A criterion of concrete failure

A. L. L. BAKER

Mr L. G. Wills, Holst and Company Ltd

The Paper draws attention to the variable nature of Poisson's ratio as well as the elastic modulus of concrete. The latter is a well known obstacle in the application of the traditional methods of calculation to the design of concrete elements. The object of the science of materials is to discover properties which are constant for the same material or sample and as the Paper points out, Poisson's ratio and the elastic modulus were selected traditionally for this reason. It might be said, therefore, that we should disregard these properties when considering a material such as concrete and seek more fundamental properties which would be constant. So far as I know there has been no success in doing this, but it is possible that by considering the material as a complex system such as the Author uses, might give the answer. No doubt the factors due to short and long term moisture movement would need separate expression as is usual in soil mechanics.

20. Immediate use of the results of the Author's work can be made in the design of structures having important stresses in more than one direction. These are not very frequently met with but the results will also apply to many components of members which it is not customary to analyse in detail, at the present time.

21. Equally important is the insight into micro-cracking and cracking which the model assists. This may lead to the design of concrete mixes which have increased resistance to cracking rather than greater compressive strength. This must be possible by modifying the cement, selecting the aggregate or including substances for the purpose. It could be of great value but, except for the special case of the development of air entrained concrete, does not seem to have received much attention.

Dr D. J. Hannant, University of Surrey

The quantitative values for stress at failure given in § 13 depend to a certain extent on personal judgement in selecting a reasonable value for Young's modulus under various stress systems.

23. It is of interest, therefore, to plot equations (3) and (4) in a form in which a comparison may be made with experimental results throughout the triaxial compression quadrant of stress space. Taking equation (3) with $\sigma_1 > \sigma_2 = \sigma_3$ yields:

$$\sigma_1 = \sigma_c + 3{\cdot}5\sigma_3 \qquad (5)$$

Taking equation (4) with $\sigma_1 = \sigma_2 > \sigma_3$ yields:

$$\sigma_1 = 1{\cdot}1\sigma_c + 5\sigma_3 \qquad (6)$$

24. When equations (5) and (6) are plotted in stress space, a surface of the form shown in Fig. 9 is obtained. This surface is very similar both in shape and in magnitude to the proposed failure envelope suggested by Hannant and Frederick[7] based on experimental data for failure stresses in multiaxial compression. This type of surface is generally considered to become more parallel to the hydrostatic axis ($\sigma_1 = \sigma_2 = \sigma_3$) at high values of triaxial compression and this suggests that the constants in Professor Baker's equations may require adjustment for values of ($\sigma_1 + \sigma_2 + \sigma_3$) in excess of, say, $16\sigma_c$.

Paper published: *Proc. Instn civ. Engrs*, 1970, **45** (Feb.) 269–278.

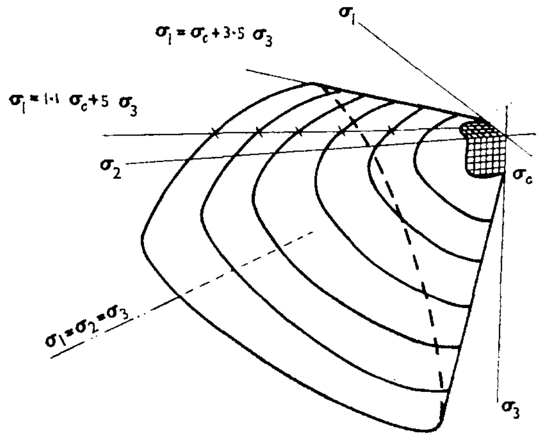


Fig. 9. Failure surface for concrete in the compression quadrant of stress space based on equations (5) and (6)

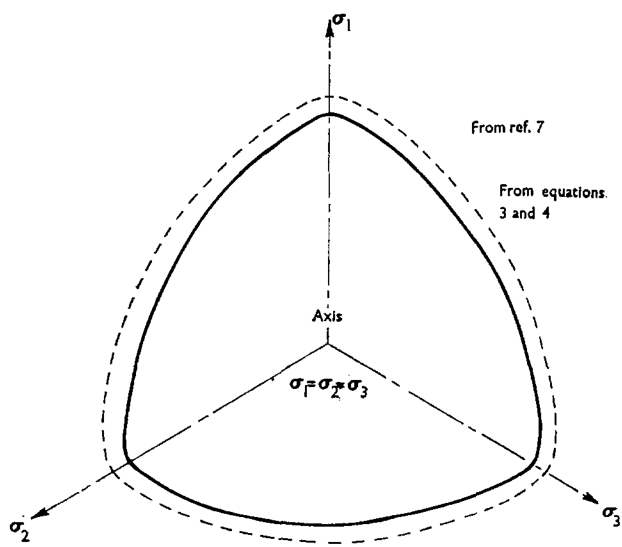


Fig. 10. Section through failure surfaces at right angles to axis $\sigma_1 = \sigma_2 = \sigma_3$

25. It is interesting to note that both the surfaces under discussion, whether produced from strain criteria or stress criteria, are three lobed curves. The maximum difference between the two approaches at stresses less than $(\sigma_1+\sigma_2+\sigma_3)=16\sigma_c$ is shown in Fig. 10. This consists of a section through the two failure surfaces on a plane normal to the axis $\sigma_1=\sigma_2=\sigma_3$ at the same hydrostatic stress. The similarity is perhaps not altogether surprising since both shapes must ultimately be deduced from experimental results which are relatively scarce.

26. It is therefore suggested that it may prove to be of little consequence in practice whether a criterion of failure for three-dimensional stress systems is based on strain or on stress. This is because the safety of each criterion will effectively depend on the factor of safety which is chosen, using the judgement of an engineer and with the help of relatively limited experimental results.

27. The experimental work in Professor Baker's own department is making an impressive contribution to improving this situation but it seems likely that the wide range of variables involved in such tests, and the time consuming nature of the work, will probably necessitate a large factor of safety for critical three-dimensional stress systems for many years to come.

Dr M. Anson, Commonwealth Scientific and Industrial Research Organization, Australia

For most states of stress a limiting extensional strain criterion certainly seems plausible (except, surely for σ_1, σ_2, σ_3 compressive and roughly equal) in terms of observed deformation and failure patterns, our knowledge of internal cracking and for its explanation of the fact that equal biaxial compressive stresses at failure are slightly higher than the uniaxial compressive stress at failure.

29. From the point of view of practical design, however, the outstanding difficulty may be the determination of what values to give E_c and ν_c since both alter very rapidly as failure is approached. Strictly, E_c should be zero according to the definition, but by assuming $E_c=1\cdot8\times10^6$ and $\nu_c=0\cdot2$ the Author has attempted to provide a reasonable lower-bound for ϵ_u. In discussing biaxial failure E_1 and E_2 were taken as 4×10^6, and by the application of equation (2) for $\sigma_1=\sigma_2$ the Author showed that:

$$\sigma_1 = \sigma_2 = 1\cdot1\times\sigma_c$$

which is typical of experimental results.

30. This relationship, however, is extremely sensitive to the assumed E_c and ν_c. For example, values of $1\cdot5\times10^6$ and $0\cdot25$ respectively would have led to:

$$\sigma_1 = \sigma_2 = 1\cdot67\times\sigma_c.$$

31. On the other hand, in this latter case, if we put E_1 and E_2 as 3×10^6 instead of 4×10^6, we find:

$$\sigma_1 = \sigma_2 = 1\cdot25\sigma_c$$

which is not untypical of experimental results.

32. In fact, using equation (2), the difficulty of knowing what E_c and ν_c to select in calculating ϵ_u would not matter too much if:

$$2\cdot0 \leqslant \frac{\nu/E^*}{\nu_1/E_1} \leqslant 2\cdot5,$$

for all values of stress between say 75% and 100% of the ultimate stress.

33. If this happens then whatever ν and E are selected as E_c and ν_c the corresponding ν_1 and E_1 would be such as to make:

$$\begin{gathered}1\cdot0\sigma_c \leqslant \sigma_1 \leqslant 1\cdot25\sigma_c \\ (=\sigma_2)\end{gathered}$$

34. As a matter of further interest, the pin-jointed tetrahedral model, with a limiting strain criterion of failure, also reflects the fact that equal biaxial compressive stresses at failure are slightly greater than the uniaxial compressive failure stress.

35. Consider only σ_1 acting (Fig. 3) and assume the stiffness of the long members AD, FC and EB is to the stiffness of all other members in the ratio 10:4. This gives a typical Poisson's ratio of 0·15. On increasing the load, members AF, FD, DC and CA fail at 30% of ultimate, as described in the Paper and their stiffness becomes zero. At 75% of ultimate the two tensile members AD, FC have their stiffness reduced to five, at $87\frac{1}{2}$% to three, at 94% to unity, and at 100% to zero.

36. The stress–strain curve under those conditions resembles typical concrete behaviour and likewise the behaviour of Poisson's ratio, which begins to increase above 0·25 at 75% of ultimate, approaches 0·5 at $87\frac{1}{2}$%, and 0·75 at 94%.

37. The strain in AD and FC at ultimate load is taken to be the failure criterion and used in the study of biaxial failure of the model.

38. With σ_1 and σ_2 acting and equal, the only member undergoing extensional strain is AD. Allowing its stiffness to reduce progressively to five, three, unity and zero at the same strains respectively as under uniaxial load the value of σ_1 $(=\sigma_2)$ at failure is 1·2 times the failure load under uniaxial compression.

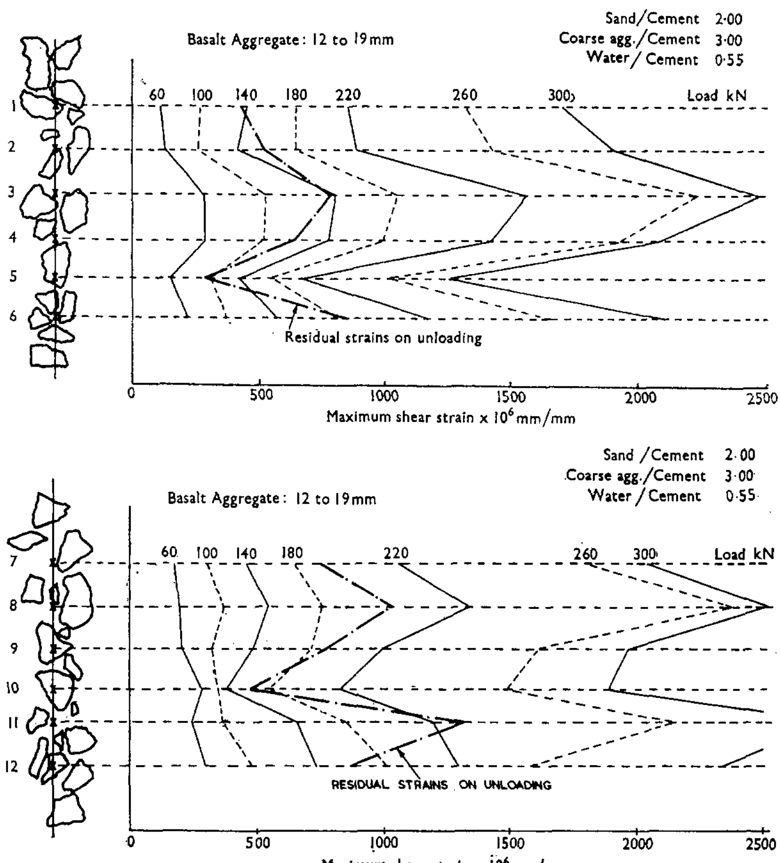


Fig. 11. Internal strain distribution in the aggregate and matrix phases in a typical sawn section of concrete under uniaxial compression

Dr R. N. Swamy, University of Sheffield

It is now well established that the principal causes of cracking and failure in the heterogeneous system of concrete are the differential stiffness between the aggregate, matrix and their interface, the bond interaction and the weakness in tension of the mortar matrix. The suggestion, however, for the basis of the model that the crack forms first in the mortar pocket and then eventually extends around the stone interfaces is highly questionable.

40. When inclusions of greater stiffness are embedded in a relatively softer matrix, the interface invariably becomes the weakest link in the system, and flaws, voids and cracks originate there. Studies of internal strain distribution using reflective photoelasticity show that this is so[8,9] and these results are in agreement with those reported by Hsu *et al.*[10] In heterogeneous materials such as concrete and fibre-reinforced composites composed of constituents of differing stiffness, random non-uniform strain distribution occurs internally (Fig. 11).

41. Although large strain concentrations also occur in the matrix, cracking does not start here since the aggregate restraint to plastic and drying shrinkage, moisture movement, and the external load imposes local biaxial and triaxial compressive stress systems in the matrix. It is the presence of these favourable complex stress conditions that enables the matrix to sustain very high strains without cracks occurring until much later in the loading history.

42. Further, the tensile–shear strength between the matrix and aggregate which influences the bond of the two phases of concrete is much less than the tensile–shear resistance of the matrix itself so that bond cracks at the interface would appear first during the hydration process and the subsequent loading long before the strength of the matrix is exceeded. Isochromatic patterns on the sawn face of a concrete specimen under uniaxial compression show conclusively that material breakdown occurs first around aggregate inclusions in spite of the matrix being highly strained. Fig. 12 shows these patterns on the sawn face of a concrete specimen made of a typical concrete mix under progressive loading.

43. It is, of course, possible to observe these large strain concentrations, and perhaps even initial cracks, occurring in the mortar matrix in idealized model tests where a single or a small array of inclusions is embedded in a large shell of the matrix. The stress distribution in such a simplified model is unlike that in an actual mix with random distribution of aggregates, and the effect of neighbouring inclusions upon the stress distribution of the surrounding matrix is completely absent.

44. One of the most important factors that needs to be considered in defining a mathematical model for the failure of concrete is the influence of shrinkage stresses. It is these stresses which determine the subsequent stress distribution within the concrete system under external load, and indeed, the origin and nature of micro-cracking in concrete. When mechanical behaviour under external load is studied these effects are completely ignored but they have a profound influence on the load-induced strains and the resulting cracking process. Our studies on shrinkage with random distribution of aggregates show that the distribution of strains and the orientation of the principal strains vary from point to point; there is random and irregular distribution of strains, and the restraints imposed by the aggregates on the matrix can create bond cracks at the interface.

45. Over a period of time, however, there is a certain release of internal stresses due to interaction of adjacent inclusions upon the stress intensity in the surrounding matrix and the strains in the core of the material become fairly uniform and compressive, although areas of severe strain concentration have been observed to persist even after a hundred days.[8,11] In an idealized model with a single or an array of aggregate inclusions embedded in a shell of matrix, shrinkage has the greatest effect creating radial compressive stresses in the aggregates and hoop tensile stresses in the surrounding matrix engendering radial cracks in the matrix normal to the interface

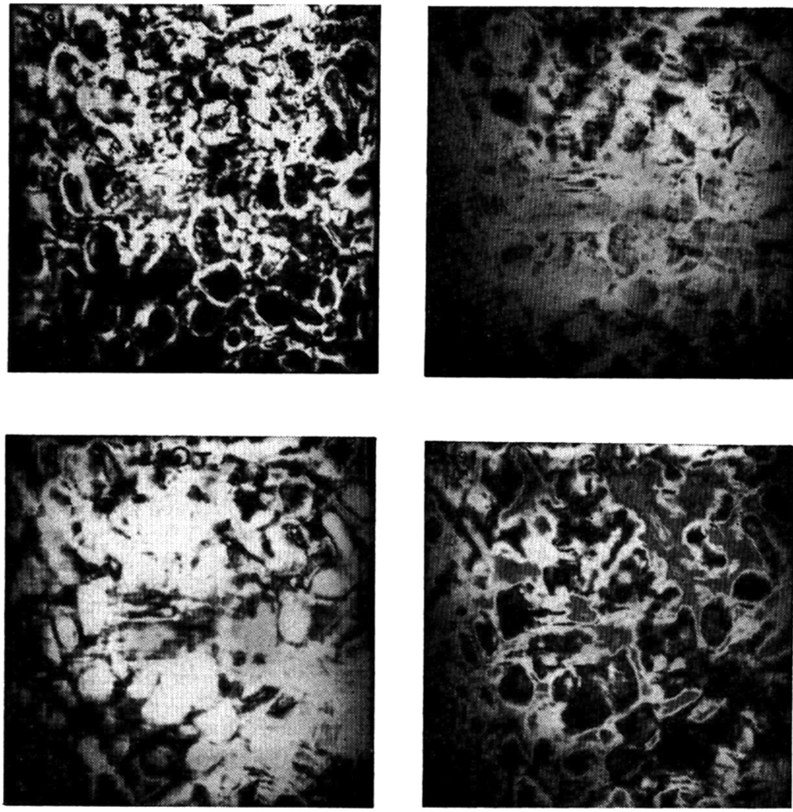


Fig. 12(a) top left: No load, shrinkage strains concentration around aggregate inclusions: aggregates (dark areas) remain relatively unstrained

Fig. 12(b) top right: Strain distribution under a stress of 3·86 N/mm^2. Aggregates and matrix are relatively unstrained whilst the aggregate–matrix interface (dark areas) are highly strained

Fig. 12(c) lower left: Strain distribution under a stress of 7·72 N/mm^2. Severe strain concentration around aggregate inclusions. Aggregates and large areas of matrix (light coloured areas) remain relatively unstrained

Fig. 12(d) lower right: Strain distribution under a stress of 15·44 N/mm^2. Aggregates (dark areas) still remain less strained whilst severe strain gradients and cracking occur around aggregate inclusions

Fig. 12. Isochromatic patterns on the sawn face of concrete specimen under uniaxial compression

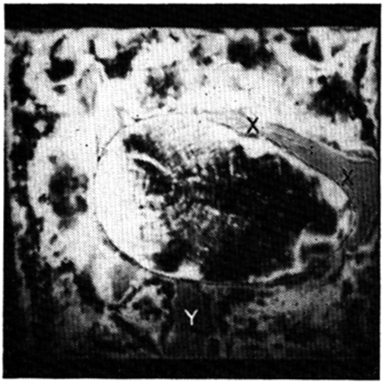


Fig. 13. Isochromatic pattern showing shrinkage cracks (marked X) at the interface of a large pear-shaped smooth gravel embedded in mortar matrix. Area marked Y shows local strain concentration without cracking

rather than at the interface. It is, therefore, not surprising that micro-cracking was observed at the points marked X in Fig. 1.

46. In Anson's tests[12] the specimens consisted of single-sized spheres or discs arranged in a diamond fashion in the matrix. When the aggregate size is comparable to that of the surrounding matrix, however, cracks are observed at the interface itself in spite of the tensile stresses in the matrix (Fig. 13). These idealized arrays[12] are far from reality and the resulting stress distribution ignores the significant influence of adjacent inclusions and of randomness occurring in practice which creates compressive stresses in the interior of the system. Models consisting of a thick shell of matrix covering and interacting with each aggregate inclusion independent of the adjacent aggregates oversimplify the state of internal stress, and can indeed portray cracking processes not perhaps truly representative of the real system.

47. There are two important parameters which influence the magnitude and distribution of shrinkage stresses, namely, the ratio of the elastic moduli of the aggregate inclusion and the matrix, and the ratio of the distance between the aggregates and their size. The former is probably the more indeterminate quantity of the two since the elastic modulus of the matrix is not an invariable quantity during the development of shrinkage. In the first few hours after casting, during setting and initial hydration, this ratio is high, but with increasing maturity and continued hydration the matrix becomes stiffer, and at 28 days the ratio of the elastic moduli of aggregate and matrix can have values ranging from about three to ten depending on the mix proportions and the type of aggregate. This differential stiffness between the aggregate inclusions and the matrix together with the spatial distribution of the former create complex stress conditions within the concrete system even before any external load is applied.

48. Photo-elastic studies of the restrained shrinkage of the matrix show that the influence of adjacent aggregates is very profound and results in a hydrostatic tension in the mortar which varies rapidly along the longest intercentre distance between the aggregates causing bond cracks, the development of which results in a relaxation of these stresses.[13,14]

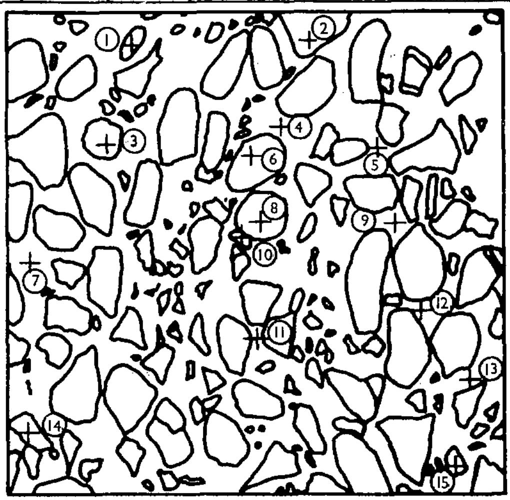
49. The hoop tensile stress, on the other hand, existing along the minimum clear distance between aggregates is responsible for the subsequent development of mortar cracks.[14] Although tests with idealized arrays of single-sized inclusions show a systematic influence of the ratio of the spacing of the aggregates to their size on the internal shrinkage stresses,[14] no such systematic variation has been found in actual concrete systems.

50. The most important change in strain distribution occurs on the application of a superimposed external load which completely alters the shrinkage strain distribution

Strains in micro mm/mm

C - Compression T - Tension

	Shear strain	Principal strain			Shear strain	Principal strain	
		ϵ_1	ϵ_2			ϵ_1	ϵ_2
1	627	135 C	762 C	9	627	193 C	820 C
2	724	101 C	825 C	10	482	193 C	675 C
3	820	97 C	917 C	11	1206	338 C	1544 C
4	521	203 C	724 C	12	772	106 T	666 C
5	579	289 C	868 C	13	579	121 C	700 C
6	482	309 C	791 C	14	531	77 T	454 C
7	675	48 T	627 C	15	579	376 C	955 C
8	724	318 T	406 C				



Crushed gravel max. agg. size 19 mm

Fig. 14. Typical shear and principal strain distribution in the aggregate and matrix constituents in a sliced concrete prism due to shrinkage and a superimposed uniaxial compressive stress of 11·7 N/mm^2

even when the load is applied for a short time.[8] Apart from the reduction of the areas of severe strain concentration, the orientation of the principal strains is completely changed, and if loading is continued for a sufficiently long time, the shrinkage effects gradually transform the internal stress distribution into localized biaxial and triaxial compressive systems of varying magnitude (Fig. 14).

51. Anson[12] also found that the paste pockets within a specimen in uniaxial compression were in a state of triaxial compression. This explains why the matrix is capable of sustaining much higher tensile and compressive strains than is observed in uniaxial tests, without cracks occurring in it. This also explains why the largest strains in the matrix are generally observed between adjacent aggregates (Fig. 11) than in other directions.[12] It does not necessarily follow that internal fracture will first occur in these regions such as X, in Fig. 1.

52. Professor Baker has suggested that when the local average extensional strain exceeds the parameter $\nu_c\sigma_c/E_c$, 'internal cracking normal to the direction of minimum

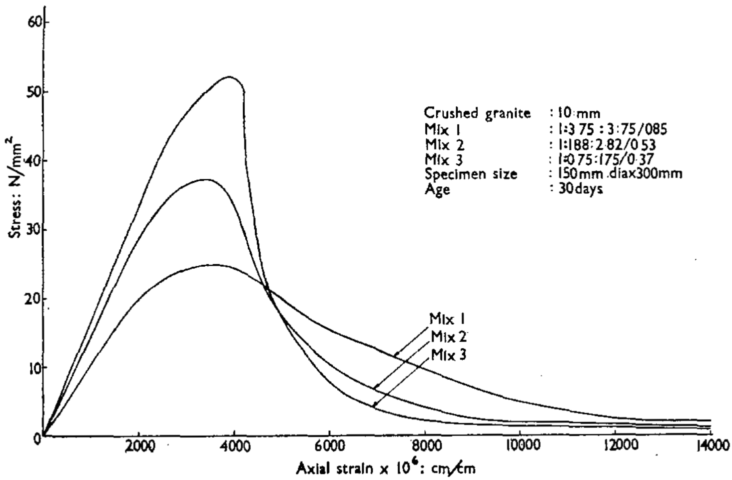


Fig. 15. Complete stress–strain curves for 150 mm by 300 mm cylinders of crushed granite aggregate at constant rate of strain

stress has extended sufficiently to distintegrate the concrete by splitting so that it is incapable of sustaining further load'. This criterion of failure obviously refers to the ultimate strength conditions (Fig. 4). It is appreciated that this parameter has a completely different meaning from its conventional usage in elastic theory, nevertheless, it would be interesting to know what physical aspect of the behaviour of concrete this term represents which leads to total collapse of the system.

53. Obviously, internal micro-cracking would have commenced very much earlier in the loading history and Poisson's ratio would also have commenced to show an increase at loads approaching the ultimate strength. A clarification of the significance of this term in relation to the physical behaviour at loads approaching failure, would be greatly appreciated.

54. Another important implication of this proposed failure criterion is that it appears to ignore completely the descending part of the load–deformation characteristic of concrete. Many tests have shown the significance of this part on inelastic deformation, micro-cracking and load-maintaining capability[8,15,16] (Fig. 15), and it may well be that there are two distinct mechanisms of micro-cracking in the ascending and descending part of the stress–strain behaviour of concrete. In the ascending part the cohesive strength of the material is probably the main load-bearing element whilst in the descending part, the change in surface energy, and the energy dissipation due to friction at the interfaces and plastic deformations at the interface discontinuities all play a vital part in sustaining the load further. This latter has an important implication on the cracking process within the system. If initial cracking occurred primarily in the matrix, the energy balance is very much concerned with changes in surface energy alone, since the mortar matrix is essentially more brittle than the heterogeneous system of concrete. The fact that concrete shows a progressive inelastic deformation, without fracture, from very low loads is probably the most significant evidence that initial micro-cracking occurs at the interfaces permitting considerable energy dissipation through friction and local yielding in the vicinity of the discontinuities.

55. Mortar cracking can therefore occur only at later stages of loading history

after considerable interfacial cracking has occurred. This is confirmed by photo-elastic studies[9] which show that progressive external loading causes strain concentration mainly at the extremities of flaws, voids or micro-cracks at the interface than within the matrix. I would therefore consider the mechanism of interface breakdown to be the most important factor that contributes to and very often initiates the cracking and failure process in cementitious materials, and indeed, in most composite materials.

56. The breakdown of the interface should therefore be considered in defining a mathematical model for the failure mechanism, and the consequent change in the stiffness of the constituents and of the interface. Failure in concrete is thus essentially a heterogeneous process and consists of a series of tensile–shear interfacial breakdowns within the system which gradually transfer the load to increasingly fewer elements sustaining the external load until complete collapse occurs.

57. I am not sure to what extent the tetrahedral model proposed by Professor Baker takes into consideration the continually changing values of stiffness and Poisson's ratio of the constituents of concrete and of the interface. Due to internal restraints to volume changes and moisture movement, local stiffness and Poisson's ratio values are quite different from those obtained from uniaxial tests even prior to application of external load. The latter causes further changes in these values. The relative stiffness of rods such as AD and FC should therefore change at every stress level; some members will lose stiffness, not necessarily at every successive stress level since micro-cracking often reaches locally a state of quasi-equilibrium with the cracks alternately stable and propagating.

58. On the other hand, it is possible that some members can show an increase in local stiffness depending upon the stress conditions there. But the numerous discrete cracking should result in progressively fewer elements carrying the external load until the stiffness of all the remaining elements reduces to zero. This does not, of course, mean that a concrete test specimen has reached its load-sustaining capability at the peak strength. The uncracked parts at this stage are by no means capable of absorbing, by themselves, the increases of strain energy due to the external load; the ability to sustain decreasing loads at increasing deformation is made possible only by the increased energy dissipation by local yielding at the interfaces and the frictional damping which occurs there.

59. This is why the differential stiffness of the aggregate and the matrix and the resulting interfacial micro-cracking are far more significant in the cracking and failure process of concrete than cracking in the mortar pockets. Indeed if the latter were more predominant in the concrete system, much of the mechanical behaviour of concrete will be quite different from that normally observed, and will impose severe limitations on its inherent advantages and use as a constructional material.

Dr J. N. Gardner, Associate Professor, University of Ottawa, Canada

Equation (1) is developed using an exact elastic analysis but Fig. 6 is plotted assuming that the quantity K_1/K_2 is a function of strain. Furthermore, the conclusion stated in § 12 that 'the classical concept of Poisson's ratio as constant material property has no meaning' cannot be drawn from a bar type elastic model. In fact the bar model could better be justified by stating that its Poisson's behaviour is similar to that measured for concrete.

61. The proposed failure criterion of critical lateral strain is a useful contribution to the literature on failure theories of concrete. However it must be remembered that equation (2) is a linear elastic formula and that concrete is not an elastic linear material. Also the physical significance of the quantities E_c and ν_c is difficult to visualize. The assumption that $E_1 = E_2 = E_3 = 2 \times 10^6$ lb/sq. in. used in equation (3) is difficult to accept.

Professor A. L. L. Baker

I greatly appreciate the work of contributors to the discussion, particularly that of **Dr R. N. Swamy,** who emphasizes the importance of attributing initiation of failure to breakdown of bond between matrix and stones rather than by tension in the mortar between the stones. Both, of course, occur, but Vile's cones in uniaxial compression and halos in biaxial compression, and McCreath's and Pigeon's photo-elastic observations seem to indicate early predominance of cracking in the mortar. Either theory explains much of the internal behaviour of concrete, and the variations of stress–strain relationships which occur with various combinations of multiaxial stress at various levels. Further research will, no doubt, produce clarification.

References

7. HANNANT D. J. and FREDERICK C. O. Failure criteria for concrete in compression. *Mag. Concr. Res.*, 1968, **20** (Sept.) No. 64, 137–144.
8. SWAMY R. N. Aggregate–matrix interaction in concrete systems. Paper 31, *Solid mechanics and engineering design in civil engineering materials.* International Conference on Structure, University of Southampton, April 1969.
9. SWAMY R. N. Application of photoelastic coating techniques to the determination of internal strain distribution in cementitious materials. Paper 36. *Fourth International Conference on Experimental Stress Analysis.* University of Cambridge, April 1970.
10. HSU T. T. C. *et al.* Micro-cracking of plain concrete and the shape of the stress–strain curve. *J. Am. Concr. Inst.*, 1963, **60** (Feb.) 209–224.
11. KEETON J. R. Photo-elastic determination of strain distribution in cement paste, mortars and concrete. Technical Report R 404, US Naval Civil Engineering Laboratory, Port Hueneme, California, 1965.
12. ANSON M. An investigation into a hypothetical deformation and failure mechanism for concrete. *Mag. Concr. Res.*, 1964, **16** (June) 73–82.
13. KOUFOPOULOS T. and THEOCARIS P. S. Shrinkage stresses in two-phase materials. *Jnl Compos. Mater.*, 1969, **3**, No. 2, 215–223.
14. THEOCARIS P. S. and KOUFOPOULOS T. Photo-elastic analysis of shrinkage micro-cracking in concrete. *Mag. Concr. Res.*, 1969, **21** (Mar.) 15–22.
15. BARNARD P. R. Researches into the complete stress–strain curve for concrete. *Mag. Concr. Res.*, 1964, **16** (Dec.) 203–210.
16. SWAMY R. N. Inelastic deformation of concrete. Symposium on Deformations of Concrete. Highway Research Board, Washington, USA.