Application of similitude theory to correlation of uniform flow data

D. I. H. BARR & A. A. SMITH

Mr A. T. Stuckey and Mr R. W. O'Garra, University of Salford

The Authors are to be congratulated on their ingenious attack from the rear, as it were, upon a very involved topic. Their discussion of the numerous diagrammatic representations of the relationship between flow rate in pipes and associated energy loss helps to clarify the somewhat confused state of present knowledge.

62. One of the major difficulties in the application of present theories is shown to be the assessment of a suitable value for the relative roughness of a pipe surface. The Authors mention the difficulty in assessing equivalent roughness size but appear to have overlooked the difficulty in assessing a suitable value for the other parameter which contributes to relative roughness—the pipe diameter. Owing to manufacturing difficulties it is neither possible to produce a number of pipes of identical cross section, nor to produce even a single pipe which is perfectly cylindrical. When a line of pipes are joined together the variation in diameter along the length of the pipeline can be considerable. It is therefore necessary to adopt an effective value for pipe diameter and this effective value may be significantly different from the nominal or even the volumetrically determined value. Should a spurious value of diameter be used when evaluating the equivalent roughness (as outlined in § 24) then an erroneous roughness value will result. Such errors exaggerate the actual scatter of experimental results when plotted for comparison purposes on the Moody or Blasius Stanton diagram. In this connexion it must be remembered that the relative error in energy loss will be about five times the relative error in diameter. It is possible that the difficulty in assessing a value for pipe diameter has led to the conclusion that certain pipes and conduits are smoother than smooth and it may well be that when the concept of effective diameter is applied to these cases they will turn out to be normal.

63. It is not practicable to determine the parameters effective diameter and equivalent roughness from physical measurements on the pipes themselves. It is possible, however, as has been shown elsewhere[22] to infer the values from observations of flow rates and corresponding energy losses. In this method the value of the effective diameter and corresponding value of equivalent roughness emerge from an application of the principle of least squares to the fitting of a combination of the Darcy–Weisbach and Colebrook–White equations to the field observations. The paper referred to also demonstrates how the Colebrook–White equation, using values of d and k established over relatively few observations at lower flows, may be extrapolated to give a remarkably good fit to observations at flow rates, in one case, 12 times as great as those used in the fitting operation. In other words, effective values for d and k have been found in the transition zone where the Colebrook–White equation effectively simulates reality. These effective values then permit accurate extrapolation towards the zone of fully developed turbulence. This suggests that even if suspect the Colebrook–White equation is very effective and may, as was demonstrated by Lamont[23] be used with some confidence.

64. Values for effective diameter and equivalent roughness cannot, of course, be determined until the pipeline has been constructed. In the future, experience may help to forecast effective values for particular types of pipe, based on observations

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made on similar existing pipes. These forecasts would be helpful to the design engineer because they could take account of the process of ageing, which is influenced by such factors as the duty imposed upon the pipe and the characteristics of the fluid it carries as well as the initial surface condition. It would appear that the mathematical model based on the Colebrook–White equation would be more effective in simulating mature or new pipelines than any model involving only artificial roughness patterns.

Mr S. Irmay, Professor of Fluid Mechanics, Technion—Israel Institute of Technology, Haifa

The Authors introduce the concept of dynamic velocities on a formal basis of dimensional analysis. I have already introduced this concept[24,25] under the name of stress velocities, on a physical basis (existence of models). I define seven such stress velocities, the best known of which is the shear velocity U (proportional to the Authors' V_λ).

66. In another paper[26] the Writer defines a non-dimensional flow coefficient c by

$$c = V/2(2gdS)^{1/2} \quad (37)$$

which is proportional to V/U. In rough flow we have Nikuradse's formula

$$c = c' = \log(d/k) + 0{\cdot}57 \quad (38)$$

While in smooth flow we have Kármán–Prandtl's formula

$$c = \log(Vd/\nu . 1/c) - 0{\cdot}70 \quad (39)$$

67. The difference $(c'-c)$ plotted versus the non-dimensional $R = Uk/\nu$ is a universal diagram. It is 0 in rough flow, and

$$c'-c = 0{\cdot}52 - \log R \quad (40)$$

in smooth turbulent flow, which gives a straight line in semi-log paper. In Nikuradse's transition zone for uniform sand

$$c'-c = (\log R - 0{\cdot}52)3{\cdot}82/R \quad (41)$$

68. Colebrook and White's results differ from Nikuradse's in the transition zone. The difference seems to be due to their different definitions of d and k. Whereas Nikuradse's k is the diameter of sand grains stuck densely on the walls of a pipe of diameter d, Colebrook and White define k as the protruding height of the sparsely distributed roughness.

Dr F. V. A. Engel

The Authors draw attention in the text of the Paper (§ 55) to 'the comparatively narrow range of Nikuradse's rough pipe experiments. Sooner or later comprehensive tests will be undertaken which will give a check on the extrapolative use of the logarithmic rough pipe law'. In the conclusion the Authors state 'that the basis of data most often used for resistance diagrams, namely the Colebrook–White equation, is suspect', and infer from their investigation that 'There is a basic case for a comprehensive experimental investigation of the effect of a standard form of roughness in a wide range of pipe sizes'. In these aims the Authors should get full support of engineers, as well as scientists, who are interested in the fluid dynamics of open channels and flow through closed conduits.

70. One of the related issues in which I am engaged is the application of 'Similitude theory'[27,28]. The reader of the Paper may get the impression that all the non-dimensional terms form merely the basis of a conveniently contrived curve fitting procedure or of plotting diagrams.

71. The composition of some of the non-dimensional terms applied by various authorities quoted in the present Paper appear to be selected at random and may detract from an understanding of the problems involved. Furthermore, the numerical evaluation of the various terms raised to fractional powers may become cumbersome.

72. From equation (6) it appears 'more obvious' to write relation (7) as follows:

$$\phi\left(\frac{V_{m}d}{\nu}, \frac{(sgd)^{0\cdot 5}}{\nu/d}, \frac{V_{m}}{(sgd)^{0\cdot 5}}\right) = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

This is obtained by a combination of the three terms of (6), omitting only those expressions which are equal to unity or terms occurring twice. Why did the Authors exclude the Reynolds number in equation (7)? In several of their diagrams the Reynolds number is used as an additional parameter.

73. The Authors, as well as the other authorities mentioned, apply a non-dimensional term, viz.:

$$\frac{(sg)^{0\cdot 5}\, d^{1\cdot 5}}{\nu} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (43a)$$

which may be expressed in more conventional terms, namely:

$$s\frac{gd^{3}}{\nu^{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (43b)$$

This is the Grashof number which comprises the component terms of the square of the Reynolds number and the Froude number, thus eliminating the velocity term. The expression $Re^{2}Fr^{-1}$ is called the Galileo number

$$gl^{3}/\nu^{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

The latter multiplied by a non-dimensional gradient, a hydraulic gradient, a density, temperature or concentration gradient forms the Grashof number.

74. The Grashof number is a well known group relating to heat transfer problems. Besides the Rayleigh number and other groups, it is a 'convection group'. These groups belong necessarily to the complete system of non-dimensional groups of dynamic similarity. In many convection problems it is often impossible or very difficult to determine the velocity. Therefore, by suitable combinations of the Reynolds number, the Weber number, and the Péclet number with the Froude number, the velocity term is eliminated from the various convection groups.

75. However, it must be understood that the convective groups do not form new laws of dynamic similarity.[28] Problems of fluid flow, heat and mass transfer are characterized exclusively by six laws of dynamic similarity and the appropriate six 'genuine' groups. The 'convection groups' must be interpreted on the basis of their component genuine groups.

76. In view of the extensive use of the Grashof number in the Paper, it is warranted to refer briefly to the meaning of convection groups. A fixed numerical value of a convection group cannot uniquely define flow conditions as the component terms of the groups are related to incompatible laws of dynamic similarity.[28] A randomly chosen and designed non-dimensional expression cannot be representative of similitude theory.

77. An elucidating example, which may explain the importance of limiting conditions, is the so called critical Rayleigh number which determines the onset of heat convection. All three component groups of the Rayleigh number, namely the Reynolds number, the Péclet number and the Froude number, are equal to zero before the onset of heat convection currents. Neither the critical Reynolds number nor the critical Froude number enter the problem. A critical Péclet number does not exist. Nevertheless, the critical Rayleigh number assumes finite values. These critical values can be calculated which distinguishes the Rayleigh number from the critical Reynolds number and the critical Weber number. In general convective groups require well defined limiting conditions when applying these groups to the interpretation of flow characteristics.

78. In an earlier paper Dr Barr[29] characterized uniform turbulent flow by 'dynamic times', extending the 'dynamic velocity' concept given in eq. (2) and (4) of the present Paper. In non-dimensional form these 'dynamic times' represent the

Froude number and the Reynolds number. The term (43a) was called the 'uniform flow Froude–Reynolds number' or 'Chézy–Reynolds number'. Regarding a similarly designed 'densimetric Froude–Reynolds number' Dr Barr observes 'that this is a logical terminology because it is the ratio of a densimetric Froudian dynamic velocity to a Reynolds dynamic velocity. It can be called a form of Grashof number, to which the Authors see no objection, but it should not be called a form of Reynolds number.' I am in full accord with the latter statement. However, there are serious objections against multiplying any of the component terms of 'genuine' groups by a factor, even if it is non-dimensional.[27] Under those circumstances the resulting expression is not uniquely defined and cannot form the basis of a law of dynamic similarity. The correct interpretation of the Grashof number, and there is no other choice, is given by the terms (43b) and (44). In related fields of application of silt transport and the stability of alluvial channels more rapid progress would have been made if the stringent rules of formation of non-dimensional groups would have been better understood. It was only recently that the Galileo number was introduced as a flow criterion of open channel hydraulics.[30] (See Ref. 30, in particular page 243, § 242.)

79. In accordance with the equations (8) and (9) the Grashof number fits very well the Prandtl relation for the smooth pipe law. Ackers[9] has shown that the Grashof number

$$sgk^3/\nu^2 \qquad (45)$$

transforms the Colebrook–White equation for the transition range into engineering terms. In view of the role of the Grashof number as a tool for representing resistance laws for smooth and rough pipes a major issue before starting any new experiments (see § 59) would be to investigate the significance of the Froude number related to 'internal flow problems' in closed conduits. The Richardson number or the 'internal Froude number' may require a more extensive interpretation regarding eddy diffusion and turbulence.

80. Furthermore, it appears essential that roughness patterns and the five regimes of turbulent flow established by Morris[31] should also be considered when planning the new experiments. The interpretation by Morris given to the roughness problem appears to differ from the Colebrook–White concept. Are there any reasons why the Authors did not refer to his paper?

R. J. Garde, Professor, and **K. G. RangaRaju,** Reader, Civil Engineering, University of Roorkee, India

The Authors are congratulated on a critical review of the various forms of resistance diagrams developed for the purpose of obtaining direct solutions for the usual pipe flow problems. The 'New Diagram V' (Fig. 13) proposed by them is of special advantage since it provides a direct solution of all six categories of problems mentioned by the Authors. However, the problems commonly encountered in engineering practice are those for which, the other relevant parameters being given, a solution for Q, D or S is required. We presented[32] a non-dimensional plot between the parameters

$$\frac{\nu^2}{2k^3gs}, \quad \frac{k\sqrt{gDS}}{\nu} \quad \text{and} \quad \frac{QS^2k^5g^2}{\nu^5}$$

which yields a direct solution for the diameter and discharge but was primarily intended for the former.

82. The usual scales to which these various diagrams are plotted are such that one cannot get very accurate values by reading from them, particularly in solving for the discharge or the slope. Hence we approached the problem from dimensional considerations and obtained a relationship as mentioned below but described in detail elsewhere.[32]

83. The functional relationship between the various parameters governing pipe friction was written as

$$\frac{Q}{\nu k} = f\left(\frac{k^3 gs}{\nu^2}, \frac{k}{D}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

It may be mentioned that the Colebrook–White equation can be written in terms of the above parameters as done by Powell.[33] We used Moody's diagram to obtain the relationship between the various parameters in equation (46) which in this instance were calculated for smooth, rough and transition boundaries over the range Re 4×10^3 to 10^7 and a range k/D of 4×10^{-2} to 6×10^{-7}. By plotting these data the following relationships were evolved:

$$\left(\frac{Q}{\nu k}\right)\left(\frac{k}{D}\right)^{8/3} = 2{\cdot}51\left(\frac{k^3 gs}{\nu^2}\right)^{0{\cdot}547} \quad \text{for} \quad \frac{k^3 gs}{\nu^2} < 0{\cdot}35 \quad \text{or} \quad \left(\frac{Q}{\nu k}\right)\left(\frac{k}{D}\right)^{8/3} < 1{\cdot}60 \quad . \quad . \quad . \quad (47)$$

and $$\left(\frac{Q}{\nu k}\right)\left(\frac{k}{D}\right)^{8/3} = 2{\cdot}40\left(\frac{k^3 gs}{\nu^2}\right)^{0{\cdot}507} \quad \text{for} \quad \frac{k^3 gs}{\nu^2} > 0{\cdot}35 \quad \text{or} \quad \left(\frac{Q}{\nu k}\right)\left(\frac{k}{D}\right)^{8/3} > 1{\cdot}60 \quad . \quad . \quad . \quad (48)$$

84. It may be noticed that the above equations apply to all three regimes of pipe flow and provide direct solutions to S, Q and D. The errors in the diameter obtained from equations (47) and (48), compared with the values obtained using Moody's diagram were found to be in the region of $\pm 3\%$. Since a standard pipe diameter is to be used ultimately, and since the procedure of successive trials by using Moody's diagram is seldom carried to the exact value, equations (47) and (48) are well suited to problems of this category. The errors in the value of Q obtained from the above equations are in the order of $\pm 5\%$. However, the errors in the values of S computed from these equations are in the order of $\pm 10\%$ and in a few cases as high as $\pm 15\%$. We feel, however, that the errors in interpolating and reading the various diagrams discussed by the Authors could be of the same order or larger and as such, equations (47) and (48) may prove preferable despite the fact that they are semi-empirical.

85. In cases where it is definitely known beforehand that the pipe is smooth one does not need to know the value of k, but can obtain a direct solution based on the smooth-pipe law

$$1/\sqrt{f} = 2 \log_{10} Re\sqrt{f} - 0{\cdot}8 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

where $Re = VD/\nu$. The Authors have presented 'New Diagram IV' (Fig. 12) to yield a direct solution for the diameter of smooth pipes. They have used the parameters $Q/(sg)^{1/3}\nu^{5/3}$ and VD/ν to obtain such a solution. However, the solution requires reading from a graph of very small scale and we should prefer to use an equation for the purpose. By re-writing the equation $hf = fLV^2/2gD$ and using equation (49) RangaRaju obtained (equation (48)) a relation between $Q^3 gs/\nu^5$ and VD/ν which can be expressed as follows:

$$\frac{Q^3 gs}{\nu^5} = 0{\cdot}055\left(\frac{VD}{\nu}\right)^{4{\cdot}79} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (50)$$

86. The above equation can be used to obtain a solution for the diameter. It is applicable to the same range of Reynolds number as equation (49). Practically the same equation, in dimensional form, was given by Powell.[35]

Mr A. A. Putnam, Battelle Memorial Institute, USA

Some years ago I suggested that there were four classes of modelling problems:[36] (*a*) diagnostic, (*b*) predictive, (*c*) developmental, (*d*) basic.

88. One may consider the presentation of phenomenological laws in dimensionless form in the same sense. One may wish to use the dimensionless presentation to (*a*) evaluate the operation of an existing system, (*b*) design a new system, (*c*) present data

in the process of determining the exact form of the 'law', and (*d*) aid in understanding the phenomenon. The optimum presentation is not necessarily the same for all the situations. In courses in advanced education, the emphasis is often on (*c*) as in heat transfer in fluids, or (*d*) as in thermodynamics. Unfortunately, this carries over into industry, where an (*a*) or (*b*) presentation would often be more pertinent. In fact, it was quite a shock in my first design position that a particular type of trial-and-error calculation was made over and over, day after day, because the presentation of the data in textbook form made this procedure necessary. A few hours spent replotting the textbook relations in a suitable form for this particular class of design problems paid off handsomely. In the same sense, thermodynamicists continue to struggle with Mollier diagrams for types of problems much more rapidly handled by the use of enthalpy-specific volume charts.

89. Messrs Barr and Smith have presented an excellent story directed toward the application engineer ((*a*) and (*b*)). They have produced a diagram (Fig. 13) that has 'no lack of sensitivity' for any of the usual types of problems one might encounter and, in addition, one that corresponds in general format to the classical curve one encounters in the literature (*c*). This is more than one could normally expect in the general scheme of things. I look forward to testing this curve on a variety of problems as they arise in our operations.

90. A further valuable suggestion by the Authors might be overlooked. With many economical processes now available for reproducing high-quality working charts, there is little excuse in routine problems for having to use 'hat pin and textbook' approaches to reading design charts. Costs will be lowered by having available adequate supplies of working charts developed for the specific needs of a company.

Mr P. Ackers, Hydraulics Research Station, Wallingford

The Authors have provided a very comprehensive study of the possible ways of representing pipe resistance in diagrammatic form, with the aim of establishing the most comprehensive arrangement. Their treatment is based—very properly—on similarity considerations, though with an approach novel in this field. They introduce the concept of dynamic velocities, and one can see the convenience of using parameters with these dimensions. The Authors' V_λ is equivalent to our old friend the friction velocity, usually denoted by V_*; the local, maximum and average velocities are obvious flow parameters; and kinematic viscosity only needs dividing by a characteristic length to have the same dimensions.

92. Whether this approach will find general favour is to some extent dependent on whether students of the subject find the approach convincing, and this will be so only if the logic of the development is apparent. In § 6, the dynamic velocity is defined on the basis of the velocity that would be attained over a representative distance as a result of an active force. Hence the student would expect the representative distance to be in the direction of the relevant force, not perpendicular to it. It seems to me that the use of d as the appropriate distance at this stage of the argument needs some explanation. Would the Authors' derivation of V_λ still lead to the same result if the representative distance were taken as the length of tube? If not, how convincing will the derivation be to a student? This is not to suggest that there is any doubt that the dimensionless groups that result are valid and complete, of course. The only doubt is whether the argument on which it is based is sufficiently convincing to warrant general adoption of this novel approach.

93. That the logarithmic form of the resistance equation is a matter of convenience is the conclusion of § 21, and the Authors are reluctant to extrapolate beyond the range of experimental confirmation. Whilst agreeing that this reluctance to extrapolate is wise, surely the logarithmic form of equation combines convenience with a wide range of application confirmed to a sufficiently close approximation for engineering purposes, both in smooth and rough conditions. If convenience alone were the criterion, the Blasius equation for smooth turbulence and the Manning equation for

rough turbulence would be preferred. In fact it is because the logarithmic equations are less convenient than exponential ones that some engineers are reluctant to make use of modern developments. But is there any doubt that the adoption of the logarithmic equations has led to improved accuracy in design? Almost ten years ago I expressed the opinion 'Hydraulicians now generally accept the Colebrook–White transition formula, perhaps not as the last word, but certainly as the closest approximation available to flow conditions in commercial conduits. It thus forms the best basis of design, and will continue to do so until some major theoretical advance occurs and is confirmed as a refinement on present knowledge'. There have been some refinements in knowledge since, particularly for 'artificial' types of roughness, but the Authors' acceptance of the Colebrook–White equation—albeit reluctant—suggests that we are still not in a position to make any appreciable improvement. The quest should continue, but it would be wrong to undermine confidence in the current design formula when we are not able to recommend anything better for the general run of engineering application.

94. There is nevertheless strong evidence that the perfect equation, when it is found, will approximate to the logarithmic form now popular. Certainly the previously accepted exponential equations, Blasius and Manning for smooth and rough respectively, have only a very limited range of application in comparison. Table 4 of Hydraulics Research Paper No. 1[9] provides strong circumstantial support for an approximately logarithmic law in the rough turbulent zone.

95. The Authors are to be congratulated on the thoroughness with which they have approached the task of representing flow data in a way that meets the needs of designers, researchers and teachers alike, but my task ten years ago was concerned with design only, and not with the representation of research data or with academic purposes. Thus recognizability, in the academic sense the Authors use the phrase, was irrelevant—but recognizability of the end product *to the engineer* was important, and was achieved.

96. Problems 5 and 6 in Tables 1 and 2 are of very minor importance. As the basic equations (transition and rough-turbulent) are explicit in respect of k, I would advise against the use of charts in determining k from friction test data—it is a simple direct computation taking only a few minutes per case. It would be foolish to use turbulent resistance as a basis for determining viscosity.

97. The consideration of the use of the Colebrook–White equation in design led me along the following train of thought ten years ago:

(i) Resistance under turbulent conditions is not very sensitive to viscosity, and hence, as the main concern of civil engineering hydraulics is with the flow of water, a set of charts based on a single typical value of ν will suffice, especially if supplemented by a method of correcting for other temperatures in the rare cases where this is necessary.

(ii) A designer usually selects his pipe or channel material from a limited range suited to the job in hand, and hence has decided upon a k value before he uses a design chart. Hence each chart should relate to a particular roughness.

(iii) Each dimensioned chart must then include the four variables V, Q, s and d. The best method of plotting leaving full flexibility of use is to separate these variables.

(iv) As V, Q and d are related through $Q=(\pi/4)d^2V$ (for pipe flow), the linings for these three parameters can be common to the whole set of charts. Such is the case if they are logarithmically spaced straight lines. Thus only the slope linings need be special to each chart, and curved.

(v) The desired form of dimensioned chart being settled, the dimensionless progenitor must follow the same pattern, namely the variables V, s and d must be separated, and the chart must be in the form of dimensionless iso-s lines plotted against dimensionless V and d.

Of course if dimensioned charts were not required to be obtainable by tracing, other dimensionless plots would be possible, as the Authors have shown.

98. The title of the Paper under review refers to uniform flow, but the treatment is restricted to pipes of circular section flowing full. My method was derived with open-channels very much in mind, and hence my dimensionless discharge parameter is $4\pi(m/p)(Q/k\nu)$ (where p is the wetted perimeter, and m the hydraulic mean depth). Readers should be reminded that the charts in the Paper apply in general only to pipes of circular cross-section flowing full.

Mr V. L. Streeter, Professor of Hydraulics, University of Michigan

The Authors have made a thorough study of the various flow charts that have been proposed for solution of steady flow problems in pipes. The study is based entirely on the concept that roughness may be defined by a single relative roughness parameter, as given in the Nikuradse artificially roughened pipe experiments.

100. That relative roughness based on the ratio of height of roughness to diameter is not an adequate measure for all roughness types is given by Fig. 15 showing test results run at Illinois Institute of Technology* in 1949. The roughness lands have a height equal to their width, and are formed in aluminium pipes as helical threads. The pipe diameter, from top of lands is about 4·5 in. The three roughness heights are 0·028 in., 0·055 in., and 0·099 in.

101. The Colebrook–White test results and the Colebrook correlation on an equivalent sand grain roughness basis indicate the relative roughness concept is quite satisfactory for most commercial surfaces. Relative spacing can be important, however, and should be kept in mind when new types of roughened pipes are being correlated.

Mr R. W. Powell, Professor Emeritus of Engineering Mechanics, Ohio State University and President of the Rocky Mountain Hydraulic Laboratory

The Authors have kindly referred to me as one of the key workers in this field, but actually I have done little but carry on the work of Johnson whom they did mention, and Bretting[37] whom they did not. Johnson's paper was presented at the June 1934 meeting of the American Society of Mechanical Engineers, but its importance was not realized and it was not printed in the *Transactions* or in *Mechanical Engineering*, and so reached a very small audience. The Authors have overlooked one of my papers,[35] but it was simply an extension of Johnson's work.

103. In 1964 Ludovico Ivanissevich Machado and Jorge Enrique Mosconi presented a paper 'Ábaco para el cálculo hidráulico de cañerías lisas' at the I Congreso Latino Americano de Hidraulica held at Porto Alegre, Brasil, which gave a log log plotting of $(Sg)^{1/2}d^{3/2}/\nu$ for smooth pipes against Re, with a scale of values of SgQ^3/ν^5 along the curve. I do not know whether this was ever published.

104. I have not checked everything in the Authors' Paper, but it seems a comprehensive treatment of the whole subject, giving the practising engineer a choice of several methods. I have noticed the obvious misprint on page 496 where the references to Figs 5 and 6 are interchanged and believe that a ρ is omitted in the right hand member of equation (2).

105. At the middle of page 490 the Authors say that equation (8) is 'for lower values of Reynolds number'. It is true that the data which Prandtl used went up to a maximum Reynolds number of only 3 230 000, but that is hardly small. Scobey[38] gives the results of tests on the 18 ft dia. tunnel of the Ontario Power Co. at Niagara Falls, which was lined with monolithic concrete. 'After the whole pipe had been erected all defects—were removed by chipping and then the whole inside was rubbed

* Figure taken from V. L. Streeter, *Fluid Mechanics*, 1st ed., 1951, McGraw-Hill Book Co., 208.

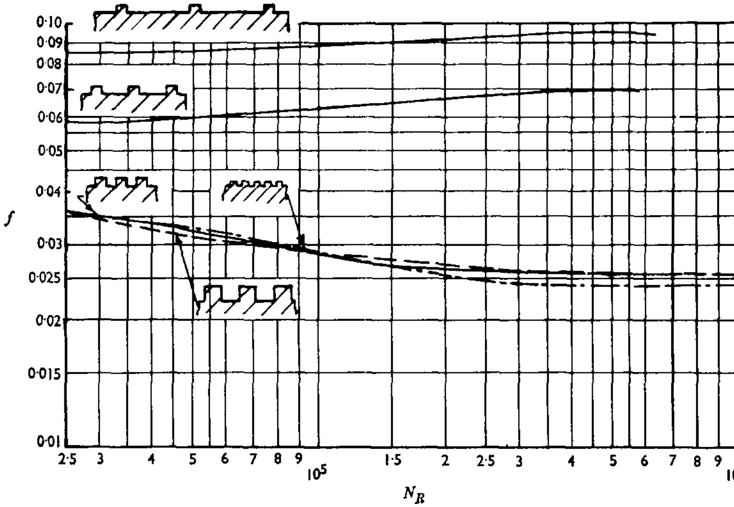

Fig. 15. Artificially roughened pipe data

down by hand with carborundum bricks.' In 1913 and 1914, 42 runs were made and from them a 'velocity–friction loss curve' developed. This goes up to a velocity of 20 ft/s, for which it gives a head loss of 2·397 ft/1000 ft. Unfortunately the temperature was not given. If it had been 61·7°F the Reynolds number would have been 30 300 000 and it would check Prandtl's equation exactly. A more probable value of 55°F gives $Re = 27\,500\,000$ and the head loss is 1·3% less than by Prandtl's equation.

106. In 1953 Burke[39] reported tests on three coated-steel penstocks at the San Gabriel Dam in California, with Reynolds numbers up to 38 070 000. From the given data I have computed f by Prandtl's formula for the 35 tests where Re was more than 16 000 000. In only 14 cases was the 'observed' f more than the formula gave, while in 21 cases it was less. The average algebraic discrepancy was −1·7% and the average absolute discrepancy was 3·5%. In discussing this Paper, Campbell[40] called attention to a test made on the outlet works conduit at Denison Dam[41] where the Reynolds number was 120 000 000. For this Prandtl's formula gives $f = 0{\cdot}00586$ while the test gave 0·0064 or 9·2% greater. These data would seem to indicate that the Ontario Power tunnel and the San Gabriel penstocks were hydraulically smooth and that at these very high Reynolds numbers Prandtl's formula is still quite satisfactory, giving losses less than 2% too high. It would seem that the Denison Dam outlet was not quite smooth and that this extremely high Reynolds number brought the flow into the transition range.

107. I cannot share the Authors' fear that further tests with Nikuradse's sand roughness where d/k is more than 1014 will show that equation (25a) no longer applies. It might be that the numerical constants may have to be revised slightly, but from the analogy with channel flow, I believe that the logarithmic form will still prove satisfactory.

Mr B. Vasudeva Rao, Senior Research Fellow, and **Professor K. Seetharamaiah,** Indian Institute of Science, Bangalore

The Authors have produced an excellent and critical review of the literature on friction factors. Some of the limitations of Colebrook's equation, which is a transition function, appear to be inherent in the modified charts too. Nevertheless the Authors' attempts to recast them seem to be most welcome in that a more rational approach is adopted.

109. While formulating the equation (10) for rough turbulent flow, the Authors used the formula of friction factor for smooth turbulent flow. Though the derivation is the same, the coefficients in equation (10) may vary slightly because of the change in the regime of flow near the boundary. It is true that the above equation is valid if the thickness of the laminar sub-layer is more than the height of roughness elements.

110. The information presented in Tables 1 and 2 provides valuable data about different types of flow problems investigated along with their dependability. This helps the designer to choose the relevant approach depending upon the solution sought.

111. In fact the Colebrook–White equation does not take the spacing of roughness elements into consideration which affects the friction factor as greatly as the height of roughness elements because of the development of different regimes of flow near the boundary.

112. The three different regimes of flow near the boundary as presented by Morris[43] are

(*a*) isolated roughness flow or semi-smooth turbulent flow;
(*b*) wake interference flow or hyperturbulent flow; and
(*c*) quasi-smooth flow or skimming flow.

113. In the case of isolated roughness flow, the wakes formed behind the roughness elements do not interfere with each other. The additional friction factor is due to the drag of the roughness elements. The friction factor due to this drag is to be added to smooth turbulent friction factor to get the actual friction factor. The formula for friction factor was given by Morris.[43]

114. One more interesting point is that in this type of isolated roughness flow, secondary flow develops and this has been reported by Jocobs and explained by Schultz–Grunov.[44] This type of secondary flow may lead to additional friction but this complicates the problem very much and it is difficult to isolate the individual effects.

115. In the case of wake interference flow the vortex generation and dissipation phenomena associated with each wake will interfere with those at the adjacent elements so that individual effects are not additive as in the case of isolated roughness flow.

116. From dimensional considerations the variables governing friction factor for pipes, can be grouped as

$$\frac{V}{\sqrt{gsd}} = \phi\left(\frac{k}{d}, \frac{\sqrt{gdsk}}{\nu}, \frac{\epsilon_l}{k}, \frac{\epsilon_p}{k}, \sigma_1, \sigma_2\right) \quad . \quad . \quad . \quad . \quad . \quad (51)$$

where ϵ_l and ϵ_p are longitudinal and peripheral spacing of roughness elements, the other parameters σ_1 and σ_2 are the shape and distribution of roughness elements, the remaining symbols are the same as those of the original Paper.

117. The exact form of the transition function depends upon the shape of the wake zones behind roughness elements and must be determined experimentally. Only the first three were taken into account in the Colebrook–Moody charts.

118. Quasi-smooth flow is produced if the spacing of the elements is so close that vortices formed within the elements cannot escape or shed into the main flow and so they are stable within the roughness elements. Here the additional friction factor is due to the additional energy consumed by vortices between the roughness elements.

119. Another interesting point in this type of flow is that the friction factor may decrease at high Reynolds numbers because of the formation of pseudo wall by the trapped vortices in which case the net surface of the contact boundary is reduced. As long as the trapped vortices do not diffuse into the main stream, they function as

a pseudo wall in reducing the net surface of the contact boundary. But when once they start diffusing into the main stream at very high Reynolds numbers, the diffusion phenomenon offers more resistance to the main flow and so the friction factor is high. This may be the reason why at some value of Reynolds number the curve of friction factor drops and then rises.

120. Lastly we feel that slightly increased efforts are needed to revise Colebrook's equation in light of the points discussed above. However, Morris was successful to some extent in this aspect.

Professor F. M. Tiller, Professor of Chemical Engineering, Director of Center for Study of Higher Education in Latin America

The Authors are to be congratulated on their Paper concerning the use of diagrams for frictional flow of fluids. Too often engineers are content to accept diagrams and formulae such as those used for frictional flow without seriously questioning either accuracy or convenience. The traditional chart involving a plot of the Fanning friction factor as a function of the Reynolds number at constant roughness factors is an example which has long dominated text books. The chart is attractive, easy to discuss, and rather limited as far as problem solving is concerned. Barr and Smith have admirably pointed out how a small amount of ingenuity can pay off in more convenient charts.

122. The Authors discuss the relevance of d/k as a means of extrapolating from one range to another. It is highly probable that a dimensional analysis of the flow problem is considerably oversimplified when one linear dimension is used to characterize roughness. It is further probable that a more sophisticated analysis would indicate that factors used to represent roughness occur in combinations other than simply d/k, thereby making extrapolation based upon d/k alone somewhat suspect.

123. A systematic approach to developing groups in dimensional analysis assists in analysing which groups may be of greatest value for problem solution. For example in the flow problem where the six variables (Q, d, s, μ, ρ, k) are involved, there are three independent groups which can be found by usual techniques. Once the first three groups have been generated, others can be compounded by systematically combining the original groups to eliminate variables.

124. Assuming flow in circular pipe with $V=4Q/\pi D^2$, a simple chart showing basic groups which can be compounded from the first three follows:

Group	Q	d	s	ρ	μ	k
(1) $DV\rho/\mu=4Q\rho/\pi D\mu$	×	×		×	×	
(2) $Sgd^2/V^2=\pi^2 sgd^5/16Q^2$	×	×	×			
(3) k/d		×				×
(4) $(1)/(3)=4\rho Q/\pi k\mu$	×			×	×	×
(5) $(2)\times(3)^5=\pi^2 sgk^5/16Q^2$	×		×			×
(6) $(1)^2\times(2)=sgd^3\rho^2/\mu^2$		×	×	×	×	
(7) $(1)^2\times(2)\times(3)^3=sgk^3\rho^2/\mu^2$			×	×	×	×

Other groups can be added to the list. Any three groups can be chosen provided they contain all six variables. Groups (1), (2) and (6) could not be used as k would be absent. If it is desired to construct a graph giving direct solutions for each of Q, d, s, three groups must be chosen such that each contains only one of the three variables. The groups (3), (4), (7) will give the desired direct solutions.[45] Those groups could have been obtained through the usual dimensional analysis by assigning independent exponents to Q, d, sg or correspondingly V, D, sg.

Mr E. O. Macagno

This discussion will not bear with equal emphasis on the different questions treated by the Authors. I have little to offer to the immediate problem of constructing the optimal diagram for turbulent uniform flow; it is rather with comments within the areas of dimensional analysis and similitude, effects of roughness and cross-sectional shape on resistance to flow, and other means of calculation than the graphical that I feel qualified to contribute.

126. The dimensionless group that we call the Reynolds number was first introduced by Lord Rayleigh[46] in the inverted form ν/Vd; this was actually done in a relation in which a Strouhal number also appeared. One can find in some of Stokes' writings an anticipation of the arguments of dimensions and of similitude for viscous flow, but Reynolds should be credited with the priority (several decades before Prandtl) in clearly confronting the roles of viscous and inertial forces to arrive at the formulation of the dimensionless number that now bears his name. Reynolds considered the equations of motion of viscous fluids, and compared the relative value of the viscous and accelerative terms.[47] He also expressed—although only in passing—the notion that the quantity $\mu/\rho d$ and the mean velocity are measures of the values of those terms. However, Reynolds did not elaborate on the meaning of $V=\mu/\rho d=\nu/d$, which is considered by the Authors as a velocity with a special significance. The introduction of 'dynamic velocities' is certainly interesting, but, when extended to all aspects of fluid flow, it seems to lead to somewhat artificial interpretations. When one looks upon $\sqrt{(gsk)}$ as a dynamic velocity, it seems hard to associate it with the velocity that a representative mass would attain when acted upon by a given force over a certain distance. Should one consider in this case a fluid mass of the order ρk^3, and a force of the order $\gamma s k^3$? If so, what do they mean? I could not find a meaningful interpretation for $\sqrt{(gsk)}$ and the associated forces. A force associated to $k^3(dp/dl)$ (assumed to correspond to $\gamma s k^3$), is due to a pressure field with gradient dp/dl acting on the fluid enclosed in the volume k^3; but this force does not seem to bear any plausible physical relation to the force applied on an obstacle with linear size of order k.

127. Perhaps a general discussion is still best formulated with the methods of dynamic similitude or of dimensional analysis. For flow in circular pipes, this would lead to statements like

$$dp/dl = f_1(d, V, \rho, \mu, k) \quad \ldots \ldots \quad (52)$$

or

$$\tau = f_2(d, V, \rho, \mu, k) \quad \ldots \ldots \quad (53)$$

in which τ represents the wall shearing stress. Equation (52) can also be expressed as

$$F(dp/dl, d, V, \rho, \mu, k) = 0 \quad \ldots \ldots \quad (54)$$

The dimensional matrix of these six variables being of order three, application of Vaschy's theorem[48] would show that the number of dimensionless groups must be three ($6-3=3$). As is well known, this can be done in many different ways; the standard procedure of selecting a set of repeating variables, and forming groups with the minus-one power of each of the remaining variables, would yield in the present case 16 forms of the dimensionless functional relationship corresponding to equation (54). This is illustrated in Fig. 16 in which the points for d and k are not joined with a segment because there is no solution for groups like $\pi=\rho^x d^y k^z V^1$. The repeating-variables procedure yields the following different dimensionless π-groups:

$$\pi_1 = \frac{(dp/dl)d}{\rho V^2} = E_d, \qquad \pi_2 = \frac{(dp/dl)k}{\rho V^2} = E_k \qquad \text{(Euler)}$$

$$\pi_3 = \frac{(dp/dl)d^2}{\mu V} = P_d, \qquad \pi_4 = \frac{(dp/dl)k^2}{\mu V} = P_k \qquad \text{(Poiseuille)}$$

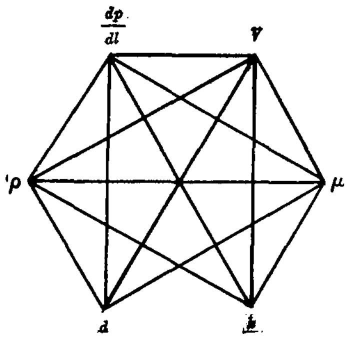

Fig. 16

$$\pi_5 = \frac{\rho(dp/dl)d^3}{\mu^2} = K_d, \quad \pi_6 = \frac{\rho(dp/dl)k^3}{\mu^2} = K_k \quad \text{(Kármán)}$$

$$\pi_7 = \frac{\rho Vd}{\mu} = R_d, \quad \pi_8 = \frac{\rho Vk}{\mu} = R_k \quad \text{(Reynolds)}$$

$$\pi_9 = \frac{\mu(dp/dl)}{\rho^2 V^3}, \quad \pi_{10} = \frac{k}{d}$$

Of course, there are relations among these groups, some of which are:

$$P_d E_d^{-1} = R_d, \quad (P_k/P_d)^{-1/2} = k/d, \quad P_d R_d = K_d$$

There is a natural question that arises from these considerations: is there a most convenient choice of the form of the dimensionless relationship to be used in constructing diagrams for turbulent flow in pipes? The Authors have come very close to a very convenient one, if it is not the optimal; but a systematic analysis of all the possibilities would seem in order. I am aware of the large number of possible forms because compounding seems to yield an almost limitless number of forms, but many of the forms would be discarded in a first scanning process based on practical reasons. It should also be possible to establish guiding lines to keep the compounding within reasonable limits.

128. In my opinion, it is worth considering the alternative of using numerical calculations with either the currently accepted formulae or simpler interpolatory formulae which lend themselves better to computations.[49,50] When a large number of calculations are necessary, one can presume that a digital computer will be available; in such a case, rather simple programs utilizing standard techniques of numerical analysis can be used for all types of problems. In this approach, modifications and additions can be made as soon as more accurate and comprehensive formulae are established. In the case of the diagrams, it would be necessary to discard the old ones and prepare, print, and distribute new ones, while a computer program can quickly and easily be updated. The same disadvantage is shared by nomographic devices.[51] I have in view the general problem of turbulent uniform flow including, at least, shapes of the cross section other than the circular, as well as non-Newtonian fluids[52] when possible. In addition, a certain amount of information becoming available on the role of the Froude number in open channel flow should be incorporated.[53] It is certainly desirable to have a general method capable of utilizing all the available information on uniform flow.

129. Even if we confine ourselves to the problem of representing the resistance in circular conduits, the influence of roughness, as the Authors point out rightly, needs further research. The Authors express the opinion that the Colebrook–White formula may have to be replaced by a more comprehensive expression, and that the

standard for roughness should be revised. The latter recommendation has also been put forward recently by other hydraulicians.[54,55] The recent and current research efforts in the field of wall roughness should lead to better established transition functions.[17] The already numerous studies of the resistance due to artificially produced regular and irregular rough walls should crystalize soon into a more accurate representation of roughness than that presently used. It is true that the question of correlating statistical parameters of the shape and distribution of roughness elements to the resistance coefficients is still an open question, but important progress is being made in this direction.[56,57] It is very likely that the new expressions will have to include at least one more parameter to represent accurately the effects of different types of roughness.

130. Without repeating quantitative arguments presented elsewhere,[58,59] a simple qualitative treatment of the Reynolds' equations of motion for uniform turbulent flow may help to understand and emphasize the role of the cross-sectional shape on the law for the discharge, or on the law for the resistance. Time averaging and a first integration of the Reynolds' differential equation yield, for the case of a perfectly smooth circular pipe, the following expression for the mean velocity $\bar{u}$:

$$\bar{u} = \frac{1}{4\mu}\frac{d\bar{p}}{dl}\left(\frac{d^2}{4}-r^2\right)-\frac{\rho}{\mu}\int_r^{d/2}\overline{u'w'}\,dr \quad \ldots\ldots \quad (55)$$

where $\bar{p}$ is the mean pressure, and u' and w' are respectively the longitudinal and the transverse velocity fluctuations.[60] A further integration of equation (55) gives

$$Q = \frac{\pi d^2}{4}V = \frac{1}{4\mu}\frac{d\bar{p}}{dl}\int_0^{d/2}2\pi r\left(\frac{d^2}{4}-r^2\right)dr-\frac{\rho}{\mu}\int_0^{d/2}2\pi r dr\int_0^{d/2}\overline{u'w'}\,dr \quad (56)$$

or

$$V = \frac{d^2}{32}\frac{d\bar{p}}{dl}-\frac{8\rho V^2 d}{\mu}\int_0^{1/2}\frac{r}{d}d\left(\frac{r}{d}\right)\int_{r/d}^{1/2}\frac{\overline{u'w'}}{V^2}d\left(\frac{r}{d}\right) \quad \ldots \quad (57)$$

from which

$$\frac{32\mu V}{\left(\frac{d\bar{p}}{dl}\right)d^2} = 1-\frac{\rho V^2}{\left(\frac{d\bar{p}}{dl}\right)d}I \quad \ldots\ldots\ldots \quad (58)$$

or

$$32P_d^{-1} = 1-E_d^{-1}I \quad \ldots\ldots\ldots \quad (59)$$

are obtained with obvious notation. The symbol I represents a dimensionless function which is not yet exactly known, although reasonably good approximations have been provided by phenomenological theories of turbulent flow. If the above considerations are extended to conduits and channels of different cross-sectional shapes, one must certainly expect that the corresponding function I depends on the shape of the domain of integration.[61] In addition, there are good reasons to expect that the integrand (in equation (55) and following) will be different for different shapes of the cross section. Experiments as well as calculations show that the influence of the cross-sectional shape is stronger in laminar flow than in turbulent flow, but the general trend is usually the same.[59] It is also known that the hydraulic radius cannot take completely into account the dependence of the resistance coefficient on the shape of the cross section.[62] Making use of the logarithmic velocity distribution, it is possible to analyse with a satisfactory degree of accuracy the effect of the cross-sectional shape on the resistance; in this way, it has been shown[59,63] that in the logarithmic expression for the resistance coefficient, both the coefficient of the logarithm and the additive term are functions of the shape of the cross section. The additive term can, if desired, be incorporated as a factor of the hydraulic radius.[59]

131. I could not agree more with the Authors' conclusions (*a*) and (*b*). Whoever tries to organize the available data on turbulent flow cannot fail to recognize that in

spite of all the progress made our knowledge of the subject is still quite incomplete. More investigations in depth of the effects of wall roughness, cross-sectional shape, and free surface, are highly desirable.

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In §§ 19, 21 and 22 the Authors seem to disparage the logarithmic velocity profile (and resistance law) to the point at which it appears to be no better than any other relation determined on a 'best-fit' basis. The deficiencies of the logarithmic velocity profile and the need for slight correction of the coefficients in the logarithmic resistance law had long been recognized; however, as the logarithmic relations had some theoretical basis they had been accepted in place of other, wholly empirical, equations. Von Kármán's part-logarithmic expression, which took into account the shearing stress variation neglected by Prandtl, had been shown by Ross[64] to be accurate, but its complexity had ruled against its adoption for general engineering use. However, both these expressions are of the same 'family' and I feel the Authors are not justified in dismissing so lightly the simple Prandtl logarithmic relations.

133. I also consider it unfortunate that in § 22 the Authors have drawn further attention to the article by Levin,[13] in which evidence was presented which appeared to show that large conduits existed which were 'smoother than smooth'. I have checked two of the sources[65,66] quoted in the article and in one case (the Apalachian tunnel system) it seemed that Levin had overlooked a second paper[67] which corrected the data originally presented. All the revised results plotted well above the Prandtl smooth turbulent flow curve.

134. When using field data, it should always be remembered that the resistance coefficient is a function of the head, the square of the discharge and the fifth power of the size, and inaccuracies in the measurement of these quantities will inevitably detract from the dependability of the results based upon them. In the second source I have checked (the USBR records),[65] it appears that current meters, colour movement and venturi meters have been used for flow measurement. A comparison of these and other commonly used field methods of flow measurement was made recently by skilled personnel under good conditions at Finlarig[68] and the results showed a scatter of 2%. It is improbable that the USBR inaccuracies are less than this. No mention was made of the determination of the size and shape of the conduits and it might be significant that it was only the concrete pipe results which deviated below the accepted smooth line. Perhaps the steel pipes were more consistent dimensionally, or easier to measure accurately.

135. I think that properly run field tests are very important and of the greatest value but they cannot compare in precision with carefully conducted laboratory experiment. Many investigators using completely smooth pipes have shown that the Prandtl logarithmic smooth turbulent law is valid and does represent the lower limit of resistance (Schlichting[69]). It can also be argued from Nikuradse's tests on pipes of varying roughness that if at a certain Reynolds number a pipe with a large roughness gives the same resistance as one with less roughness, then roughness has ceased to influence the flow and the same resistance will be offered by completely smooth pipes. This is borne out by experiment. There is certainly no justification at present for supporting Levin's call for a new smooth turbulent equation, least of all on the basis of field tests of suspect accuracy.

136. I agree with one of the Authors' points in §§ 50–51 that errors will occur in the determination of roughness size if the rough turbulent equation is inaccurate outside the limits of Nikuradse's tests. However, it seems that the Authors are also questioning the basic principle of geometric similarity. I might have misunderstood their argument but I see little to choose between different roughness sizes (of a given type) in one pipe, and one roughness size (of the same type) in pipes of different sizes. Perhaps the Authors would amplify their argument.

137. I appreciate that the Authors are principally concerned to develop their new

correlation and were pressed for space, but I feel it is a pity they have omitted all mention of the inadequacies of the 'equivalent sand' definition of roughness. The proper description of rough surfaces in terms which enable their resistance to flow to be predicted at a given flow is one of the most challenging and urgent of hydraulic problems, as noted by Clauser,[70] and the current lack of understanding about this creates difficulties for every user of the logarithmic equations (Prandtl or Colebrook–White), or the various charts based upon them. A proper description has to take account not only of the height of the roughening element, but also of its shape, the pattern of the elements on the surface and their spacing. Further complications are created if the rough surface contains a range of sizes.

138. Another important point is that the rough surfaces on which the Colebrook–White and Nikuradse transitions are based are only two of a large family, ranging from a virtually smooth surface with isolated roughness projections to a surface consisting of deep and regular undulations. The resistance coefficient transitions of such surfaces are markedly different (see Robertson[17]). Thus it cannot be too strongly emphasized that the Authors' charts, like those before, are suitable only for the typically 'isolated roughness' surface found in steel and cast-iron pipes. Since such surfaces have higher values of resistance than most other surfaces, any errors in size calculation would be on the safe side for capacity, but the wrong side economically.

139. I am glad that the Authors have stressed the severe limitations of the data now available on the resistance of rough pipes. There is a clear need for comprehensive tests to confirm and extend Nikuradse's original data. A standard form of roughness element will need to be adopted and I should like to know how the Authors view Rouse's recent proposal[71] to use a cube.

Dr Barr and Mr Smith

First we thank the contributors to the discussion for their expressions of interest, for the valuable increase in the scope of the survey which has been provided and for raising many points which require further elucidation. The reply is under two general headings followed by consideration of the matter of the contributors in turn.

141. *Scope of paper*. There was recognition of the need for separate treatments of the possible forms of resistance diagrams and of the resistance laws which should be shown thereon. The present Paper has dealt with the first topic, with restriction to certain fundamentals in respect of the second. In particular, we thought it desirable to show why we did not consider the logarithmic form of resistance law to be a necessary corollary of the logarithmic nature of the velocity distribution expression in turbulent pipe flow. Thus many references to previous work which, in fact, existed in drafts on the second topic were not included. Examples were the well known papers on the effect of various types of roughness element by Morris,[31,42,43] to which several contributors referred, and the pioneering questionings of the logarithmic forms of resistance law by Williamson[72] and by Blench.[73]

142. *Linear functional equations*. The recent introduction of linear functional equations,[74,75] cannot but influence any further search for forms of resistance diagram. Using the notation of the Paper, the equation for pipe flow is

$$\phi\left[d, k, \underbrace{Q^{2/5}/(sg)^{1/5},\ Q/\nu,\ \nu^{2/3}/(sg)^{1/3}}_{\text{any two from}}\right] = 0 \quad . \quad . \quad . \quad . \quad (60)$$

The basis of such equations was the isolation of the proportionality of length (to the first power) in the non-dimensional numbers which could be formed as velocity ratios and where the velocities were dynamic velocities in respect of the actions of active force, and velocity proportionalities in the case boundary actions. Thus linear proportionalities are formed to cover pairs of actions and in the case of pipe flow with three actions, there are three linear proportionalities, one of which is redundant over and above the necessary inclusion of a single redundancy in respect of either an action or an actual length. The linear functional equation is, of course, completed by the necessary lengths for definition of the geometry of the system.

143. Given equation (60), the non-dimensional groups for all the recognized forms of diagram can be selected directly, although the powers of the groups may be unfamiliar. If the problem of determination of unknown d is posed, it is found that there are many potential solutions and variations of solution. The following alternatives are open.

(i) To explore the possibilities without regard for recognizability.

(ii) To take the well recognized arrangement, which is basically

$$\phi\left\{\left|\begin{matrix}\dfrac{V}{(sgd)^{1/2}},\\ \text{or}\\ \dfrac{Q}{(sg)^{1/2}d^{5/2}}\end{matrix}\right|,\left|\begin{matrix}\dfrac{V}{\nu/d}\\ \text{or}\\ \dfrac{Q}{\nu d}\end{matrix}\right|,\frac{d}{K}\right\}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (61)$$

with the first group as ordinate, the second as abscissa, and to search for additional linings such as given in New Diagram V.

(iii) To accept that recognizability remains with either $(sg)^{1/2}d^{3/2}/\nu$ or $Q(sg)^{1/3}/\nu^{5/3}$ substituted as main abscissa, and again search for additional linings.

144. Linear proportionalities and linear functional equations are emphasized because of the aid given to systematic choice as outlined above which effectively supersedes the use of the velocity form; because it now appears that the device would be helpful in any future examination of resistance laws and of the effect of different roughness forms; and because the linear form of functional equation will be used to substantiate the logic of the velocity form which was used in the Paper.

145. With reference to the contribution from Messrs Stuckey and O'Garra, if the values of two lengths, such as d and k, completely specify the geometry of the various pipes in a test series, the purposes of correlation will be served no matter how d and k are defined provided that they can actually be measured, and that the definition is consistent. The evidence provided by Nikuradse shows that in the circumstances of his tests, the volumetric definition of diameter brought results for smooth turbulent flow in a rough pipe to be more or less along the line of the results for turbulent flow in a smooth pipe. It seems probable that this superimposition becomes less likely for any constant definition of diameter; the more recessed is the roughness form and this applies especially to the range of larger k/d values. It appears rather early to reject the possibility of direct measurement of diameter and of roughness at a cross section. However, we agree that practical circumstances become dominant in those cases where there is variation of these dimensions with longitudinal position. We nevertheless stress the difference between using the Colebrook–White equation in the form of equation (28a) and of equations (28c) and (28d). Our criticism concerns the assumptions implicit in its use in conventional form, and we have no criticism of its value in defining the form of the transition routes commonly obtaining in flow in commercial pipes. The Paper is much concerned with procedures depending on absolute values of k.

146. In reply to Professor Irmay, there is a parallel between the development of the concepts of dynamic velocities and of stress velocities. Velocity groupings were used in discussions early in 1965[76,77] and the concept of dynamic velocity defined in October 1965.[78] It is interesting that Bogardi[79-81] took up and developed the use of dynamic velocities starting from the discussions mentioned,[76-77] and with particular reference to sediment transportation. In fact, the progenitor of the dynamic velocity concept was Keulegan's concept of densimetric velocity.[82] Thereafter there was a gradual enlargement of scope and increase in variety leading eventually to the

generalization of the method of synthesis including the use of linear $(n+1)$ term equations.[75] Then it was found that White[83] had provided a lead in respect of the linear proportionality concept in 1947, which had not been taken up or developed. So there have been many potential clues to the development of the method of synthesis for incompressible viscous flow, but, as already mentioned, it was Keulegan's lead that was acted upon.

147. The use of a friction stress velocity in explicit form in correlations, precludes direct solutions to the unknown d problem, and it is also true that sediment transportation work has been bedevilled by this device. We have emphasized dimensional homogeneity in the $(n+1)$ term equation, but stress that dynamic velocities should not be mixed with less complex variables which may be components of the velocities.

148. With reference to Dr Engel's comments, we regret any impression of lack of realization of the fundamental importance of similitude arguments. Many of the important issues raised by him have been covered in a subsequent paper,[74] and for reasons of space it is expedient to refer first to this in its entirety. It is shown that the initial grouping of variables in an $(n+1)$ term functional equation can be in the form contained in the governing equations; in the case of the Navier–Stokes equations in acceleration or force per unit mass form. This is at one end of the scale; at the other end is the linear term equation. But all the consequent forms of non-dimensional state the same conditions for similarity; the variation is merely in how these conditions are defined. There is a distinction between analytical capability and physical significance. Thus we agree that in any given circumstance there is only one law of dynamic similarity, but also insist that there may be many ways of writing it.

149. We are fully in agreement with Dr Engel regarding the dangers of isolation of non-dimensional groups out of the context of their complete functional equation. But we would not consider such a step and therefore cannot admit the existence of 'genuine' groups as opposed to non-genuine groups.

150. It so happens that the use of dynamic times was also particularly appropriate to the circumstances of experiments described in two recent papers.[84,85]

151. Dr Engel is concerned at the practice of the multiplying of two non-dimensional groups and the subsequent elimination of one of these without very detailed examination of the case but this most valid objection should not hold against the multiplication of groups provided there is still independent retention of two groups. In fact, any form of complete functional equation can be obtained from any other form by successive compounding. When compared with the examination of the potentialities from the position of one or two $(n+1)$ term functional equations, including a linear term form, compounding has the same deficiency as has conventional dimensional analysis. It can give any arrangement, including the best one for the purpose in hand, but so often, in practice, it does not.

152. In this connexion, we would argue that Dr Engel's equation (42) contains a superfluous term since any one is merely a product of the other two. In certain cases it is preferable to separate the variables of velocity V and diameter d, and this leads automatically to the rejection of the Reynolds number.

153. A treatment of silt transport correlation has been given elsewhere.[75] Dr Engel's strictures on similitude in heat transfer will be borne in mind when this stage is reached in the extension of the scope of the method of synthesis. We have perhaps been as guilty as others in the past of coining names for non-dimensional numbers with the best of intentions regarding the clarification of their significance. It might in the long run aid true understanding, if neither names nor notations were used for these groups but the complete grouping of variables were always given. There are many examples of faulty argument being supported by implications that unique significance is possessed by certain named groups.

154. We agree with Professor Garde and Mr RangaRaju that accuracy of interpolation is important, and it is emphasized that the diagrams in the Paper

were in outline form. Diagrams for actual use may be prepared later. The alternative approach made by the discussors is complementary to the use of diagrams which are essentially capable of receiving the first plots of new data, and it may be that equation (60) can lead to yet better forms. However, with increasing realization of new forms of functional equation on which an approximate law might be based, there is also increasing difficulty in making a survey and selecting the best. We are grateful for the discussors' survey and hope that the combined treatments may aid future workers.

155. Mr Putnam recognizes that the onset of digital computer techniques including direct plotting allows the preparation and exploration of new forms of data plot with a facility which was not available only a few years ago. Computers can also be used for direct solutions, but there will always be advantage in having the diagram solution available in the most convenient form.

156. The Authors agree with Mr Ackers that V_λ is equivalent to V_*, but consider that V_* has unfortunate connotations, because it has so often been used together with more basic variables, whereas V_λ has been used, together with other velocity measures, to emphasize the dimensional homogeneity of $(n+1)$ term equations.

157. The basis of similitude arguments is comparison between geometrically similar systems, thus it is immaterial in which direction lies the length chosen as representative. This is demonstrated clearly by equation (60). It would be appropriate to adopt the length of tube from an entry in an analysis of entry conditions, and this length could be entered in a velocity matrix or in a linear functional equation. But the use of the piezometric gradient, s, requires the prior assumption of established flow, and it is then appropriate to adopt d as representative length, i.e. as diameter and length of a representative element.

158. We are in general agreement with § 93, but there still remains the tendency not to recognize that the Colebrook-White equation is essentially a delineation of a transition route. Although the second topic of study mentioned earlier is still underway, we believe that examination of past results may yet result in a reappraisal of the significance of the Colebrook–White equation, even without new experimental work.

159. Recognizability does appear to affect the degree of adoption of new diagram arrangements, regrettable though this may be. Regarding the determination of k from test data using charts, we agree that this should not be done, but the determination of acceptable roughness is a design problem, particularly in open channel flow.

160. We have already stated our appreciation of Mr Ackers's work. We have tried to show the potential advantages of an alternative approach, where dimensional working is possible without the existence of a separate diagram for each value of k. Following from this, single supplementary diagrams can be made to allow direct solutions for unknown size of standard forms of channel. For example, Fig. 17 shows a diagram for trapezoidal channels. The proportions, y/b and a/b, are chosen and Q^* is given where Q^* is the factor by which the desired channel flow must be multiplied to give the flow in the equivalent pipe. This allows direct solution for unknown diameter of equivalent pipe in either dimensional or non-dimensional working, and the return to channel dimensions is by a simple computation.

161. We accept the limitation mentioned by Professor Streeter that our present study was confined to cases where a single roughness size can define the whole roughness form. In future studies and in the reappraisals of past results which may be combined to allow further progress, the whole geometry of artificial roughness must be included in correlation, and again equation (60) can be readily modified to this end.

162. The demonstration of deficiencies in coverage of past work in the Paper is most valuable, and Professor Powell is also thanked for noting misprints. It is

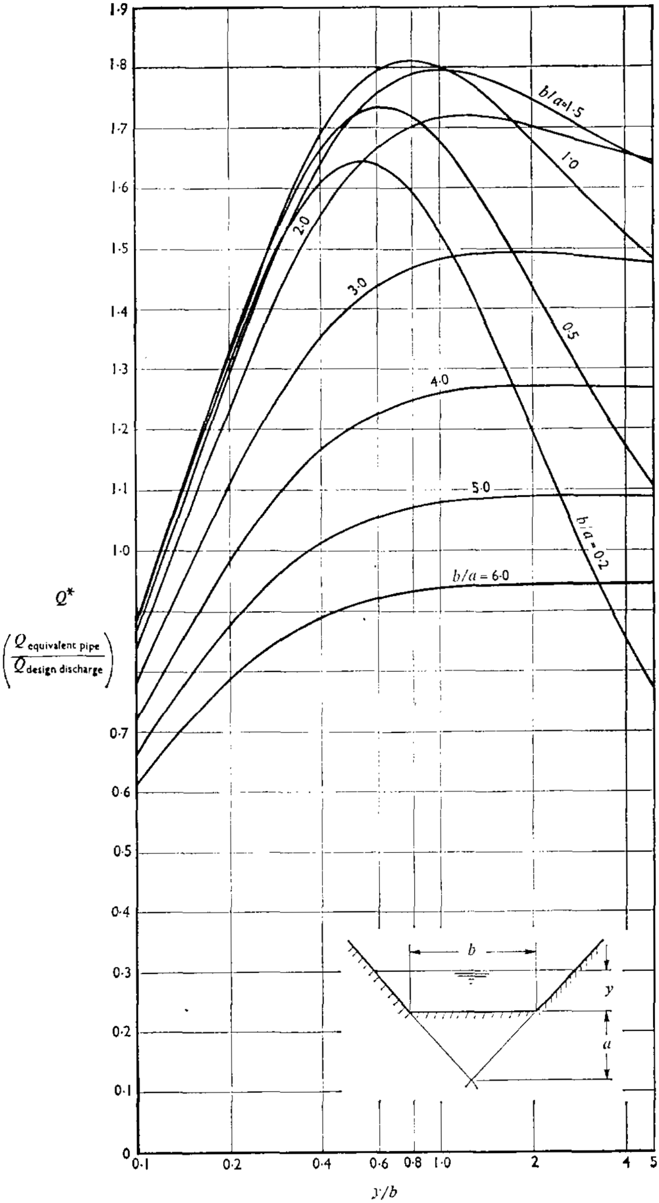

Fig. 17. Outline of diagram for obtaining discharge in equivalent pipe given discharge in trapezoidal channel of selected y/b and b/a ratios

certainly the case that Johnson's work[5] should have been more widely published and indeed the Authors have only recently obtained direct access to it.

163. Regarding our statement that equation (8) was 'for lower values of Reynolds number', this is in fact Prandtl's[1] opinion, and he gives a second formula

$$1/\sqrt{\lambda} = 1{\cdot}95 \log (R_e\sqrt{\lambda}) - 0{\cdot}55 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8a)$$

'which is more accurate for large values of Reynolds number'.

164. The survey of results at higher values of Reynolds number is most useful and we hope to make use of the sources in the future. In the Paper we were mainly critical of the present extrapolative practice regarding the logarithmic rough pipe formula. It is agreed that successive adjustments of constants will allow coverage of any practical range, but the same can be said for exponential formulae.

165. Mr Vasudeva Rao and Professor K. Seetharamaiah are thanked for their valuable remarks, which will influence our future work. At the same time, we have wondered whether too much emphasis can be placed on continuing generation of turbulence by roughness. Once the turbulence of the main stream is triggered, it is self-preserving above the critical Reynolds number, no matter how smooth the pipe. Turbulence generated in the core must be considered, and it is a fact that, while resistance of a given mean velocity is affected by roughness size, it is affected much more by conduit size.

166. Professor Tiller has joined other contributors in noting the shortcomings of assuming the roughness to be uniquely and sufficiently described by a single linear dimension. It is possible that refinement of resistance laws with respect to roughness geometry will find realization not in a more sophisticated form of diagram but by the development of digital computer procedures to link established resistance laws with theoretical or semi-empirical correlations between friction coefficients and other boundary or flow parameters.

167. One interesting result of our study is the demonstration of the number of workers who quite independently have derived the set of parameters which separate the variables Q, d and s. Professor Tiller's method of compounding may not altogether find favour with some of the contributors to this discussion. However, when linked with conventional dimensional analysis or with the use of similitude concepts based on $(n+1)$ force term equations, it is in many ways superior to the use of dimensional analysis alone. It does appear that the use of the $(n+1)$ linear term equation (60) is even more direct.

168. Although publication was long delayed, Professor Tiller and his colleagues in 1948 made use of the parameters of equation (30), the first example of this device of which we are aware.

169. It may be that Professor Macagno's and our positions are closer than at first appears. If kinematic similarity with velocity defined by V is presupposed in an accelerative system, of size defined by l, affected by viscous force, the velocity quantity ν/l is obtained as a viscous dynamic velocity and if the procedure is carried through for inertial force, V is obtained as a dynamic velocity for inertial force. This is compatible with Reynolds' passing hypothesis. Dynamic velocities are not so much velocities with special significance as velocity measures with special significance, providing a basis of comparison between systems.

170. To us, $(sgk)^{1/2}$ has as much or as little significance as $(sgd)^{1/2}$. After all, similarity is concerned with geometrically similar systems, and the criterion of usefulness of a measure is not its absolute value. We have stressed the value of a dynamic velocity as a measure provided it be applied consistently. An alternative substantiation is given by the concept of the linear functional equation, where the actual lengths are isolated from the linear proportionalities which arise from the ratios of pairs of velocity (or other homogeneous) measures of actions. Then the initial step involves the assumption of the need for only one actual length measure.

For systems involving the need for further definition of geometry, a second and a third length, and so on, can be added, but the conditions for dynamical similarity will always include geometrical similarity.

171. Our reply to Dr Engel is relevant here, and equation (60), in amended form, appears to be the most explicit manner by which equation (52) can lead to all the possible varieties of complete non-dimensional equations which contain the non-dimensional groups listed by Professor Macagno and the other relevant forms. There is, however, a proviso here: the Poiseuille form of non-dimensional group is rather a special case in that it cannot in itself pertain to accelerative (turbulent) flow and is in fact the basis of the functional form of laminar flow equation. A relationship containing the Poiseuille form of group is best obtained from a force term dimensionally homogeneous equation. Of course, a non-dimensional equation containing a Poiseuille form of group can cover the accelerative flow case and equation (58), apart from the term I, is a good example of this form of equation and of how the terms may be predicted.

172. Professor Macagno raised the question of using numerical calculations incorporated in a digital computer program as an alternative to diagrams. We are keenly aware that for the purpose of design calculations, the design chart and nomograph are already being replaced by the remote time-sharing interrogating typewriter.[86]

173. It can be shown that of the variables involved in the functional relation

$$\phi(Q, \text{section, depth}, s, \text{coeff.}, \nu) = 0 \quad . \; . \; . \; . \; . \quad (62)$$

only the discharge may be expressed explicitly for all possible resistance laws. It therefore follows that the most flexible way of introducing a resistance law into any computer program is to define a series of function or real procedure segments having an identical parameter list and which evaluate the normal discharge for a particular formula. Any of the other variables which are defined implicitly can be found by iterating on a function of the explicit quantity Q. For example, the unknown s may be found by successive approximations for the solution of the equation

$$Q(A, P, \text{coeff.}, s, g) - Q \text{ spec.} = 0 \quad . \; . \; . \; . \; . \quad (63)$$

in which the area, wetted perimeter, resistance coefficient, gravitational acceleration and specified discharge are known and constant.

174. An extension of this technique is to substitute for the coefficient of resistance a further real procedure or function which interpolates the value of coefficient as defined by some empirical curve. In this way shape factors, surface wave Froude number effects or any transitional route in the Blasius–Stanton diagram may be introduced into the computation at the whim of the user. Perhaps most important in such a system is that the choice of resistance law may be left to the user and not built into the program at its inception.

175. The transformation of formulae relating to full-pipe flow to equivalent formulae for open channels by means of the relation is undoubtedly an over-simplification. The significance of shape effects has been of interest to us for some time,[87] and Professor Macagno demonstrated how the integration of the velocity distribution law yields a constant of integration which is a measure of the mean velocity defect. The corresponding quantity in laminar flow is the Boussinesq number. A similar approach has been made by Sundin[88] and a more recent publication on the subject by Bock has given weight to this treatment[89] by giving an equivalent hydraulic mean depth as a function of relative depth for various cross-sectional shapes.

176. Dr Wright has concluded that we regard both the velocity and resistance logarithmic laws to be entirely empirical. That this impression has been given is due in part to the necessary condensation of the original argument but we recognize and accept the theoretical basis for a logarithmic velocity distribution in turbulent flow. What has not been satisfactorily demonstrated by theoretical

argument, however, is that the resistance law must necessarily be of a similar functional form and this view has apparently been based on von Kármán's assumption that only one linear dimension, i.e. the distance y, could be relevant to the similarity reasoning relating to velocity distribution in a boundary layer of unlimited thickness. The concept of established flow implicitly recognizes the existence of a second dimension, the conduit size d, necessary to define the resistance. Inclusion of both y and d in the functional relation immediately opens the way for a quite arbitrary functional form ϕ_b in equation (22) although it is accepted that ϕ_d is logarithmic.

177. With reference to 'smoother than smooth' test results, Dr Wright's plea for accuracy in and careful scrutiny of field test results is laudable. One of the instances in our minds was in fact a laboratory test of pitch-fibre pipes reported by Ackers[90] who suggested that the low resistance implied a coefficient of 2·04 rather than 2·0 in equation (26a). If no sound justification exists for seeking a new smooth

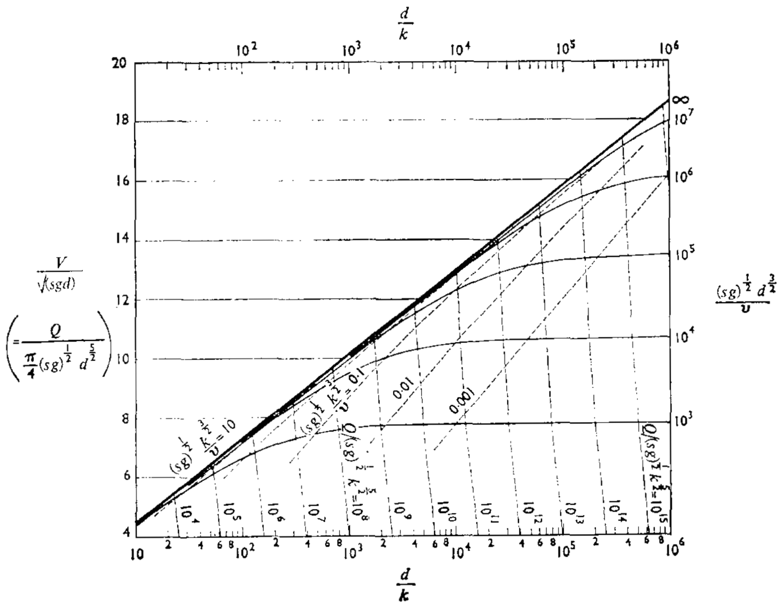

Fig. 18. New Diagram VI for solution of rough pipe problems

turbulent equation, it is at least desirable to maintain an open mind with respect to the interpretation of results which do not agree exactly with currently accepted laws.

178. The definition of equivalent sand roughness is a problem recognized by many contributors including Dr Wright. We are of the opinion that in any proposed series of tests to examine roughness height in relation to the terminal value of $V/(sgd)^{1/2}$, it is very desirable that a particular surface be examined in different conditions of relative roughness in order that the uncertainty of surface measurement be eliminated. If the logarithmic rough tubulent law is correct, a semi-log plot of tests results conducted in this way should give the same slope irrespective of the absolute value of k used to define the surface. We did not, of course, question the basic principle of geometrical similarity. We showed that present practice could be a contradiction of this principle where k was not directly measured, if the law used to assess k was not completely accurate.

179. Rouse's suggestion of using a cube as a basic roughness element is of interest and most of the contributors have raised the question of roughness shape, spacing and concentration. We feel, however, that a detailed review of the relative merits of the various research programmes actively being pursued towards this end is outside the scope of this discussion.

180. We are glad that Dr Wright has re-emphasized the great variety of transition routes which are possible. This point was implied if not stated quite explicitly in §§ 25, 49 and 59(*c*). The matter is, however, sufficiently important to warrant reiteration.

181. One type of diagram which does not appear to have been investigated previously is shown in Fig. 18, although Rajaratnam[91] has given a graphical solution of equation (25) for unknown d. Essentially this diagram starts from a relationship for rough turbulent flow, giving a unique value of $V/(sgd)^{1/2}$ for any value of d/k. The additional linings required for transition zone solutions were obtained using the Colebrook–White equation. Just as in the normal form of diagram, independence of $V/(sgd)^{1/2}$ from viscous influence is demonstrated at higher values of Vd/ν or of $(sg)^{1/2}d^{3/2}/\nu$. This diagram shows a zone of independence of $V/(sgd)^{1/2}$ from d/k. The laminar line has not been entered on this outline diagram.

182. Dr Hunter Rouse has drawn our attention to the prior claim of Blasius as originator of the conventional pipe resistance diagram. Although the terminology 'Stanton diagram' was used in the Paper, attention was drawn to Robertson's proposal of 'Blasius–Stanton diagram', which corresponds to the analogous case of the Hagen–Poiseuille law for laminar flow resistance. Again Reech–Froude number has been proposed instead of Froude number, to correct the record, but has not been generally adopted. On balance, Blasius–Stanton appears to offer the best combination of clarity and justice. The Blasius–Stanton diagram published by Moody,[16] on which the Colebrook–White equation is delineated, was preceded by the more comprehensive though less detailed arrangement given by Rouse[6] where the Colebrook–White equation was also used. In turn, Rouse, although working independently, had been preceded by Johnson[5] in respect of the combination of Vd/ν and $(sg)^{1/2}d^{3/2}/\nu$ as alternative abscissae, but Johnson worked before the existence of the Colebrook–White equation.

Acknowledgement

183. We wish to thank Mr J. J. Sharp for help in respect of Fig. 18.

Corrigenda

Page 488, § 1, line 3: for $V/_{\mathrm{m}}d/\nu$ read $V_{\mathrm{m}}d/\nu$.
Pages 489 and 490, equations (2) and (3): for d^3 read ρd^3.
Pages 498 and 499: transpose Figs 5 and 6.

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