

**CORRESPONDENCE**  
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“ The Computation of Shrinkage and Thermal Stresses in Massive Structures ” †

by

Olgierd Cecil Zienkiewicz, Ph.D., B.Sc.(Eng.), A.M.I.C.E.

**Correspondence**

Mr T. K. Chaplin (Scientific Officer, Soil Mechanics Division, Building Research Station, Watford) observed that the values of  $\theta$  used in Fig. 4a gave very irregular values of  $\nabla^2\theta$ , probably as a result of their being interpolated from the drawn isotherms of Carslaw and Jaeger.<sup>6</sup>

Since  $\nabla^2\theta$  controlled equation (3), namely,  $\nabla^4\psi + \nabla^2\theta = 0$ , Mr Chaplin would have expected the irregularities in  $\nabla^2\theta$  (plotted in Fig. 9) to have produced some approximately corresponding irregularities in the stresses  $\delta^2\psi/\delta y^2$  in Fig. 4c, but they scarcely appeared.

To show the effect of using a very coarse mesh he had calculated the stresses given by a mesh length double that used in Fig. 4b. Although in that mesh there were only three independent values of  $\psi$ , surprisingly good values were obtained for the stresses in the central region (Fig. 10), and at the outer boundary the difference was about a quarter of that given by the finest mesh. The trend of the successive stress values at the outer boundary suggested that even the Author's finest mesh might be under-estimating the maximum stress by as much as 10%.

In Fig. 4a there appeared to be errors in the printed  $\theta$ -values of 0.510 and 0.703, which he thought should be 0.570 and 0.730.

The Author, in reply, thanked Mr Chaplin for his detailed contribution to the Paper and the labour he had put into the extension and checking of some calculations. The error which had been pointed out in Fig. 4a had occurred during tracing and the correct values, as given by Mr Chaplin, had in fact been used in the computations.

† Proc. Instn Civ. Engrs, Pt I, vol. 4, p. 88 (Jan. 1955).

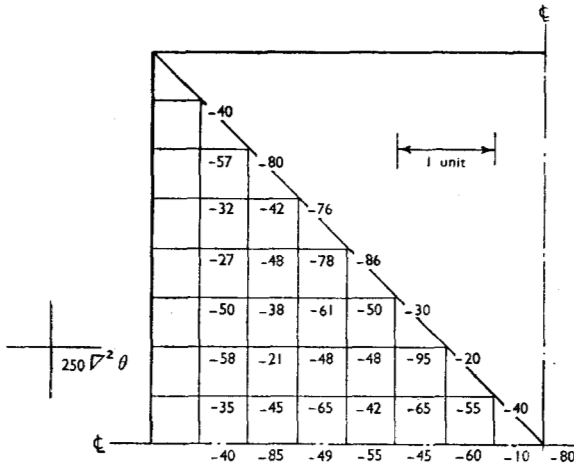


FIG. 9.—VALUES OF  $250D^2\theta$  DURING COOLING OF A SQUARE PRISM

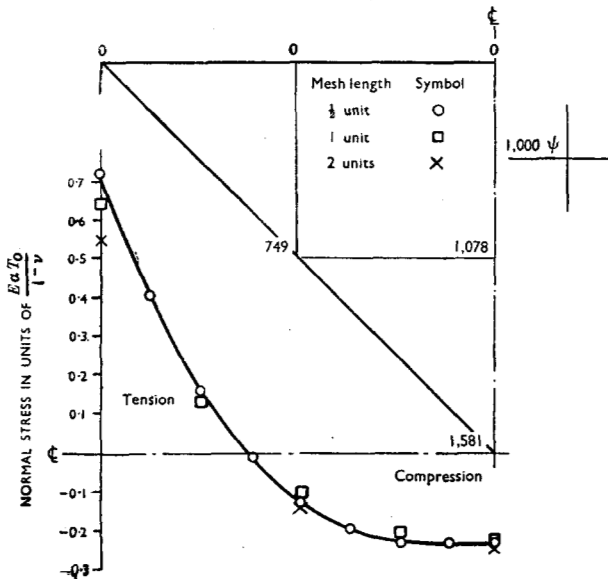


FIG. 10.—SOLUTION FOR A VERY COARSE MESH (UPPER LINE) AND STRESSES GIVEN BY VARIOUS MESH SIZES (LOWER LINE)

As Mr Chaplin had rightly remarked, the values of temperatures taken in Fig. 4 were only approximate—the problem having been introduced mainly to illustrate the general methods of solution. The reasonably smooth distribution of stresses resulting from rather irregular values of  $\nabla^2\theta$  was even more striking in the other examples, where isolated values of that residual on a particular line gave smooth and continuous stress distributions at least on one side of such a line.

There was no method known to the Author by which the degree of approximation reach could be estimated with mathematical precision. Comparison with problems for which known analytical solutions existed served often as a useful guide to the accuracy attainable and in specific cases the advance to finer meshes gave an indication of the rapidity of convergence. In the example of Fig. 4 it did indeed appear that the approximation to the boundary stresses might be of the order of 10% with a much closer estimate within the interior of the prism.

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