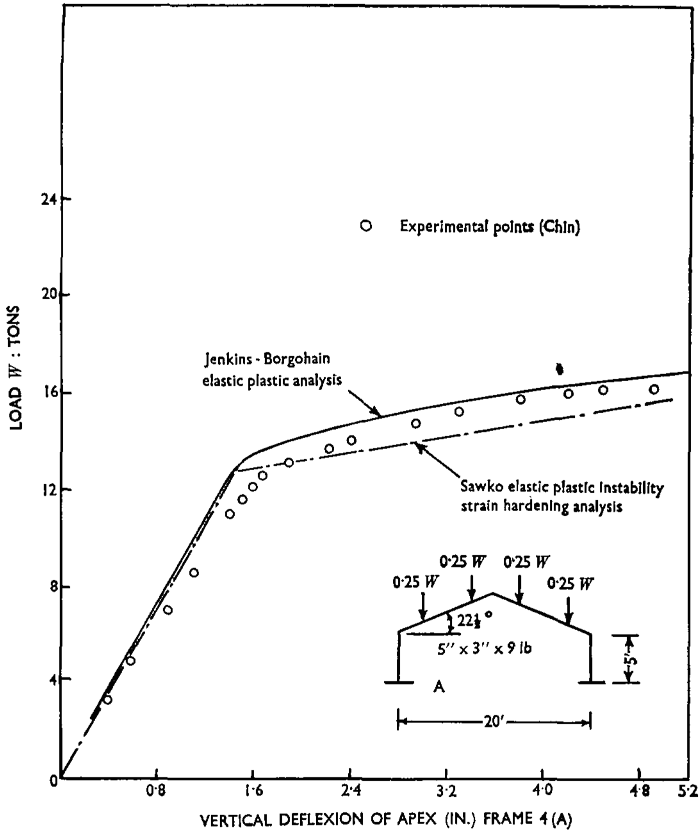
We were most interested to hear of the Contributors' work on strain hardening, and look forward to the forthcoming publications with full details of the method. It seems to be similar to the approach developed by Chin[28] who adapted the displacement method of analysis using the finite element technique and actual moment curvature relationships.

46. The assumption that $k \simeq K_s$ in our approach does not seem to lead to any appreciable error in the analysis. The factor k, which of course depends on the



bending moment distribution in the vicinity of the hinge, has been investigated experimentally[24] on a series of simply supported beams of steel to B.S. 968 and shown to be:

$$k = 30{\cdot}345 - 22{\cdot}066 \frac{Mp}{M}$$

47. The value of $k = 9{\cdot}51$ was an average obtained from the series of tests, and does compare well with $K_s = 9{\cdot}55$.

48. In defending our approach, we would comment that in no analysis carried out so far has there been any discernible error due to the neglect of spread of plasticity along the member. We realize that the approach advocated by Messrs Jenkins and Borgohain is probably more 'accurate' if moment-curvature relationships are specified with precision, but that it suffers from many disadvantages of a commercial nature. Computer time, specification of data and number of runs will be very much increased in the alternative approach suggested, since the step by step iteration with spread of plasticity throughout the structure will require very small load increments leading to an excessive number of computer runs.

49. Although we do not know the exact approach of the Contributors to this problem, we feel that the moment curvature relationship for steel I sections is more

in the form of Fig. 3 than that of Fig. 9, and would modify somewhat the Contributors' results in Figs 10–11. It is unfortunate that the wrong curves were compared in Figs 10–11, since the comparison seems to suggest some error in their approach. Thus in Fig. 10 the elastic plastic analysis curve does not incorporate strain hardening, the effect of which is to increase the slope of load-deflexion curve, thus approaching the Jenkins/Borgohain curve. In Fig. 11 including instability effects would lead to larger deflexions again, making the two sets of curves very close.

50. One of the main objects of the Paper was to emphasize the importance of strain hardening in the plastic theory of structures. Once this is recognized many alternative approaches to the analysis are possible, and we concentrated on making their method automatic and therefore suitable for commercial use.