An analysis of corrugated plates under bending conditions

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The Authors are to be congratulated on their excellent Paper and the extremely interesting experimental results they have presented. Perhaps the work might have been extended to give it more generality, since although a particular profile may be quite general, it is limited to identical corrugations and can therefore rely on the boundary conditions due to symmetry to simplify the problem.

57. Perhaps the Authors would care to comment on this and also whether their method may be extended to deal with various forms of surface loading such as is used in the following example.

58. I have recently been pursuing a somewhat similar investigation of the elastic stresses and deflexions of corrugated sheeting subjected to dead and live surface loading, a condition more realized in structural practice than in bending due to terminal moments.

59. The method is an extension of my previous work on multi cylindrical shells[7] and involves the coupling up of cylindrical shells in a corrugated form as shown in Fig. 16.

60. The individual shells forming the corrugated sheet may have any half angle ϕ_1, radius R_1, thickness t_1, loading P_1, and inclination β_1 of the centre line of the shell to the vertical. Suitable variation of these parameters can thus produce quite a general shape of profile and overall corrugated form.

61. The analysis uses the Donnell equation and thus produces a reasonably general state of stress, the actual stress resultants being shown in Fig. 17.

62. The moment and stress resultants for each individual shell are in terms of eight arbitrary constants and thus in the case given in Fig. 16, since there are five individual shells, 5×8 or 40 arbitrary constants have to be determined to satisfy the boundary conditions of the problem. At the free edges A and F the four edge moment and forces, when equated to zero, provide eight equations, whilst at each junction B, C, D and E there exist four equilibrium and four compatibility (displacement) equations to be satisfied, providing in all 4×8 or 32 equations. Thus, in all, 40 equations exist to uniquely determine the arbitrary constants.

63. As an example of this method, the results for one of the initial trial examples are given for the problem shown in Fig. 1 in which all radii are 30 in., half angles 40°, thickness 0·25 in., $\beta_1 = 0°$, $\beta_2 = -180°$, $\beta_3 = 0°$, $\beta_4 = -180°$ and $\beta_5 = 0°$ whilst the external loading is 10 lb/sq. in., on all shells.

64. The results are given in Table 2 for a limited number of stress resultants and deflexions though the complete set may be generated by the program if required. Due to symmetry only the results for shells (i), (ii) and (iii) need be presented. It is interesting to observe that some of the Authors' assumptions are reasonably verified by this example.

(*a*) That even for only five shells the three inner shells (ii), (iii) and (iv) are behaving almost identically with the edge effects limited to the outer shells;

(*b*) Even in this case, zero shear ($Nx\phi$) occurs at the centres ($\phi = 0$) i.e., *b, d, f* in Fig. 4 of the Paper.

65. A more useful program is now being developed which involves two shell

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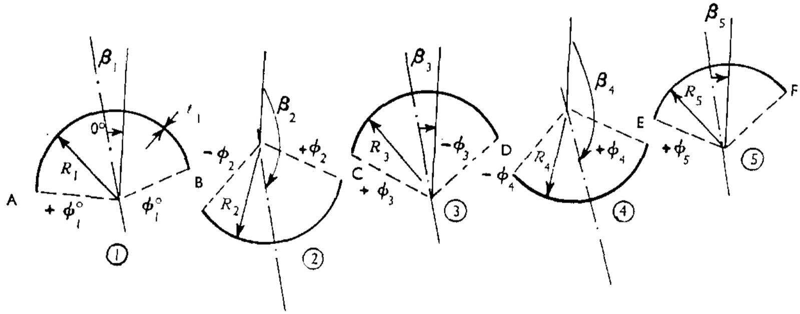

Fig. 16. General corrugated profile

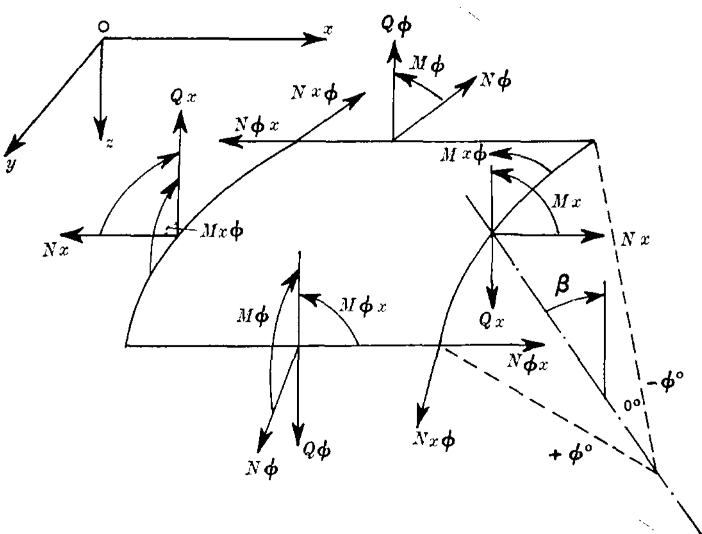

Fig. 17. Stress resultants acting on shell element

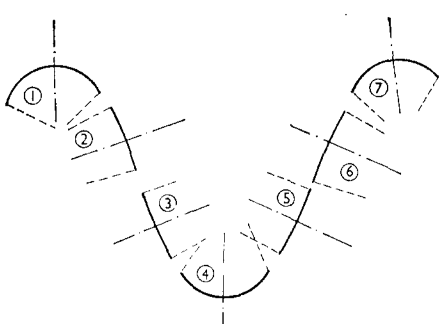

Fig. 18. More generalized profile

segments for every half corrugation, as shown in Fig. 18. By increasing the radius of shells (ii) and (iii), and suitably decreasing their half angles, the profiles of these will become flat thus generating the type of profile[8] analysed by the Authors in Fig. 10 of their Paper.

66. Whilst the above methods of analysis, since they apply to both symmetrical and asymmetrical conditions, will probably give a more accurate stress analysis than the Authors' in the linear elastic range, they will not be able to predict buckling.

67. The Authors have produced an extremely interesting technique for the prediction of such buckling and it is hoped they will continue their investigations further.

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We would like to thank Professor Gibson for his detailed appraisal of our Paper. We were particularly pleased that he was able to support our main assumptions. We would agree also with Professor Gibson's main criticism of the Paper—namely that it did not go far enough—but there were valid reasons for terminating this approach at the stage we indicated.

69. It is a straightforward exercise to extend this approach to consider a complete profile consisting of many similar or dissimilar corrugations, but one is ultimately

Table 2. Stress resultants and deflexions for corrugated shell

	$M\phi$, *lb in./in.*	$N\phi$, *lb/in.*	Nx, *lb/in.*	$wxEI$	$Nx\phi$, *lb/in.*
$+\phi_1$	0·0	0·0	11 400·7	−22 420 745·8	−0·0
$-\phi_1$	246·8	10·3	−3909·4	−273 066·2	1726·9
$+\phi_1/2$	−161·2	−551·5	−2873·4	−4 439 670·8	−252·8
$-\phi_1/2$	19·4	−437·0	−1067·1	−279 288·8	−56·3
0	−119·4	−410·1	1008·9	2 525 664·4	218·9
	−119·4	−410·1	1008·9	2 525 664·4	218·9

Shell (i)

	$M\phi$, *lb in./in.*	$N\phi$, *lb/in.*	Nx, *lb/in.*	$wxEI$	$Nx\phi$, *lb/in.*
$+\phi_2$	229·0	−0·0	−3833·5	−167·9	−1733·1
$-\phi_2$	246·8	−10·3	−3909·4	273 065·8	1726·9
$+\phi_2/2$	−0·2	−472·5	−1234·5	−248 724·7	13·7
$-\phi_2/2$	1·6	−475·2	−1217·3	−231 156·3	−21·9
0	−42·8	−394·9	727·1	1 414 496·8	−1·0
	−42·8	−394·9	727·1	1 414 496·8	−1·0

Shell (ii)

	$M\phi$, *lb in./in.*	$N\phi$, *lb/in.*	Nx, *lb/in.*	$wxEI$	$Nx\phi$, *lb/in.*
$+\phi_3$	229·0	0·0	−3833·5	167·5	−1733·1
$-\phi_3$	229·0	0·0	−3833·5	167·4	1733·2
$+\phi_3/2$	−0·2	−472·5	−1234·4	−247 488·3	13·6
$-\phi_3/2$	−0·2	−472·5	−1234·4	−247 488·0	−13·6
0	−43·0	−394·8	722·2	1 416 443·5	−0·0
	−43·0	−394·8	722·2	1 416 443·5	−0·0

Shell (iii)

faced with a real boundary at either end of the region and to match both conditions would require a further level of iteration within the existing level. This would increase the time necessary for solution and would also require much greater storage availability within the machine. One of the advantages of the method set out in the Paper is its great speed and limited storage requirement. The other way this approach could be generalized is to consider that each successive profile strip is deforming under a different longitudinal moment from those on either side of it. This would take into the area of solvable problems such situations as 'out of plane loading' mentioned by Professor Gibson, 'cantilever loading', and any other type of non-uniform bending. To deal with such situations would require the analysis of the mutual effects of each profile strip on its two neighbours and would also necessitate the consideration of each profile strip separately, with the accompanying increase in computer time and storage. In viewing this enlarged situation, the Authors could find no real justification in maintaining this approach when compared with the much more general and basic approach stemming from shell theory and so at this point, having obtained some useful results, we decided to return to basic principles and approach the enlarged problem from the standpoint of shell analysis.

70. The conclusion of this new approach was the formation of three highly non-linear partial differential equations which described the behaviour of any developable shell (including corrugated plates and closed cylinders) and took into account all types of surface loading, any arbitrary profile shape, in-plane loads, membrane stresses and bending moments. Various edge conditions were examined and the numerical solutions were arrived at through finite difference procedures and variations on a theme by Gauss–Sidel. The numerical solutions compared extremely well with experiments performed on aluminium shells in the laboratory and highly unstable profiles were successfully analysed near to the point of instability. This work formed part of a thesis[6] and is shortly to be published.

71. We are, of course, aware of Professor Gibson's long-standing association with this and allied topics and were most interested to see how he intends to generalize his well known shell roof analysis to cope with different profile shapes. This is a tried and tested method and it will be interesting to compare the results of his future work with our own. We are no longer actively engaged in research work on this but the computer program could easily be made available for comparative studies.

References

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