The threshold of surge damage for moored ships

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Algiers harbour is notably sensitive to surge action. The harbour consists of a number of basins, the natural surge periods of which are from 60 to 180 s.

66. The waves generated by northern depressions flying across the Western Mediterranean enter Algiers bay, and by diffracting against the beaches all around the coastline, generate long period swell that excitates these basins and results in a strong surge action even in the most sheltered ones.

67. In December 1957–January 1958, numerous rope breakages and casualties were observed in the harbour:

(*a*) number of observed casualties: 24
(*b*) ship displacements: about 400–10 000 long tons
(*c*) seiche characteristics:

$$60\text{ s} < \text{period} < 160\text{ s}$$
$$0{\cdot}7\text{ ft} < \text{amplitude} < 3{\cdot}1\text{ ft}$$
(vertical)

(*d*) ship motions:

surge, 7–53 ft
sway, 0–16 ft

(*e*) number of rope breakages: 20

with slack or taut moorings, wire, nylon and fibre ropes of 1–3 in. dia. (see Table 4 for summary of observations).

68. A threshold for surge action on moored ships appears to be 0·7 ft for seiche amplitude, the maximum observed seiche amplitude being about 3·2 ft. Correlations between seiche amplitude, the ship surge and the number of ropes broken are given in Fig. 16. This number appears to be roughly proportional to seiche amplitude. The seiche periods critical for moored ships lie in this case between 60 and 160 s (for rather small ships).

69. These results generally agree with the Author's conclusions and confirm some of his quantitative results.

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The Author is to be complimented for his interesting Paper on ship surging and in particular for the criterion developed which deals with rendering a harbour safe from surge-action damage. This latter problem is, of course, one of the major objectives of studies of ship-surging.

71. The Writer is involved at present in an investigation of wave-induced motions of small moored boats which has provided some results which are considered pertinent to the Author's study. Although these small boats have lengths which vary only from approximately 15 to 17 ft, compared to the large vessels discussed by the Author which have lengths from 220 to 759 ft (see Table 1), the dynamical equations which describe the forced oscillation of the small moored boats are essentially the

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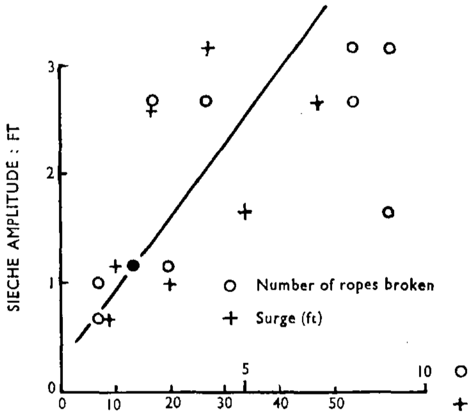

Fig. 16

Table 4

Ship displacements *long tons*	Seiche		Surge	Sway	Number of ropes broken	Observations
	ampl. *ft*	period *s*	*ft*	*ft*		
2000 . . .	2·7	80–110	18	10	7	
1400 . . .	2·7		50		4	
4400 . . .	1·3 0·8	100 135	20	3	1	fibre ropes
400 . . .	1·5		13		2	nylon
1000 . . .	0·8 1·5	110 150	10		3	
2800 . . .	1·7	110–160	33	13	9	
10 000 . .	0·8	70–110(?)	50	7	6	
1200 . . .	0·8		8	5	1	
6000 . . . 9600 . . .	1·1 3·2	75 90	20–23	7–10	8 9	

same as that of large moored ships. However, there are certain differences between small and large vessel mooring systems which make it important to examine in detail particular aspects of the dynamics of the mooring of small boats which are usually neglected in the case of large vessels.

72. The major difference between the two cases is related to the nature of the restraining forces. For large ships usually many mooring lines are used fore and aft (see Fig. 12), which on the average probably result in symmetrical non-linear restraining forces resisting surge motion (longitudinal travel). On the contrary, small boats generally are moored with fewer lines; in most cases a four point mooring system is used which consists of two bow lines and two stern lines (occasionally for larger boats spring lines are used). As a result of this type of mooring system and some features of the slips in which the small boats are moored, the restraining forces fore and aft on the average are usually highly asymmetrical. Therefore, displacements and line stresses may be quite different for the two directions of the surge motions of the boat. Hence, the asymmetrical nature of the nonlinear elastic restraining force system is an important feature of the dynamical problem for small boats and may influence any damage criterion which is developed for harbours where these craft are moored.

73. This effect may also be important in the case of some large vessels where, due to asymmetrical restraining forces, one group of lines may be stressed more than another for a given wave period, causing these to part first. An obvious example of the asymmetrical nature of restraining forces for large vessels is related to the lateral motion (sway) of a ship moored to a fixed dock. In this case the restraining force in one direction is provided by fenders which are much stiffer than the lines which restrain the ship against movement away from the dock.

74. To determine the effect of the asymmetrical non-linear restraining forces on the mooring dynamics of small boats, the Writer has used the 'block-body' approach of the Author, considering the body moored in a standing wave environment. In this discussion, the method of solution will be described and some results of this analysis for one particular moored boat will be presented.

75. Consider the dynamical analysis of a moored block-body described by equation (1) along with all of the assumptions which are inherent in developing this expression. For the case of no damping, equation (1) can be rewritten as:

$$\ddot{x} + \frac{F(x)}{M_x} = \zeta\sigma \cos \sigma t \quad . \; . \; . \; . \; . \; . \; . \; . \quad (30)$$

where the same notation is used as was used by the Author and where $M_x = M + M'_x$, M is the mass of the boat and M'_x is the hydrodynamic added mass for surge. The restraining force $F(x)$ is expressed as:

$$F(x) = F_1(x) \quad \text{for} \quad X > 0 \quad . \; . \; . \; . \; . \; . \; . \quad (31a)$$

$$F(x) = F_2(x) \quad \text{for} \quad X < 0 \quad . \; . \; . \; . \; . \; . \; . \; . \quad (31b)$$

where X is defined as positive when the horizontal surge displacement of the ship is toward the bow and negative when it is toward the stern. If the restraining forces $F_1(x)$ and $F_2(x)$ are different, then the mean position of the ship when it is in motion in horizontal surge will not be the at-rest position of the ship, but it will be displaced in the direction of the softer spring. This feature will be discussed in detail later.

76. The solution to equation (30) for the case of forced undamped oscillation is assumed to have the form:

$$X = \delta + A \cos(\theta - \alpha) \quad . \; . \; . \; . \; . \; . \; . \; . \quad (32)$$

where $\theta = \sigma t$, δ is the distance from the at-rest position of the vessel to the mean position of the vessel when it is surging, A is the amplitude of motion of the ship about the mean such that $X_{max.} = \delta + A$, and α is a phase angle (for the undamped case it can be shown that $\alpha = 0, \pi$). The restraining forces and the motion of the ship are shown schematically in Fig. 17.

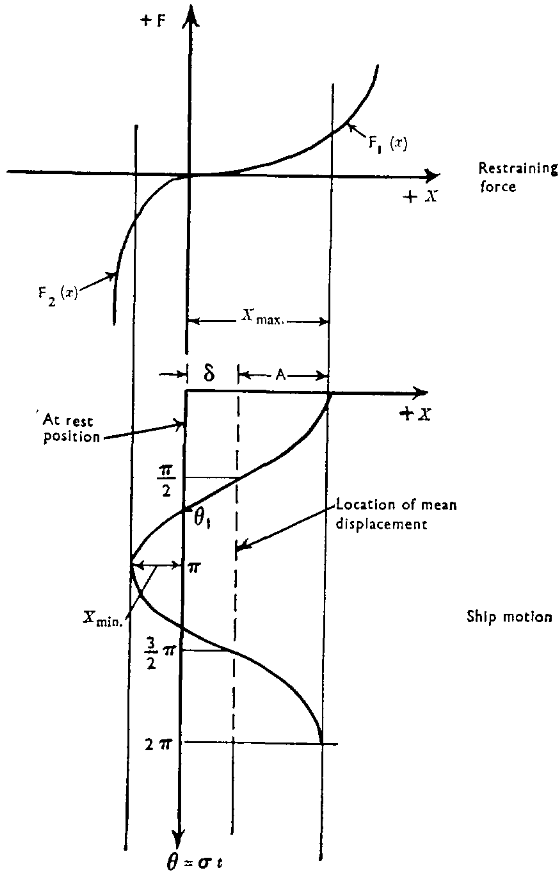

Fig. 17. Schematic representation of restraining forces and ship motion

77. The averaging method of W. Ritz[23] is used to solve this problem (the same approach used by the Author). From equation (30) this method provides the following two expressions:

$$\int_0^{2\pi}\left[\ddot{x}+\frac{F(x)}{M_x}-\zeta\sigma\cos\theta\right]\cos\theta d\theta = 0 \quad . \quad . \quad . \quad . \quad . \quad (33a)$$

$$\int_0^{2\pi}\left[\ddot{x}+\frac{F(x)}{M_x}-\zeta\sigma\cos\theta\right]\sin\theta d\theta = 0. \quad . \quad . \quad . \quad . \quad . \quad (33b)$$

In order to determine the quantity δ (or θ_1 in Fig. 17) equation (30) is averaged over the period 2π, hence:

$$\int_0^{2\pi} F(x)d\theta = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

equations (33) and (34) are solved in a simple fashion to evaluate the response of the moored boat using the following approximations for the restraining forces:

$$F_1(x) = aX+bX^3+cX^5 \quad . \quad . \quad . \quad . \quad . \quad . \quad (35a)$$

$$F_2(x) = rX+sX^3+tX^5. \quad . \quad . \quad . \quad . \quad . \quad . \quad (35b)$$

When small moored boats have free travel associated with them due to slack mooring lines, equations (35) have been found by the Writer to represent the restraining force as well and at times better than the expression $F(x)=CX^n$.

78. Substituting equations (32) and (35) into equations (33) the following expression results, which describes the response of the unsymmetrically moored block-body in a standing wave system:

$$\sigma^2-\frac{\zeta(1-\cos\theta_1)}{X_{\text{max.}}}\sigma-\frac{(1-\cos\theta_1)}{\pi X_{\text{max.}}M_x}\int_0^{2\pi}F(x)\cos\theta d\theta = 0 \quad . \quad . \quad (36)$$

where: $$X_{\text{max.}} = \delta+A, \quad . \quad . \quad . \quad . \quad . \quad . \quad (37a)$$

or since: $$\delta = -A\cos\theta_1, \quad . \quad . \quad . \quad . \quad . \quad . \quad (37b)$$

hence, $$X_{\text{max.}} = A(1-\cos\theta_1). \quad . \quad . \quad . \quad . \quad . \quad . \quad (37c)$$

79. Figure 17 shows that, for a given forcing frequency σ, the time that it takes the moored body to travel in surge from its maximum displacement to the at-rest position of the body is proportional to θ_1. Hence, to some extent, the quantity θ_1 describes the degree of asymmetry of the motion. If $\theta_1=\pi/2$ then the motion of the body is symmetrical about the at-rest position, since the time that it takes the body to travel from its maximum displacement to the at-rest position is simply $T/4$. For the hypothetical system of restraining forces shown in Fig. 17, the other extreme is reached when $\theta_1=\pi$. This is the case when the restraining force $F_2(X)$ is infinitely stiff compared to $F_1(X)$, and therefore, the body does not travel past the at-rest position. An example of the case of $F_2(X)\gg F_1(X)$ mentioned previously is the sway motion of a large vessel which is moored by lines to a stiff fender system. If roll is neglected, the motion of the vessel will be mainly away from the dock and this motion will be highly asymmetrical. For that case the mean position of the boat is approximately a distance A from the dock and the maximum lateral distance the boat moves is about $2A$.

80. In Figs 5 and 7 it would be interesting to know if the zero value of ship movement for both longitudinal and lateral ship movement refers to the at-rest position of the ship. If it is correct to assume this, these figures show that whereas, in general, the longitudinal movement of the ships which are shown is relatively symmetrical, the lateral movement is highly asymmetrical.

81. Returning to the analytical development it can be seen that the integral shown in equation (36) is obtained directly from the definitions of the restraining force (equations (35)) and the assumed movement (equation (32)) after first evaluating θ_1 for given values of $X_{\text{max.}}$ (or A) from equation (34) and equations (32) and (35). Although the integrations are straightforward, the result both for θ_1 and the integral of equation (36) consists of numerous terms in powers of sines and cosines of θ_1, and other system parameters, and for the sake of brevity will not be given explicitly in this discussion.

82. To demonstrate some features of the solution the problem is simplified by letting $b=c=s=t=0$ in equations (35). The restraining forces become bi-linear and unsymmetrical and equation (36) reduces to:

$$\sigma^2-\frac{\zeta(1-\cos\theta_1)}{X_{\text{max.}}}\sigma-\frac{R}{M_x} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

where:

$$R = (a-r)\left[\frac{\theta_1}{\pi}-1-\frac{\sin 2\theta_1}{2\pi}\right]+a$$

Fig. 18. 25 ft pleasure boat moored in slip at Marina del Rey, Los Angeles, California, USA

and θ_1 is given by:

$$\tan \theta_1 = \frac{r\pi + (a-r)\theta_1}{a-r}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

The extreme values of θ_1 can readily be seen from equation (39); i.e. for $a=r$, $\theta_1=\pi/2$ and for $r \gg a$, $\theta_1=\pi$, and for the case of $\theta_1=\pi/2$, equation (38) simply describes the response of a symmetrical linearly restrained block-body to a periodic force.

83. The method described by equations (32)–(37) has been applied to the problem of determining the dynamic response in surge of a number of small moored boats and one example will be described in detail to demonstrate the importance, for that boat, of considering the unsymmetrical non-linear nature of the mooring system.

84. The small boat being considered is shown in Fig. 18, and the dimensions are length 25 ft, a maximum beam of 9 ft, and a maximum draft of approximately 2·5 ft. The displaced weight of this boat is approximately 5200 lb. The mooring system is partially shown in Fig. 18; the boat is moored to a floating slip using a four point mooring system with relatively new nylon lines of $\frac{1}{2}$ in. diameter. This mooring system is fairly typical for this particular small craft harbour; i.e. the stern lines go from the boat to the dock inclined in a direction toward the bow and the bow lines go from the boat to the dock in the direction of the stern. Therefore, when the boat moves in horizontal surge the stern lines restrain the motion when the motion is toward the stern and the bow lines restrain the motion when it is toward the bow.

85. The restraining forces for surge motion toward the bow and toward the stern can be determined in a simple fashion from the three-dimensional co-ordinates which locate the mooring point on the boat relative to the mooring point on the dock and the elastic characteristics of the lines. The nylon line with which this boat is moored is three strand twisted-standard lay rope having an approximate average breaking strength of 6400 lb.

86. The restraining forces determined from these data are presented in Fig. 19 as the solid curves. It is interesting to note in this figure the degree of asymmetry of these forces. To emphasize this asymmetry, symmetrical curves are shown which were determined by computing the average displacement between the actual restraining force curves for a given applied force (after plotting both curves in the positive F vs X quadrant). In accordance with equations (35) approximate curves of the restraining force have been fitted to the actual curves. (The approximate curves shown in Fig. 19 are composed of short dashed lines.) The coefficients for these approximate relations are: $a=161{\cdot}5$, $b=99{\cdot}5$, $c=-6{\cdot}37$, $r=440$, $s=2860$, and $t=-532$, which provide a reasonable fit over the range of displacement shown in Fig. 19.

87. The procedure described here is relatively simple to apply for small boats where the number of mooring lines may range from 4 to 6 depending upon boat size. However, it is realized that for large vessels the mooring geometry is extremely complex due to both the large number of lines involved (shown in Fig. 12) and the different elastic characteristics of the lines. Nevertheless, it would be interesting to determine the degree of asymmetry of the restraining forces for longitudinal movements of ships such as the *Pasteur* and the *Charles Paddock*, whose wave induced movements are shown in Fig. 5 and indicate some asymmetry.

88. The variation of θ_1 with $X_{max.}$ for the small boat under consideration is shown

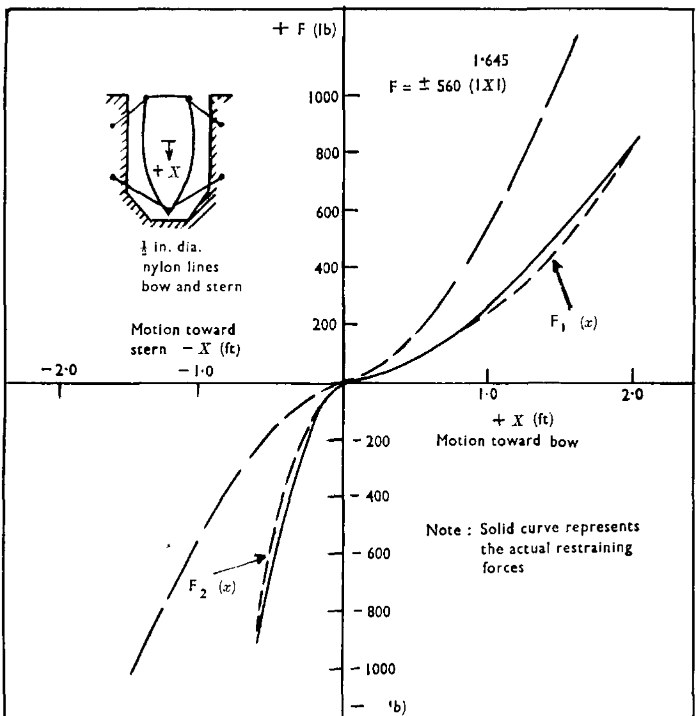

Fig. 19. Restraining force in surge as a function of boat displacement

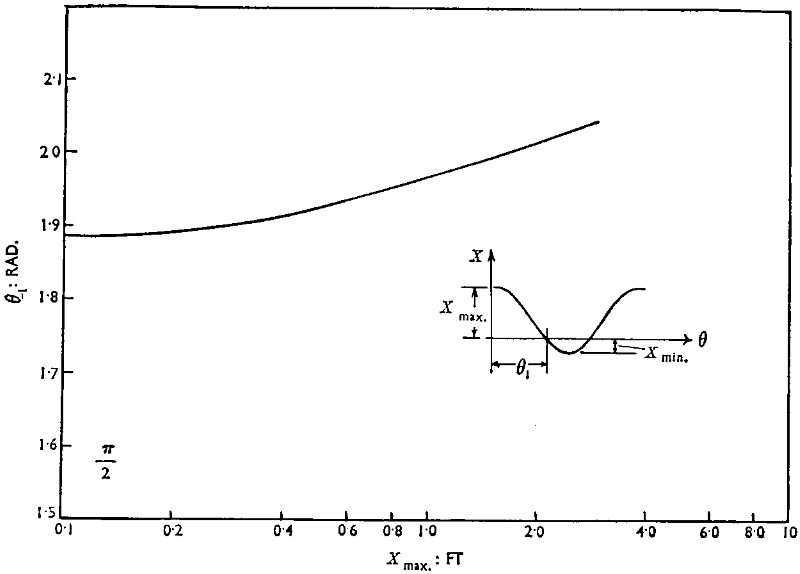

Fig. 20. Variation of θ_1 with $X_{max.}$

in Fig. 20. With reference to the schematic sketch of body motion shown in Fig. 17, it is seen that Fig. 20 also depicts the asymmetry of the restraining forces. (If the forces were symmetrical about $X=0$ then θ_1 would be $\pi/2$.) The increase in θ_1 with maximum displacement of the moored body shown in Fig. 20 indicates that as the boat displacement increases the motion of the body becomes more asymmetric.

89. The response curves of the horizontal surge motion are shown in Fig. 21 for two conditions of the restraining forces: asymmetrical and averaged-symmetrical forces. In both cases, for the forced oscillations, the response curves have been evaluated for a value of the quantity ζ of 2 ft/s (as defined by equation (4)) and for a virtual mass coefficient $M_x/M=1{\cdot}34$. It is seen that these curves are plotted as the maximum and the minimum boat displacement $X_{max.}$ and $X_{min.}$ respectively as a function of wave period. In addition to the forced response curves, the 'backbone' curves which describe free oscillations ($\zeta=0$) are shown for both restraining systems. The free oscillations, for the asymmetrical case (Fig. 21(a)), show that this particular boat has a natural period of approximately 3·7 s for a 2 ft displacement toward the bow which, of course, is considerably smaller than the expected periods of the large ships discussed by the Author. Also it is noted that due to the approximations used (equations (35)), for small X the natural period becomes constant.

90. Consider first the forced oscillations shown for the asymmetrical case, Fig. 21(a). It is seen that for a given wave period the amplitude of motion toward the bow ($X_{max.}$) is much greater than that toward the stern ($X_{min.}$). In fact for a wave period of 12 s and for a wave amplitude and basin and body geometry such that $\zeta=2$ ft/s, the displacement of the body from its at-rest position in a direction toward the bow is nearly 2·5 times the corresponding displacement toward the stern.

91. Referring to Fig. 19 and the displacements determined from Fig. 21(a) at $T=12$ s, it is seen that the restraining force for motion toward the bow is approximately 250 lb while that for motion toward the stern is 500 lb. If the geometry of

the mooring system is considered and it is assumed that the restraining force in the horizontal direction is equally divided between the two lines at either the bow or the stern, then it is found that the bow lines each have a tension of approximately 490 lb and the stern lines each have a tension of about 1000 lb for the given displacements. Therefore, not only is the boat displacement in surge asymmetrical, but the maximum line stresses are also asymmetrical.

92. If, on the other hand, the restraining forces of the moored body had been considered to be symmetrical and described by the symmetrical restraining force curves shown in Fig. 19 ($F = \pm 560(|X|)^{1.645}$), the response curve of Fig. 21(b) would

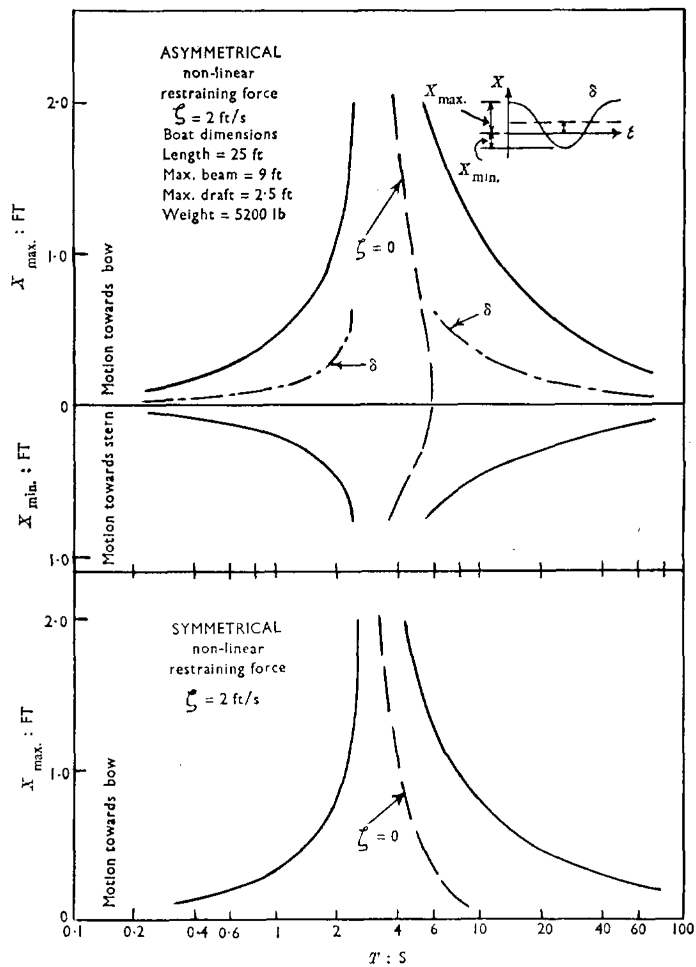

Fig. 21. Response curves of small moored boat: (a) asymmetrical non-linear restraint; (b) symmetrical non-linear restraint

result. In this case, the response curve is symmetrical about the at-rest position ($\theta_1 = \pi/2$, $X_{max.} = -X_{min.}$) and, therefore, only the branches are shown which describe the displacement in the direction toward the bow. This figure shows that for a wave period of 12 s the amplitude of motion is $\pm 0{\cdot}68$ ft. The indicated restraining force for this case for motion either toward the bow or the stern, from Fig. 19, is approximately 290 lb. Since the mooring geometry does not really correspond to a symmetric system, no attempt has been made to determine the induced line stress.

93. The floating slips to which a large percentage of small pleasure boats are moored are usually U shaped and restrained to move with the tide only in the vertical direction. Hence, the boat when moored is surrounded on three sides by the slip (bow, starboard and port) and since the boat owner rents his slip by length he will usually try to moor his boat in as small a slip as possible. This of course tends to reduce to a minimum the clearance between the bow of the boat and the slip and thereby introduces the problem of damage to the boat by impact with the slip when it is surging. Considering the response curves determined and presented in Fig. 21(b), it is seen that if the boat were considered to be symmetrically restrained one might conclude, for a particular wave period and system geometry, that the boat would be safe from impact damage. However, the more realistic case shown in Fig. 21(a) indicates that in fact, due to the unsymmetrical restraining forces, the boat could suffer impact damage at that wave period. Therefore, for small boat harbours the unsymmetrical nature of the mooring systems and potential impact damage could bear directly on a threshold damage criterion.

94. With reference to the example described, in many small boat harbours the breaking of lines may be unimportant considering the natural period of the moored boat, the period range of storm waves, and the breaking strength of the lines. However, it has been found that under conditions of excessive surgings of small boats, even though lines do not fail, it is possible that fittings either on the dock or on the boat can fail, thereby leading to serious boat damage. It is felt that this problem must be considered in developing damage criterion for small boat harbours and may also necessitate considerations of the asymmetrical nature of small boat moorings.

95. In summary, it is concluded that the threshold damage criterion developed by the Author appears reasonable for the surge motion of most large ships as long as the restraining forces are relatively symmetrical. However, due to the generally asymmetrical restraining forces for small boats and the corresponding small natural periods of these boats this criterion may not be directly applicable to harbours where small boats are moored.

96. This investigation was sponsored by the US Army Corps of Engineers under Contract DA-22-076-CIVENG-64-11.

Mr W. C. Q. Joosting, South African Railways

The Author appears to have drawn, without much alteration, on his early theories as presented in his thesis[8] of 1951, although he also mentions the correct formula for the movement of moored ships due to seiche action in equation (7). It would seem that the Author made no attempts to check his original theory against the correct one; now that it has been taken out of the dark corner where it belongs the Writer can only point out the mathematical and physical errors on which it is based.

98. This basis is to be found in equations (12) and (13), which the Author combines to give equations (14) and (15); from the latter the rest of his work is derived.

99. Equation (12) is correct in itself but, by substituting X_0 of equation (13), the Author appears to have assumed X_0 to be an independent variable. This is not permissible since the Author himself has stated in equation (10) that the ship movement amplitude X is related directly to the mooring force T by: $T = CX^n$. This reduces equation (12) to:

$$CX_0{}^n = T = \left(C_M \frac{W}{g} \sigma^2\right)^{n/n-1} \qquad (40)$$

It is clear that substitution of X_0 from equation (13) would not give any useful results now!

100. However, equation (13) is physically absurd in two ways in any case. In the first place, X_0 is the free ship movement amplitude *without* seiche action; how can it now be related to the seiche amplitude? Did the Author perhaps mean to introduce, at this stage, the ship movement amplitude, X_1 *with* seiche action? But in that case it is not permissible to substitute directly X_1 for X_0 in equation (12).

101. In the second place, the right-hand term of equation (13) simply represents the amplitude H of the horizontal water movement at the node of the seiche so that, assuming X_0 to mean X_1, equation (13) would become:

$$\frac{A}{\sigma}\left(\frac{g}{d}\right)^{1/2} = H = X_1, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

meaning that the ship movement amplitude X_1 would be equal to that of the water, H, which is absurd in any spring system. Actually, it can be shown that, whereas the ship movement amplitude X_1 may be 2–4 ft during surging, the seiche amplitude H, which causes it, can vary from about $\frac{1}{2}$–12 ft. In addition, the movements are in opposition during strong surging! This would be impossible in equation (13) or (41).

102. Of course, the Author should have used the later, correct solution, developed independently by himself[13] and the Writer,[17] which is given in equation (7). Converted to simple physical units this equation becomes:

$$T = CX_1^n = \frac{M_x\sigma^2}{\Delta n}(X_1 - H) = \frac{4\pi^2 C_M W}{g\Delta n\tau^2}\left[X_1 - \frac{A\tau}{2\pi}\left(\frac{g}{d}\right)^{1/2}\right], \quad . \quad . \quad . \quad (42)$$

where H and A have to be taken negative for strong surging in opposite phase.

103. When applying this equation, it is found that the relationship between T or X_1, A or H and τ is highly non-linear so that it must be clear that equation (15) cannot give an even approximate reflexion of the true conditions.

104. Consequently, the subsequently derived points in Figs 13 and 14 must be equally subject to correction. These corrections would apply only to the values of $1/\nu$; as a result, the Author's conclusions might, perhaps, not have to be changed much. However, it must be pointed out that the values of A in these figures should really have been corrected for the positions of the ships and their lengths, with respect to the actual, shorter period seiches that caused the ropes to break. As these surges are not known with any certainty, little can be done about it except to state that the effective values of A would be relatively smaller than shown, even if the periods were reduced back to the standard value used.

105. However, the Writer's main objection to these figures is that, whereas the Author states in § 46 (equation (23)) that A should be approximately proportional to ν, the mean lines drawn do not at all conform to this criterion, as a close examination of the values will show. Actually, these lines follow more closely the expression:

$$A - A_0 = c\nu, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

where c is a constant and A_0 is of the order of 1·5–2·5 in. for the Duncan Dock, which is by no means negligible.

106. The Writer, in his studies, has approached the problem of the reduction of surge damage in a much simpler and more direct manner. It was realized that the susceptibility of a harbour basin to surge damage must be directly proportional to the number of occasions on which seiches of various amplitudes occur. He has, therefore, made a statistical analysis of the numbers of days per year on which seiches occurred since 1941. From this analysis a simple relationship could be derived between the number of 'seiche-days' of various amplitudes and any reduction of the seiche amplitudes that might be achieved by the construction of protective works or harbour alterations. As an example it may be mentioned that, while the average total number of seiche-days was found to be 58 in the Duncan Dock at Cape

Town, this number would be reduced to 10 and 1 days if the seiche amplitudes can be reduced to 60 and 40%, respectively, of their present values. The reduction in severe surging would be even greater. Obviously, it will not be necessary to reduce the seiche amplitudes to 25% as suggested by the Author!

107. Space is not available for a full exposition of this analysis, nor for a description of the development by the Writer of a revised formula, and its application, similar to equation (42) above, in which the slack of the mooring ropes has been introduced as a separate variable. This made it possible to keep the rope constants C and n in equation (10) constant for all cases if C is taken per rope and the number of ropes of various sizes is reduced to the equivalent number of ropes of one unit size. The Writer hopes to be able to present his studies of these subjects in a separate paper.

108. Further, some minor objections of the Writer are that, in his opinion, the factor N/W in Fig. 12 can hardly be called approximately constant as it varies to the extent of 1 in 10; also that the layout of a protective outer basin, as shown in Fig. 15, is not likely to be acceptable as the outer basin will be subject to the direct attack of short and long waves which would make it less suitable for shipping accommodation, and therefore uneconomic. In any case, the old model was not suitable for testing with the shorter, more dangerous periods; with these, different results might have been obtained. Finally, the Writer offers his apologies to Dr Wilson for the serious objections that had to be raised to his theories.

Mr S. M. Fisher, Napier, New Zealand

The harbour at Napier, New Zealand, is protected by one breakwater which gives adequate protection to four piers from waves with periods up to 12 s, but not from the waves of longer period which cause dangerous movements of berthed ships. A typical water level record at a pier after the passage of a deep depression is shown in Fig. 22 with a quantitative record of the movement of a berthed ship, plotted from a spoken description from a portable tape recorder. From such records it was concluded that waves with periods between 30 and 60 s are the most dangerous to berthed ships of up to 30 000 tons displacement at Napier and a lee breakwater is being designed to reduce these waves.

110. While I acknowledge the superiority of Dr Wilson's study of conditions at Table Bay harbour, and congratulate him on this work, I am surprised and disappointed that his conclusions omit reference to the effects of virtual mass which appears to be least at a bulkhead berth of ample depth, the motion of the ship being limited to surge.

111. Considering the effect on a berthed ship of harbour waves of equal maximum slope (i.e. A/t approximately constant). Fig. 23 shows on a time base the slope of monochromatic sinusoidal waves of periods ranging from half to six times the presumed natural period of motion of a moored ship in the direction of the wave (T_s). Neglecting for the moment the effect of bottom clearance, the force on the ship is proportional to the wave slope, and the velocity of the ship after time $T_s/2$ is proportional to the area below the slope curve. The effect of a wave in accelerating a berthed ship could therefore be expressed in the form:

$$Z = SPT/T_s \quad \text{where} \quad T < T_s$$
$$Z = SP \quad\quad T_s < T < 2T_s$$
$$Z = SP2T_s/T \quad\quad 2T_s < T$$

where Z is a ship disturbance factor for the wave,
S, maximum slope of the wave,
P, a factor related to the virtual mass of the ship and dependent on its bottom clearance and its alignment to the wave direction,
T_s, the natural period of motion of the moored ship in the wave direction, and
T, the wave period.

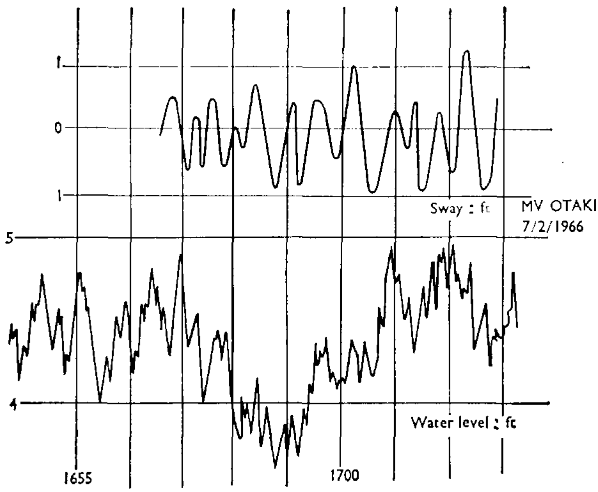

Fig. 22

112. These formulae neglect resonance, but could give a better guide to the relative potential of waves to cause damage. In illustration of this approach to the problem, Fig. 24 shows, on a base of wave period, the apparent height of the highest waves at a berth, selected from the water level record. Calculated from this curve and from information from a harbour model, the maximum water slope is shown, and thence the ship disturbance factor for the condition $T_s = 30$ and $P = 1$.

Dr B. W. Wilson

The Author is most gratified that his Paper has stimulated such interest on an international level and evoked such important discussions, as would seem to have wholly justified its presentation in the first instance. Dr Raichlen's discussion alone is an excellent short paper in itself which contributes very valuable new information to the subject. For this he is to be especially complimented. Thus, despite Mr Joosting's gloomy rebuke that the theory of the Paper had been taken 'out of a dark corner where it belongs', it seems that the Paper has had the opposite effect of promoting a healthy exchange of ideas, even leading to the promise of further developments in later publications.

114. Since Mr Joosting's comments are most damaging to the whole theme, it seems appropriate to deal first with the implications of his remarks. He has taken the Author to task for not having used the correct theory for the surge movement of ships, as outlined in equations (1)–(10), instead of an original theory that had been followed in the Author's thesis[8] of 1951. Mr Joosting, who has been a colleague and assistant of the Author in the research study on surge in Table Bay Harbour, Cape Town, South Africa, may recall that the best mathematicians in South Africa at the time were unable to indicate how the Author's non-linear equation (1) could be solved. In desperation the Author has followed a procedure of dimensional analysis[1,8] to reach part way for the truth. This approach has, of course, been superseded by the correct solutions found in 1955[13] and 1957[14,17], as reviewed in equations

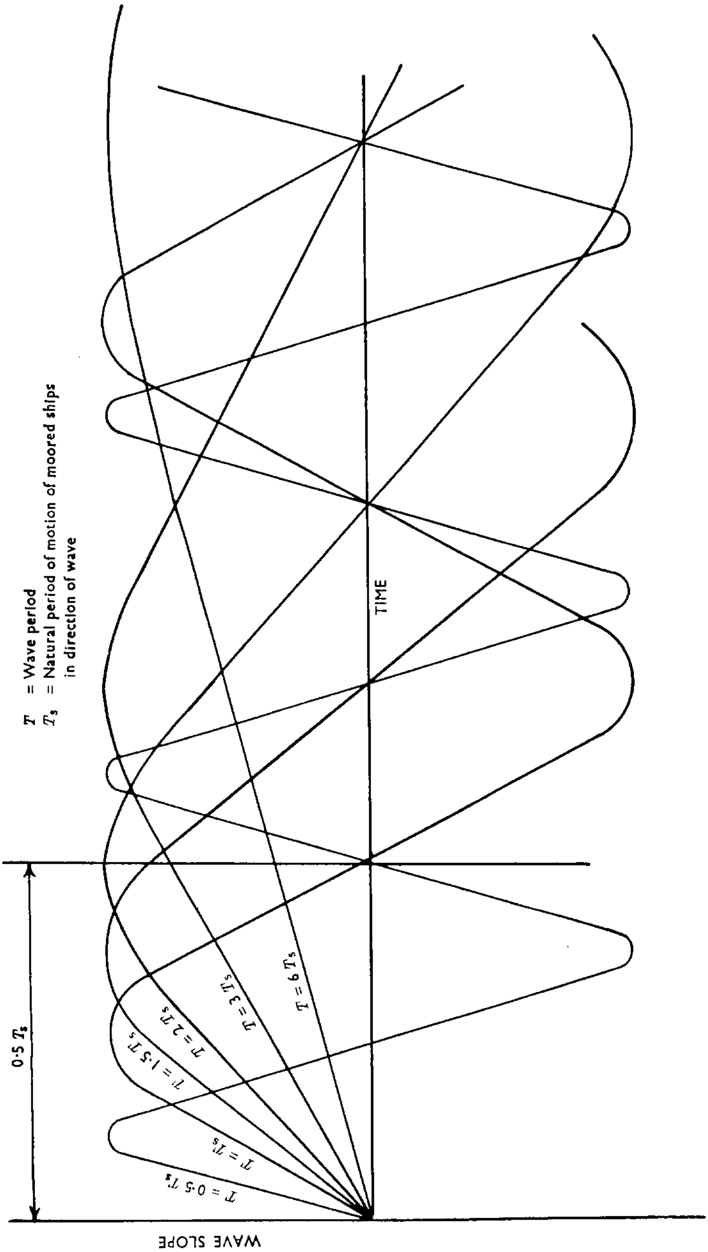
T = Wave period
T_s = Natural period of motion of moored ships in direction of wave
TIME
0·5 T_s
WAVE SLOPE
T = 0·5 T_s
T = T_s
T = 1·5 T_s
T = 2 T_s
T = 3 T_s
T = 6 T_s

Fig. 23

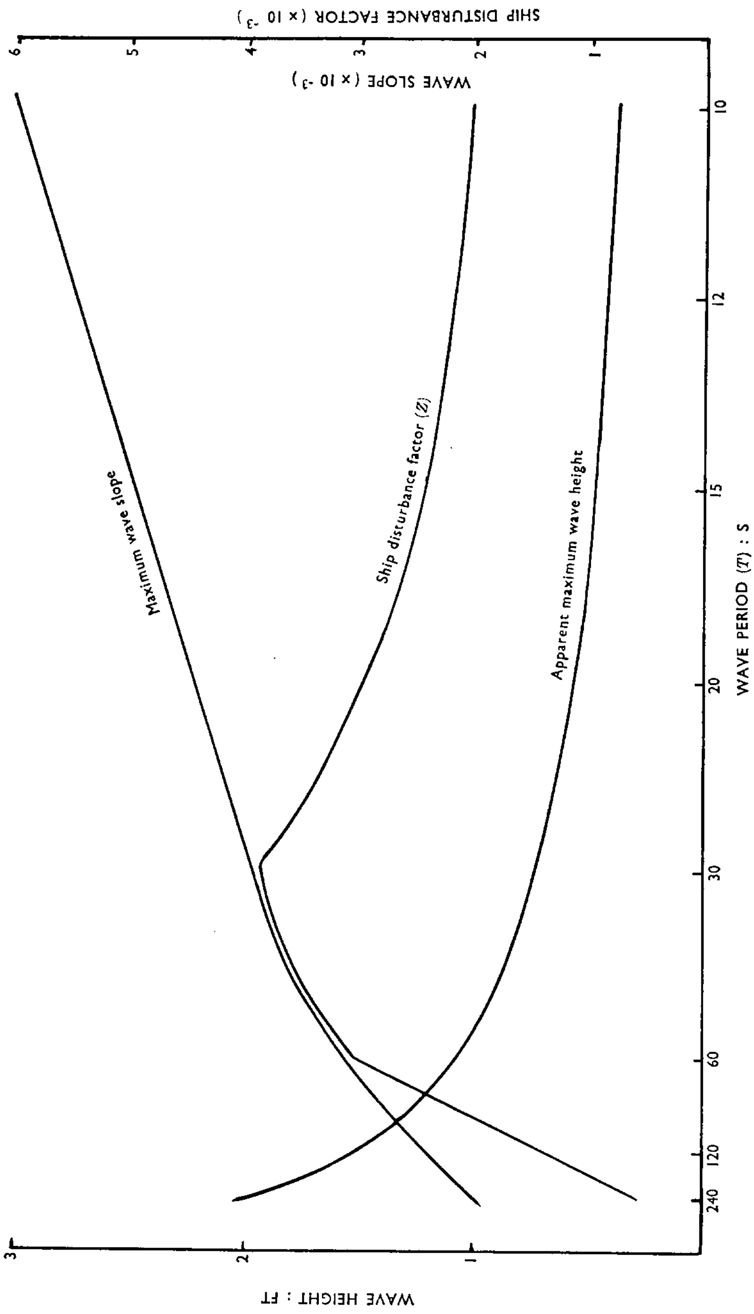
SHIP DISTURBANCE FACTOR (× 10 -3)
6
5
4
3
2
1
WAVE SLOPE (× 10 -3)
Maximum wave slope
Ship disturbance factor (Z)
Apparent maximum wave height
10
12
15
20
30
60
120
240
WAVE PERIOD (T) : S
3
2
1
WAVE HEIGHT : FT

Fig. 24

(1)–(10). The dimensional analysis approach, although sadly wanting in many respects, was nevertheless capable of channelling most conclusions in the right direction, as seems to have been proved by subsequent literature [2-22].

115. In presenting the material of the thesis which bore on the threshold of surge damage for moored ships in the present Paper, the Author no longer had access to all the statistical data upon which that original work was based and was therefore dissuaded from attempting any radical change. Possibly then he can be pardoned for having overlooked some of the fundamentally erroneous premises to which Mr Joosting has drawn attention. The Author is unafraid of the fact that errors were made in a pioneering effort and is both willing and ready to acknowledge them, and is pleased that Mr Joosting has drawn attention to these errors. On the other hand, Mr Joosting appears reluctant to acknowledge that any good whatever has been achieved in the effort itself, allowing for the circumstances, even though it probably steered his own thinking on this (on a path of avoidance) at a later time.

116. However, before accepting that the statistical part of the Paper (from § 37 to the end) is necessarily totally invalidated, as Mr Joosting implies, let us see how the application of the correct theory of equations (1)–(10) to the situation will change the results.

117. We require to establish the maximum horizontal amplitude of motion $X_{max.}$ that a moored ship can have under the conditions that lead to equation (1), for any given amplitude A of the exciting seiche. The general solution to equation (1) may be stated as:

$$[X^{n-1}\Delta(n)-\xi^2]^2+(2\beta)^2\xi^2=\left(\frac{\zeta}{\omega}X^{-1}\right)^2[(2\beta)^2+\xi^2] \quad . \quad . \quad . \quad . \quad (44)$$

For the case of no damping ($2\beta=0$), this reduces to equation (7) and is thus the generalization of equation (7).

118. Equation (44) yields a family of curves in a plot of X versus ξ^2 for different values of seiche amplitude A. This family is shown schematically in Fig. 25, from which it is clear that for any particular constant value of A, the maximum value of X will occur at or near the intersection point of equation (44) with the 'backbone' curve

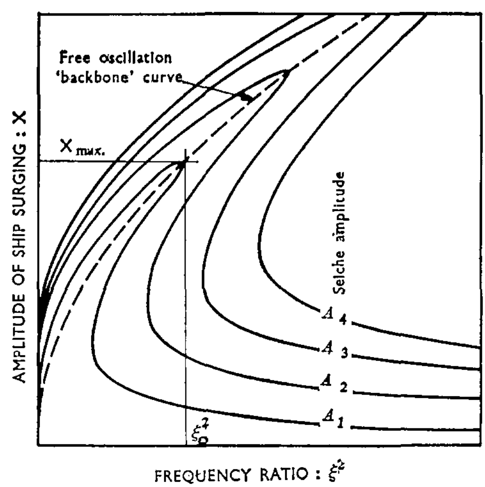

Fig. 25. Schematic response curves for longitudinal ship surging under influence of seiche

defining the free oscillation of the ship in the absence of excitation. This 'backbone' curve is expressed by setting $\zeta=0$ in equation (44), namely

$$[X^{n-1}\Delta(n)-\xi^2]^2+(2\beta)^2\xi^2 = 0 \qquad (45)$$

The relationship between $X_{max.}$ and ξ^2 at or near this intersection point may be established by differentiating equation (44) with respect to ξ^2 and setting $dX/d(\xi^2)=0$. In performing this operation it may be noted that, for the long waves normally involved in the excitation,[11] ζ is approximately constant, namely

$$\zeta \simeq A\sqrt{(g/d)} \qquad (46)$$

so that $d\zeta/d(\xi^2)=0$. Differentiation thus yields

$$2[\xi^2 - X_{max.}^{n-1}\,\Delta(n)]+(2\beta)^2-\left(\frac{\zeta}{\omega}\,X_{max.}^{-1}\right)^2 = 0 \qquad (47)$$

119. With sufficient approximation for small damping (2β small), equation (45) reduces to equation (8), which may then be used in equation (47) to render the latter in the simple form

$$X_{max.} \simeq \frac{\zeta}{2\beta\omega} \qquad (48)$$

Thus amplitude motion of the ship exceeds the motion of the water, H (given by equation (13) or equation (42)), by the factor

$$X_{max.}/H = \xi/(2\beta) \qquad (49)$$

which, of course, exemplifies Mr Joosting's correct contention that the use of equation (13) as a measure of $X_{max.}$ is really invalid.

120. At this point it is desirable to consider the nature of the damping factor 2β in equation (1). The drag force from which the second term of equation (1) (in which X should really be $\dot{X}$) is derived, is strictly a fluid drag, proportional to $\dot{X}^2$. For a ship, the longitudinal drag, according to Landweber[25] conforms reasonably well to the empirical law

$$F_d = K\dot{X}^2(lW)^{1/2} \qquad (50)$$

in which K is a constant. Since the block length $2l$ of a ship is roughly proportional to its displacement tonnage W, equation (50) may be generalized as

$$F_d \simeq K'W\dot{X}^2 \qquad (51)$$

in which K' is a modification of the constant K.

121. For the very low velocities of surge motion,[1] equation (51) can be linearized to

$$F_d \simeq N_x\dot{X} \qquad (52)$$

in which N_x, the coefficient of linear damping in surge will be directly proportional to the tonnage of the ship, via a modified constant K''

$$N_x = K''W \qquad (53)$$

In consequence of this, equation (2a) shows that

$$2\beta\omega = \frac{K''W}{M_x} = \frac{K''g}{C_M} \qquad (54)$$

Because the coefficient of virtual mass in surge, C_M, tends to be constant, equation (54) shows that the product $(2\beta\omega)$ also tends to be constant. Accordingly in equation (48) maximum ship motion $X_{max.}$ is directly proportional to ζ, which by equation (46), shows

$$X_{max.} \propto Ad^{-1/2} \qquad (55)$$

For a harbour in which water depth does not vary greatly with the tide, equation (55) suggests a direct proportionality between ship movement and amplitude of seiche, as is verified by the trend of Commander Sommet's observations in Fig. 16.

122. If we now invoke the fact that the point of maximum ship amplitude lies practically on the 'backbone' curve of Fig. 25, equation (44) yields the result

$$[X_{\max.}^{n-1}\Delta(n)-\xi^2]^2+(2\beta)^2\xi^2 = 0 \qquad . \ . \ . \ . \ . \ . \quad (56)$$

It is considered sufficiently accurate to ignore the effect of small damping in equation (56) and use the result, comparable to equation (8), that

$$\xi^2 = X_{\max.}^{n-1}\Delta(n) \qquad . \ . \ . \ . \ . \ . \ . \ . \quad (57)$$

From equations (10), (2b), and (57), maximum rope tension developing from the ship motion $X_{\max.}$ is then

$$T_{\max.} = C_M \frac{W}{g}\sigma^2 \frac{X_{\max.}}{\Delta(n)} \qquad . \ . \ . \ . \ . \ . \ . \quad (58)$$

which, of course, is the modification of the dimensional analysis result, equation (12), in the light of the correct surge theory. It was pointed out in the Paper that the assumption of $\Delta(n)=1$ normally introduces an error of less than 18%. Here, however, we avoid this error.

123. By eliminating $X_{\max.}$ between equations (48) and (58) and by using equation (46), the maximum rope tension developing for a seiche amplitude A is given by

$$T_{\max.} = \frac{4\pi^2 C_M}{\Delta(n)(2\beta\omega)\sqrt{(gd)}}\left(\frac{AW}{\tau^2}\right) \qquad . \ . \ . \ . \ . \ . \quad (59)$$

This, then, is the correct statement which should supplant equation (14). On the basis that C_M, $\Delta(n)$, $(2\beta\omega)$ and $\sqrt{(gd)}$ are all largely invariable, equation (59) shows that equation (15) should strictly be

$$T \propto \frac{AW}{\tau^2} \qquad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (60)$$

124. Because the statistical data developed in the Paper is used for a constant value of seiche period τ, the further reasoning in §§ (40)–(52) remains quite valid, but, to concede Mr Joosting's point made in equation (43), the results from Figs 13 and 14 show that

$$(A-A_p) \propto \nu \qquad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (61)$$

in which A_p is the permissible seiche amplitude. This proportionality of amplitude difference, $A-A_p$, to the number of ropes broken at a particular mooring location, ν, is verified independently by the trend of Commander Sommet's results in Fig. 16. Commander Sommet's observations even support quite closely the Author's conclusion expressed in equation (24), in which ρ was found to be about 25%. Commander Sommet's result, based on more limited data, would give a value of ρ of 22%.

125. Mr Joosting's objection to the ratio N/W in Fig. 12 being regarded as constant is not considered very serious. The data of Figs 13 and 14, as shown by Table 3, included actual values of N/W and made no absolute assumption of constancy. The fact of approximate proportionality between N and W, demonstrated by Fig. 12, was initially used only as an argument for interpreting the meaning of ν. Later it was assumed constant in equation (25), but in the light of all the great inherent difficulties of this problem and the aim for a gross engineering criterion, the assumption is surely permissible.

126. The Author is fully aware of the shortcomings of the statistical argument in relating rope breakages in a harbour, only partly occupied by ships of varying tonnage in different locations, to the amplitudes of a single seiche period. The influences of location, λ, of ship size W and of number of mooring ropes, N, have actually been taken into account and related to amplitudes, A, of a meaningful seiche period

($\tau = 1{\cdot}8$ min), shown earlier to be within the range of periods causing strong surging. The inadequacies of all this, pointed to by Mr Joosting, are held to be absorbed in the statistical scatter of plotted results. The mean trend is assumed to validate the argument that the relationship found will apply for the standard seiche period of reference ($\tau = 1{\cdot}8$ min).

127. The final effect of the non-linear dependence of rope tension T on seiche period τ, expressed by equation (60), is to amend equation (28) to the form

$$\frac{A}{A_{\text{max.}}} = \rho\left(\frac{\tau}{\tau_p}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (62)$$

This changes the final criterion expressed by equation (29) to a corrected criterion

$$\frac{A}{\tau^2} \gtrsim 2\times10^{-5}\ \text{ft/s}^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (63)$$

It may be noted that Commander Sommet's more limited data and treatment give a threshold value of A/τ^2 of about 6×10^{-5} ft/s^2.

128. Mr Joosting claims that his own method for deriving a threshold value of surge damage indicates that a mere 40% reduction of seiche amplitude would virtually eliminate any trouble. This conclusion, however, seems hardly supported by the facts. Referring to Fig. 11, it will be noted that the average amplitude of the 1·8 min seiche in the Duncan Basin at Table Bay Harbour was about 5 in. over the first six months of 1940. At the end of that period the 750 ft wide entrance to the Duncan Basin was narrowed to 400 ft, with a noticeable lowering of average amplitude of the 1·8 min seiche to about 2·5 in. This 50% reduction of seiche amplitude failed to eliminate surge troubles, which remained serious throughout 1941 and 1942, as shown by the curve of rope breakages in Fig. 11.

129. Mr Joosting's criticisms of the Paper, though seemingly harsh, have had the very beneficial effect of prompting a revision of the Author's original criterion for a threshold of surge damage to that of equation (63). The Author is thus really grateful to Mr Joosting for his free and frank objections and believes that the value of engineering discussion has hereby been clearly demonstrated. The Author looks forward particularly to Mr Joosting's further discussion of this topic in a forthcoming paper.

130. Turning now to the discussion of Dr Raichlen, the Author finds this to be of extreme interest and value as reflecting on the mooring of small boats in marinas and yacht harbours. Obviously Dr Raichlen had approached his subject in a most commendable fashion leading to the clear conclusion that the criterion of the Paper (or its modified form of equation (63)) could not be extrapolated to conditions of small boat mooring, at least as normally practised in the United States. The Author suspected that this would be the case and, as Dr Raichlen has shown, this aspect has needed its own special treatment.

131. Dr Raichlen's conclusion in regard to Figs 5 and 7 is correct. The lateral (sway) movement of ships measured at Cape Town was largely asymmetrical, as a result of the hard fenders controlling motion towards the dock. In investigating the asymmetry of surging of small boats, Dr Raichlen has in effect also solved the main elements of the problem of sway motion of large ships at their moorings, which up to the present has not been adequately broached.

132. As pointed out by Dr Raichlen in § 87, there is strong asymmetry in the longitudinal movements of the *Charles Paddock* and the *Pasteur*. In the case of the *Pasteur* measurements were made of the extension of a 'Monarch' mechanical spring to which a large bow wire was fastened. This spring has strongly non-linear tension-extension characteristics. The asymmetry in longitudinal mooring line restraints of large ships is readily determinable by methods such as outlined by O'Brien and Kuchenreuther[14] and the Author in an earlier paper.[11]

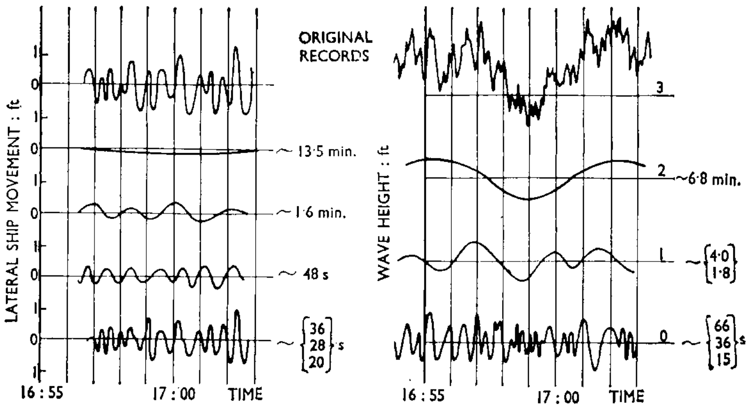

Fig. 26. Subjective analysis of component oscillations in sea and ship motions (m.v. *Otaki*, 12 570 dwt), Geddis Wharf, Napier, New Zealand

133. Mr Fisher's remarks are of interest in that they provide further corroborative evidence that the main source for moored ship disturbances are long waves in the period range between 30 and 60 s. However, Mr Fisher's Fig. 22 may be analysed subjectively, as in Fig. 26, to show that the lateral ship motion was quite strongly composite of oscillations of about 1·6 min period, besides others of less than 1 min period. Moreover, there is evidence of a 1·8 min excitation in the wave record. This suggests that a nominal range of exciting periods of 20 s to 2 min (see § 16) should be the main concern of harbour designers.

134. Mr Fisher expresses surprise and disappointment that the Author failed to consider the effects of virtual mass (presumably in sway motion). It should be apparent to any reader that the Author has, in fact, taken into account the virtual mass of a moored ship subject to surge. The problem of sway motion requires separate treatment and cannot be handled in a single paper devoted to the problem of longitudinal ship surging.

135. The Author finds interesting Mr Fisher's development of a ship disturbance factor *Z* for a wave of given height and period. Intuitively, however, the Author feels that the dynamics of the problem cannot be neglected and that some resort must ultimately be made to the non-linear theory of surge and sway.

References

23. KLOTTER K. Non-linear vibration problems treated by the averaging method of W. Ritz. *Proc. 1st US Nat. Congr. appl. Mech.*, 1951, Chicago.
24. HYDRAULICS RESEARCH STATION. *Wave model investigation of harbour improvement schemes at Napier, New Zealand.* Report No. 363. Wallingford, Hydraulics Research Station.
25. LANDWEBER, L. *Hydrodynamic forces on an anchor cable*, Tech. Report R-317 David Taylor Model Basin, US Navy, Washington, DC, Nov. 1947, 13 pp.