Punching failure of reinforced concrete slabs

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Bond and Long's theory gives means of calculating the load on a slab at three alternative stages characterized by the following:

(*a*) the extreme fibre strain in the concrete near the column reaches a value corresponding to a maximum permissible stress;
(*b*) the maximum stress in the reinforcement near the column reaches the yield value;
(*c*) a shear stress at some distance from the column reaches a limiting value.

70. Of these, the first is the least important, as over-reinforced slabs are not common. With some doubts concerning the factor β, the analysis of this case is not unacceptable but there is no apparent justification for increasing the load C_1P_1 calculated from the analysis by multiplication by 1·33, since in the absence of shear cracks no dowel action is likely.

71. The second case is a reversal to working stress design methods and cannot be expected to predict ultimate loads with the same accuracy as yield-line theory. Again there seems to be no justification for the dowel factor of 1·33. Possibly the factor is used to scale up 'first-yield' loads to ultimate loads. This appears to be the case in the calculations for Moe's slab SI–60. For this slab $P_{calc}/1{\cdot}33 = 62$ kips, which is in agreement with the observed load at first yield, i.e. 66 kips. Multiplication by 1·33 increases the calculated load to 82·5 kips, i.e. to very nearly the ultimate load of 87·5 kips. In fact, the analysis used is not applicable to the stages between first yield and failure since it is based on an 'elastic' distribution of moments. The ultimate load by yield-line theory for this slab is 88·4 kips.

72. The third case is the only one relating to punching shear, and unfortunately the treatment of it contains the most serious errors. The load calculated is presumably intended to be that at which shear cracking commences since this, and not failure, is the obvious result of a critical condition being reached with respect to shear stresses near the neutral axis. The criterion of punching failure must be one applied to the concrete at the compressed surface of the slab near the column face, as it is here that such failure occurs. This fact is shown clearly by the crack patterns of Moe's series H tests in which holes in the slab permitted the development of shear cracking to be observed. The only relation between shear cracking and punching failure is that the former is a prerequisite for the latter. No single method of calculation can predict both events.

73. An example of the error in Long and Bond's method is seen in Kinnunen and Nylander's slabs IA30b, 28 and 29. The calculated load for both of these slabs was 92·0 kips, while the actual shear cracking loads were 52·8 and 60·5 kips, and the ultimate loads were 82·5 and 93·5 kips. Thus applied to these tests the dowel factor of 1·33 enables the shear cracking formula to give reasonable agreement with actual ultimate (but not shear cracking) loads. Theoretically, this is objectionable and empirically it is not of general use, as the ratio P_{ult}/P shear cracking is not a constant: in Moe's series H tests, it varied between 1·29 and 2·13.

74. As is shown by Kinnunen, the main effect of shear cracking is to reduce the area of concrete available to resist the inclined compressive forces at the column face,

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and failure occurs within this concrete. The area of concrete concerned cannot be determined by a conventional analysis in which shear cracks are ignored, nor can a consideration of the wrong stresses in the wrong part of the slab be of much use. Another point is that the dowel force cannot be expected to act until the shear cracks have formed, i.e. until after the stage considered in Long and Bond's analysis.

75. Even if the basic calculation of C_1P_1 for the 'shear' cases is viewed simply as a shear cracking calculation, it appears to be erroneous. Firstly, while the first equation in § 29 is consistent with the assumptions stated there, equation (7) is not. From the assumptions, the shear stress τ_n at any radius r_1 can be obtained simply from the condition of vertical equilibrium as $\tau_n = P/2\pi r_1 L_*$. Thus for any given value of P, τ_n is independent of the distribution of moments. According to equation (7), τ_n is dependent on $\partial\sigma_r/\partial r$ and thence on β and the distribution of moments.

76. The assumptions of § 29 are, in themselves, questionable. The shear stress distribution on a vertical plane is assumed to be constant and equal to τ_n between the neutral axis and the reinforcement. Thus the larger part of the shear is taken to be supported by the tensile zone. If the slab is cracked in flexure, it is impossible for the air in the flexural cracks to support this shear. In most of Moe's series H tests shear cracks were observed to form from earlier flexural ones, which leads to the conclusion that it is the shear stresses in the compressive zone above flexural cracks that are critical.

77. In view of the above, it may no longer matter very much, but the concept behind the factor β is a puzzling one. The factor appears to be a device to account for a reduction of the radial stiffness of the slab at the column face caused by the occurrence of a circumferential flexural crack. If it is necessary to take account of this crack, it is hard to see why the effects of radial cracks are ignored. From an elastic theory point of view, these latter cracks must transform the major part of the slab to an anisotropic body with a tangential stiffness lower than its radial one. The whole process of elastic analysis seems in any case to be uncalled for, as the simple rigid body deformations treated by Kinnunen are in good agreement with the observed behaviour of slabs.

78. There is no reference made to the work of Andersson, who has made considerable simplifications and extensions of Kinnunen and Nylander's methods.[25-28] Although the proposals made in reference 28 must be regarded as tentative, the work contained in these four papers appears to be of far greater value than Long and Bond's which offers nothing new that is not subject to grave doubts as to its validity. This last comment applies even to their test results, as the test pieces were so small that if the scale effects observed in shear tests of beams also influence the behaviour of slabs the results obtained could well be meaningless.

Professor S. D. Lash and Mr P. Y. Tong, Queen's University, Kingston, Ontario

It is difficult to predict the ultimate load capacity of slabs subjected to concentrated loads and the theoretical method proposed by the Authors is a welcome addition to the scanty literature on this topic. Unfortunately it does not appear to work very well when applied to the results of some recent tests carried out at Queen's University at Kingston, Canada. These tests were part of a study of the behaviour of highway bridge slabs, and loads were applied by a circular steel disc rather than by a column stub. The slabs were square and their edges were simply supported. The chief variable was the span of the slab.

80. Strain measurements on the concrete showed that at low loads tangential and radial strains were of the same order of magnitude, an assumption made by the Authors, but as the failure load was approached, the tangential strains became several times as great as the radial strains. It appears that cracking of the concrete and yield of steel cause substantial redistribution of moments and as a consequence the equations of elasticity are of doubtful relevance.

81. The 'direct method' proposed by the Authors has been applied to six test

Table 4

Slab	*in.*	σ_c *lb/sq. in.*	P_T *kips*	P_P *kips*	P_T/P_P	Mode of failure
2 S . . .	15	5620	33·1	32·4	1·02	Y+S
3 S . . .	21	6000	37·7	46·4	0·82	Y
R 3 S . . .	21	5250	30·9	26·6	1·16	Y+S
S 3 S . . .	21	4180	31·7	21·8	1·45	S
4 S . . .	27	5780	32·6	21·6	1·51	Y+S
R 5 S . . .	33	5580	30·4	20·0	1·52	Y+S

Notes: (*a*) In all slabs $d_1 = 2{\cdot}43$ in., $b = 3$ in., $p = 1{\cdot}10\%$, $\sigma_{SY} = 80$ kip/sq. in.
(*b*) P_T = punching load from test
P_P = punching load from calculation
(*c*) Mode of failure according to calculation.

slabs, which failed in punching shear with the results indicated in Table 4. It will be noted that in slab 2 S there is close agreement, in slab 3 S the load capacity is less than that given by calculation, and in the remainder test loads are substantially greater.

82. The Kingston tests indicate that concrete strength is not a major factor in determining ultimate load capacity. This is to be expected, since for under-reinforced beams the ultimate moment of resistance is largely determined by steel strength. As far as shear is concerned, Kani,[29] on the basis of many tests of beams, has concluded that the effect of variation of concrete strength is negligible. This may be an overstatement but certainly the effect is small.

Mr Long and Dr Bond

The Authors, in reply, thank the contributors to the discussion of their Paper but regret that there appear to have been some misunderstandings. In § 69, Mr Regan states that a method was presented which enables the load on a slab to be calculated at three alternative stages. In fact, as described by the Authors in § 12, the stress at the most critical position in each slab was calculated by approximate methods in order to predict the punching load, which in this case was defined as the ultimate load carrying capacity. When the failure stress is reached at the critical position in a slab, wherever it may be, and provided that a complete yield-line mechanism has not formed, then a progressive failure will occur in the compression zone of the slab. As this failure progresses there will be a movement of the column relative to the slab, with the result that dowel and tensile membrane action will be mobilized, before ultimately the column will punch through the slab. In particular, even in cases where the reinforcement has yielded near the column edge but the reinforcement index of the slab is not sufficiently low for a complete yield-line mechanism to form, punching cannot take place until failure occurs in the concrete in the compression zone. In slabs where the critical position was found to occur at the neutral plane the Authors were not attempting to calculate the load at which shearing cracks initially formed since such cracks will be in evidence above the neutral plane well before the punching load has been reached, as has been shown by Moe's[13] Series H tests. In addition, the discontinuity envelopes of Vile[17] and Newman[19] suggest that cracks will have developed in the compression zone before the ultimate biaxial stress criterion has been reached in this region.

84. In § 75, Mr Regan proposes an alternative simplification of the first equation in § 29 which is also consistent with equation (7):

Since $$M_r = \frac{\sigma_r}{2} \,.\, d_n L_a,$$

then $$\frac{d\sigma_r}{dr} = \frac{dM_r}{dr} \cdot \frac{2}{d_n L_a} = S \cdot \frac{2}{d_n L_a}$$

where S = shearing force per unit width of slab.

85. If the distribution of moments in the slab is symmetrical about the column, then $S = P/2\pi r$ and equation (7) becomes

$$\tau_n = P/2\pi r L_a$$

In deriving an approximate expression for the shearing stress at the neutral plane the Authors had in mind also the combined loading case where there is a transfer of moment between column and slab in addition to a direct load. They apologize for not making this clear in their Paper and for the error in Fig. 5. The horizontal angle $\delta\theta$ should have the dimension δt.

86. Mr Regan is concerned in § 77 about the influence of anisotropic stiffness on the distribution of bending moments in a slab. The moment/rotation characteristics of reinforced concrete sections near failure are no longer linear and a great difference of radial and tangential strains would not necessarily indicate a corresponding difference of moments. Reimann[15] analysed anisotropic slabs with tangential stiffnesses which were much lower than their radial stiffnesses. A comparison of the distribution of moments in these slabs with that in isotropic slabs shows that the differences are only about 10%. It is the opinion of the Authors that although this would not influence the calculated punching loads to any great extent, it might have a considerable effect on the predictions of total deflexion at ultimate load.

87. The Authors note in § 78 the criticism by Mr Regan of the scale of their model slabs. It has been recognized for some time by those concerned with reinforced concrete research that one-quarter scale models are adequate for such tests. In any event the Authors made a serious attempt to compare the method of analysis with experimental results from many different sources, as is demonstrated in Appendix 2.

88. The Authors noted with great interest the results of tests which were reported by Professor Lash and Mr Tong in §§ 79–82 and in Table 4. In order to calculate bending stresses in a slab by the direct method, the Authors estimated the boundary conditions at the column edge using a linear relationship between a/b, β and k where

a = radius to line of contraflexure
b = radius of equivalent circular column
β = interpolation factor
k = intermediate dimensionless parameter $= a\sigma_c/E_c d_n$

In most of the slabs which were analysed the ratio a/d_1 varied between 5 and 8. Professor Lash and Mr Tong measured the punching resistance of slabs which had a wider range of a/d_1 ratios as shown in Table 4. Their valuable contribution to this subject has caused the Authors to re-examine the characteristics of the parameter k.

89. The relationship $k' = d_1\sigma_c/E_c d_n$ is a measure of the strain in the concrete and steel and represents more truly the slab bending characteristics at failure. This parameter has been used to relate β with a/b and slab characteristics as shown in Fig. 15. It has been used to calculate values of P_P by the direct method for the slabs which were tested by Professor Lash and Mr Tong and the results are presented in Table 5. It will be seen that there is still a difference between calculated and measured values of punching load. Part of this difference could be due to the stress/strain characteristics of the reinforcing steel which was used in these experiments.[30] The steel did not have a clearly defined yield point, and the value of E_s was constant only up to approximately half the final yield stress of 79 000 lb/sq. in.

90. In addition, the new parameter k' was applied to the analysis of the 52 slabs which are described in Appendix 2 of the Paper, and in general yielded a good correlation between test and calculated results. However, the results of a few slabs which have a/d_1 ratios which are 5 or less and which would be expected to give the greatest discrepancies, are given in Table 6 for comparison. This table indicates

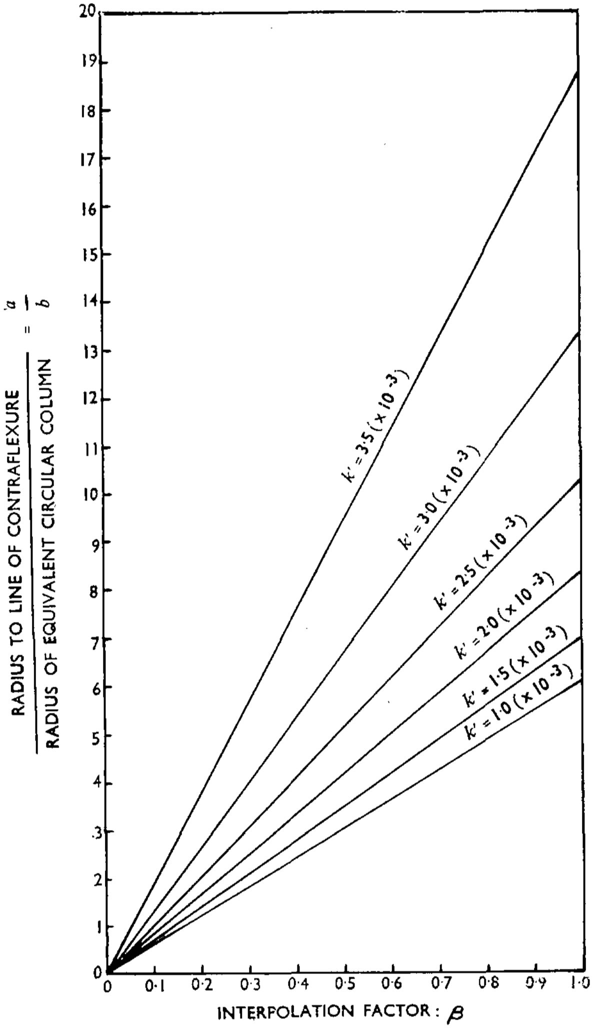

Fig. 15. Relationship between interpolation factor β and a/b ratio

that even these most unfavourable results still lie with acceptable limits and the values of P_P are generally conservative.

91. Finally, the Authors wish to comment particularly on the remarks of Mr Regan in §§ 77 and 78 and on the doubts of Professor Lash and Mr Tong of the relevance of an elastic analysis as a solution to this problem. The mechanism of punching failure of reinforced concrete slabs is extremely complex. Even if concrete were an ideally elastic or plastic material (which it is not) it would be difficult to produce an ideal, rigid solution to this problem. Since in the region of the joint, as

Table 5. Comparison of test and calculated values of punching loads for slabs of Lash and Tong using k' to evaluate β

Slab no.	d_1 *in.*	b *in.*	a *in.*	a/d_1	σ_c *lb/sq. in.*	p %	σ_{SY} *lb/sq. in.*	P_T $lb \times 10^3$	P_P $lb \times 10^3$	P_T/P_P	Mode failure
2 S .	2·43	3·0	15	6·18	5620	1·10	79 000	33·1	32·3	1·03	S and Y
3 S .	2·43	3·0	21	8·63	6000	1·10	79 000	37·7	32·5	1·16	Y
R 3 S .	2·43	3·0	21	8·63	5250	1·10	79 000	30·9	27·6	1·12	S and Y
S 3 S .	2·43	3·0	21	8·63	4180	1·10	79 000	31·7	27·4	1·15	S and Y
4 S .	2·43	3·0	27	11·10	5780	1·10	79 000	32·6	25·2	1·29	S and Y
R 5 S .	2·43	3·0	33	13·60	5580	1·10	79 000	30·4	24·3	1·25	S and Y

Table 6. Comparison of test and calculated values of punching loads for slabs in Table 2 but using k' to evaluate β

Test	Slab no.	d_1 *in.*	b *in.*	a *in.*	σ_o *lb/sq. in.*	p %	σ_{SY} *lb/sq. in.*	P_T $lb \times 10^3$	P_P $lb \times 10^3$	P_T/P_P	Mode of failure
Base . . .	G and H	2·25	2·4	11	3920	1·63	50 000	23·8	20·5	1·16	S
	J	2·25	2·4	11	3970	3·27	50 000	26·4	23·6	1·12	S
Richart. . .	207(a)	8·0	8·4	40	4250	1·25	62 400	335·0	320·0	1·04	S and Y
	207(b)	8·0	8·4	40	4040	1·25	62 400	319·0	315·0	1·01	S and Y
	213(a)	8·0	8·4	40	4500	1·25	62 400	317·0	325·0	0·97	S and Y
	201(a) and (b)	10·0	8·4	40	2660	1·00	62 400	293·0	255·0	1·15	S

progressive failure occurs, the concrete is nowhere ideally elastic or plastic and in the joint there are cracks and inclusions of steel which cause non-homogeneity, it cannot be stated that either an elastic or a plastic solution is ideal. The choice between an elastic or a plastic solution for estimating the strength of such a joint in a structure depends to a considerable extent on the performance of the structure as a whole. In flat-slab multi-storey buildings, for example, due to a combination of wind loading and unsymmetric floor loading, there will be acting on each joint not only a concentric force, which is the condition which has been analysed in the Paper, but a combination of this and a transfer of moment between column and slab. It is this situation which has for some time been of interest to the Authors, the present Paper having been written as an intermediate stage.[31]

92. In order to calculate the magnitudes of concentric forces and moments which are transferred between column and slab it is necessary to analyse the complete structure either plastically as it collapses, as a mechanism, or elastically. It was the opinion of the Authors that when punching failure was critical in a structure it would normally occur before the columns or slabs failed plastically as mechanisms. The reasons for assuming that an elastic, lower-bound solution could be applied to the slabs are described in the Paper. For these reasons and because it is intended eventually to analyse multi-storey buildings elastically by computer and to calculate also the load factor of each joint against punching failure, an elastic method was used.

93. It is the opinion of the Authors that much work remains to be done, particularly on the combined loading case. They hope that the ideas which they have presented so far will be a useful addition to the valuable contributions (both elastic and plastic) of others to the better understanding of this problem.

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