The oscillations of large circular stacks in wind

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Mr J. R. Chaplin and Dr T. L. Shaw, University of Bristol

The Author has provided much useful experimental evidence of the behaviour of cylindrical structures in three-dimensional flow at Reynolds numbers typically experienced in engineering practice. The Paper is therefore welcomed as a substantial addition to a field in need of supplement. Perhaps because of our welcome for the Paper we are particularly critical of the Author's repeated assertion that oscillatory forces on structures are caused by vortex shedding. The connexion between these two effects remains obscure; the concept of one causing the other has led to a misunderstanding of the mechanism resulting in the structural vibrations.

Dr T. A. Wyatt, Imperial College of Science and Technology

It is a noteworthy advance in the prediction of the behaviour of circular chimneys or towers to be able to use results obtained from wind tunnel tests at correct values of Reynolds numbers up to quite large sizes; it would seem that most stacks less than 5 ft in diameter, and some up to 8 or 10 ft in diameter, lie within the range of the tests described and, even for larger sizes, it will now be possible to make much more confident estimates than hitherto.

56. The use made of the size number ND^2/ν in presentation of the results gives clear expression to the dependence of the Reynolds number for the wind speed at which the critical dynamic response may occur on the size and dynamic properties of the structure.

57. The distinction made between the effect of parameter $2m\sqrt{\delta}/\rho D^2$ for uncorrelated shedding and of parameter $2m\delta/\rho D^2$ for correlated shedding is of great importance. It would be a welcome reinforcement of the general and theoretical support for this distinction if the Author could quote some pairs of comparable experimental results of behaviour from tests with differing values of $m/\rho D^2$ but equal values of the parameter that is expected to govern; it is likely that the necessary overlap of results exists in some cases, perhaps for example with models 10 and 11.

58. This is particularly significant for the supercritical range, for which the data were obtained using high pressures and thus correspondingly high air densities, ρ, in the compressed air tunnel. The model masses were not correspondingly adjusted, so that in these cases the value of $2m\delta/\rho D^2$ or $2m\sqrt{\delta}/\rho D^2$ required for prediction for a full size structure will have been obtained by combination of lower relative mass with higher damping than will be the case at full size. It is thus important to ensure that the correct parameter is used.

59. The Author has shown that when motion of the structure causes correlation of the vortex shedding, there is a rapid increase to an amplitude η exceeding 0·01, either as a jump as in Fig. 12, or at least with only a small change of the governing parameters. Consideration of the stresses caused by the larger amplitude oscillation shows that for most stacks in the supercritical range it will be necessary in practice to ensure that the vortex shedding remains uncorrelated at the resonant wind speed.

60. It can readily be shown that the stress amplitude, σ, near the base of a stack oscillating with non-dimensional amplitude η is given by

$$\sigma \simeq E\eta(D/L)^2$$

where E is Young's modulus. The exact relationship depends on the tapers and

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mass distribution of the stack. It follows from this that the strain amplitude σ/E corresponding to $\eta=0{\cdot}13$, $L/D=11{\cdot}5$, is about $1{\cdot}0\times10^{-3}$. Now for the larger stacks the resonant wind speed can be expected to be about 20 mile/h, and in an exposed location in the UK a mean speed of this value $\pm10\%$ (cf. the width of critical speed range in Fig. 12), from the prevailing wind direction $\pm30°$, may occur perhaps 4% of the time at 100 ft above ground level. At a frequency of 1 Hz this would amount to over 10^6 cycles per annum, which would be unacceptable structurally at anything approaching the strain amplitude quoted. However, has the rather high value of damping in the tests broadened the critical wind speed range for the larger sizes?

61. The first vital information for design is thus the critical value of the parameter $2m\sqrt{\delta}/\rho D^2$ that is necessary to ensure staying in the low amplitude regime, and how to achieve such damping in practice. For large stacks (size number $1{\cdot}5\times10^5$ to 4×10^5) the critical values would seem to be about 12 for $L/D=8$ and 19 for $L/D=10$. It is a pity that this size range is not covered for more slender stacks as the trend would seem to be upwards. Could the Author comment on the behaviour expected for more slender stacks?

62. The behaviour at the maximum design wind speed will also be important for large stacks, for which the Author has suggested prediction by proportion of the r.m.s. amplitude to the square of the speed (§ 43). This leads to high values of response that might be limiting in design; for example, if the resonant speed is 20 mile/h and the uncorrelated shedding amplitude is then $\eta=0{\cdot}007$, at a design mean wind velocity of 80 mile/h the corresponding value would be $\eta=0{\cdot}11$. This, however, is the r.m.s. of a normal random process and it might be appropriate to design for a maximum of three times this value, which would give a maximum stress due to this cross-wind motion of about twice the maximum given by conventional quasi-static design for a 110 mile/h gust in the downwind direction.

63. Figure 20 referring to a sub-critical example suggests this procedure may be somewhat conservative. Could the Author provide some additional data for the super-critical range? It would be reassuring to have confirmation that this procedure remains conservative when such large amplitudes may occur.

64. The Author must be commended on the large amount of information he has obtained and presented, which has given him problems of evaluation and condensation. It is strongly hoped that the National Physical Laboratory is continuing to use its unique facility to fill in the gaps left at the large sizes, particularly for somewhat more slender stacks.

Dr A. R. Flint and Dr C. S. Duns, Flint and Neill

The complexity of the prediction of the behaviour of tall stacks under wind excitation has been clearly illustrated in the Paper and the first reactions of many engineers to the results of the wind tunnel tests may be those of despair.

66. It would seem, however, from comparisons between field observations and the results of simplified calculations that the mode shapes and amplitudes may be determined with reasonable accuracy for cylindrical stacks subject to moderate winds. We have made a study of the vibrations of two such stacks. The theoretical solutions to the dynamic behaviour have been based on the conventional methods of vibration analysis adapted to the uncoupled equation for the ith mode

$$M_i^*\ddot{a}_i+2\lambda_i\omega_i M_i^*\dot{a}_i+\omega_i^2 M_i^* a_i=Q_i^*$$

where M_i^* generalized mass for the ith mode of vibration, $\{U_i\}^T[M]\{U_i\}$
λ_i fraction of critical damping for the ith mode of vibration
ω_i natural frequency (rad/s) corresponding to the ith mode of vibration
Q_i^* generalized force for the ith mode of vibration, $\{U_i\}^T\{P\}\cos(\omega t)$
$\{U_i\}$ ith mode shape
$[M]$ mass matrix for the stack

$\{P\}$ listing of magnitudes for the aerodynamic forces acting at the node points on the lumped parameter idealization of the stack
ω frequency of vortex shedding (rad/s)
t time

67. This equation may be solved using either a deterministic or a probabilistic treatment. A former approach adopts a precise definition of the excitation, whereas the latter uses a statistical model to represent the dynamic forces. The solution obtained for the amplitude under resonant conditions adopting the deterministic approach yields

$$a_i = \frac{C_L \rho D^3 \{U_i\}^T \{\delta L\}}{16\pi^2 S^2 M_i^* \lambda_i}$$

where a_i deterministic amplitude for the ith mode
C_L Karman lift coefficient
ρ mass density of air
S Strouhal number
$\{\delta L\}$ listing of lengths on the stack with diameter D

68. By means of the statistical approach the root mean square amplitude is given by

$$\bar{a}_i = \frac{C_L \rho D^3 \{U_i\}^T \{\delta L\}}{16\pi^2 S^2 M_i^* \sqrt{(2\lambda_i)}}$$

where $\bar{a}_i$ probabilistic amplitude for the ith mode.

69. It may be expected that the first of these solutions predicts the maximum amplitudes which are likely to be attained, although they may be of rare occurrence. The probabilistic solution may be more appropriate when estimating amplitudes which are of regular and frequent occurrence as, for example, when designing against fatigue failure.

70. The Author has shown that the Strouhal number may be taken as 0·2 for super-critical flow such as may be experienced around full scale stacks. The appropriate value of the lift coefficient in the range of Reynolds number from $1–2 \times 10^6$ is of the order of 0·27. This value would seem to be in reasonable agreement with the results shown in Table 1 which have been extracted from Fig. 6.

71. We have applied these parameters to the two stacks, the details of which are shown in Table 2.

72. The Hamburg stack consists of a self-supporting cylinder of welded steel construction whereas the Wigton stack is supported by a friction bolted lattice structure. The chosen values of the logarithmic decrement for both stacks are based on previous knowledge of similar structures.

73. The mode shapes and corresponding natural frequencies were obtained for lumped mass idealizations and the observed and calculated frequencies may be seen

Table 1. Variation of Karman lift coefficient as obtained from Fig. 6

C_L	R	Source
0·36	$1·79 \times 10^6$	Graph 7, Fig. 6
0·29	$1·50 \times 10^6$	Graph 6, Fig. 6
0·20	$1·24 \times 10^6$	Graph 5, Fig. 6
0·13	$1·05 \times 10^6$	Graph 4, Fig. 6
0·12	$0·81 \times 10^6$	Graph 3, Fig. 6
0·25	$0·58 \times 10^6$	Graph 2, Fig. 6

Table 2. Details of full scale stacks

Stack	Height, *ft*	Diameter, *ft*	δ_S
Hamburg	270	8·5	0·01
Wigton	320	8·5	0·06

Table 3. Observed and calculated vibration of full scale stacks

Stack	Natural frequency, R/S		Amplitude, ft		Wind speed, ft/s	
	Observed	Calculated	Observed	Calculated	Observed	Calculated
Hamburg	0·42	0·50	4·1	3·0	18	21
Wigton	0·90	0·85	1·0	0·9	—	36

from Table 3 to be in reasonable agreement. The maximum model amplitudes for the fundamental modes of vibration are also given in Table 3 and these were calculated assuming $S=0{\cdot}2$ and $C_L=0{\cdot}27$. The correlation between the observed and calculated amplitudes may be seen to be tolerable. The amplitudes given in Table 3 were of rare occurrence and the amplitudes of frequent occurrence were in agreement with those calculated using a probabilistic approach. The vibration of the Hamburg stack has been subsequently suppressed by the attachment of helical strakes and guy ropes.

74. It would appear from these comparisons that within the range of Reynolds number appropriate to large stacks such a simplified approach is sufficient to give reasonable guidance to the designer of the risk of serious amplitudes occurring.

75. We would be pleased to have the Author's observations on the assumptions made in such a treatment, particularly with reference to the value of the lift coefficient.

Mr Wootton

The value of the ordinate of Fig. 23 should be multiplied by a factor of 10.

77. The point raised by **Mr Chaplin** and **Dr Shaw** is akin to 'the chicken and the egg' discussion. The Paper does not discuss the origin of the fluctuating force and the description 'vortex induced' was not intended as an assertion that the subject is fully understood. The instability of the wake behind a cylinder, the vortex shedding and the fluctuating force are closely interlinked but the precise relationship between the three will continue to interest fluid dynamicists for some time.

78. **Dr Wyatt** raised several pertinent points. The results of recent experiments (Figs 24 and 25) show that the use of $2m\delta/\rho D^2$ at large amplitudes and $2m\sqrt{\delta}/\rho D^2$ at small amplitudes is reasonably justified. The use of the former parameter should be restricted to the peak amplitudes, whereas the latter has wider application. Clearly, the general parameters are $2m/\rho D^2$ and δ. The use of such parameters would introduce another variable and further complicate the analysis of the data. It was not possible to maintain $2m/\rho D^2$ constant at all tunnel pressures since an increase in m commensurate with ρ would reduce N and hence large size numbers, ND^2/ν, could not be attained. The Paper attempted to simplify the problem by dividing the response into two distinct parts.

79. Dr Wyatt asks whether the highest values of damping increased the critical wind speed range. The largest value of damping used was $\delta=0{\cdot}30$ which gives a bandwidth of 0·096. In practice, the values of bandwidth used in the experiments where appreciable peaks were attained were much lower and, although greater at the larger size numbers, do not significantly affect the wind speed range for instability. The point raised on fatigue is interesting. As far as I know, this rather than instantaneous overstressing is usually the reason for damage of stacks due to wind-induced oscillations. The calculation of the number of cycles given by Dr Wyatt may be pessimistic because if the wind speed passes relatively quickly through the critical wind speed range for instability, the oscillation will not have sufficient time to build up.

80. More slender stacks were not studied because of the difficulties of making

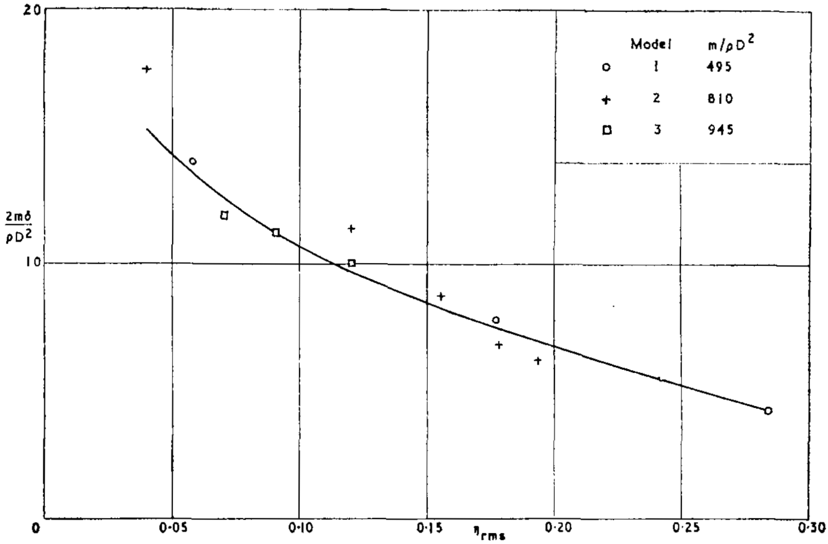

Fig. 24. The effect of increasing mass and damping on the peak amplitude of oscillation of a stack with $L/D = 11{\cdot}5$; $ND^2/\nu = 82 \times 10^3$, $V/ND = 6{\cdot}8$

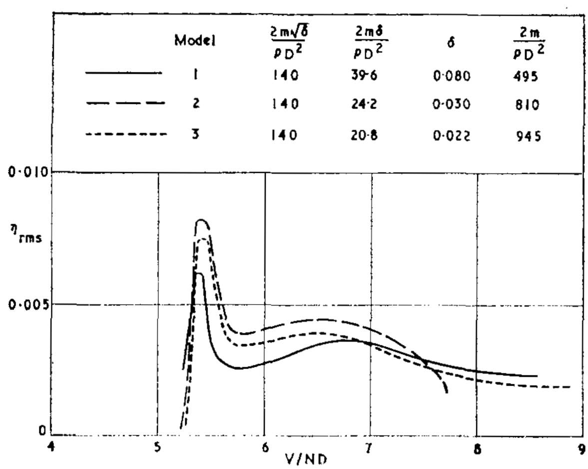

Fig. 25. The effect of increasing mass and damping on the amplitudes of oscillation of a stack with $L/D = 11{\cdot}5$, $2m\sqrt{\delta}/\rho D^2$ held constant

Table 4

C_L	R	Graph no., Fig. 6
0·23	$1·78\times10^6$	7
0·15	$1·50\times10^6$	6
0·083	$1·24\times10^6$	5
0·052	$1·05\times10^6$	4
0·042	$0·81\times10^6$	3
0·087	$0·58\times10^6$	2

satisfactory linear mode models of length to diameters greater than about 12. This is, however, an important topic of research.

81. The lift coefficients that **Dr Flint** and **Dr Duns** quote were based on the assumption that the response is random. In fact, at the peak amplitudes this is not true and the deterministic approach should have been used. The values of C_L thus calculated are shown in Table 4. The comparison of the amplitudes predicted and observed on the stacks at Hamburg and Wigton is unaltered (since the statistical approach was also used in this calculation) and provides an interesting correlation between model and full scale.

82. The statistical approach suggested by Dr Flint and Dr Duns is of particular interest if the amplitudes of the critical wind speed are required. At large velocities these may be greater than those attained at $V \simeq 5ND$.