

Structural design for minimum cost using the method of geometric programming

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The following contribution has been received.

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The Paper is a most interesting one, covering a technique which, although documented in standard text books, is comparatively unknown in this country. It is very pleasant to find a theoretical paper that seeks to demonstrate applications in a simple manner and the following observations might add to its value as a means of awakening interest in the technique.

107. Although a generalized suffix notation seems necessary for a full description of the mathematics, it might help people like myself, whom it completely baffles, to rephrase the formulation of the dual function as below, which owes its manner of presentation to the paper,⁹ and is appropriate to positive coefficient polynomial primal functions.

108. To set up the dual function:

(a) identify with each of the *total* of *T* terms, in the primal objective function together with the *M* constraint inequalities, a 'weighting' δ_t , where $t=1, 2, 3, \dots T$

(b) the dual function, *d*, is then given by:

$$d = \bar{d} \cdot d_1 \cdot d_2 \cdot d_3 \dots d_M$$

where:

$$\bar{d} = \prod_{t=1, 2, \dots T} \left(\frac{C_t}{\delta_t} \right)^{\delta_t}$$

and:

$$d_m = \lambda^m, \quad m = 1, 2, 3, \dots M$$

with:

$$\lambda = \sum \delta_t$$

the summation being over the 'weights' in the *m*th constraint inequality

and: C_t = coefficient of the *t*th term.

109. In words of as few syllables as possible it may be stated that the dual function, *d*, is the product throughout the terms, of the coefficient per unit weight raised to the power of the weight, this product being modified by the application of further multiplying factors. There is one multiplying factor for each constraint inequality and each multiplying factor is equal to the sum of the weights of the terms in the particular inequality, when raised to the power of this sum.

110. Turning, now, to the second example in the Paper, and still exploring the implications of the technique in the particular application, it is useful to extend the example somewhat further.

111. Paragraph 25 indicates the wisdom of adopting the cheaper solution of having the beams span the shorter length. Noting that equation (27) was derived with the assumption that the deflexion constraint was set as *A*/360, it might be inferred that the conclusion is valid only in this situation. If the solution is reworked allowing a general deflexion constraint such that the maximum deflexion shall not exceed *D*,

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say, then the general result replacing equation (27) is as below, where d_D implies the minimum cost solution in the presence of this deflexion constraint:

$$d_D = 0.39229\gamma A^{27/11} B \left\{ \frac{K_F^3 K_B^8 K_1^8 W^6}{D^5 E^3 K_2^4} \right\}^{1/11} \dots \dots \dots (49)$$

Substitution of $D = A/360$ in this result yields the Paper's equation (27) when allowance is made for a minor inaccuracy in Dr Templeman's arithmetic arising from the approximate nature of the second coefficient in his equation (24). It will be observed from equation (49) that, whatever the form of the deflexion constraint, the conclusion concerning the span of the beams is valid.

112. The numerical example chosen in the Paper provides a design which is governed by the deflexion constraint. A clearer view of the situation arises if limiting stress constraints are allowed in addition to deflexion constraints. Denoting the maximum allowable bending stresses in beams and floor plates by σ_B and σ_F respectively, the additional constraint inequalities are obtained:

$$\frac{WB A^2}{8nZ\sigma_B} \leq 1 \dots \dots \dots (50)$$

and:

$$\frac{3WB^2}{4n^2 t^2 \sigma_F} \leq 1 \dots \dots \dots (51)$$

113. Solution of the geometric programming problem posed by the addition of these constraints involves a problem with two degrees of difficulty, since the number of terms is increased by two, with no change in the number of variables. However, the problem is easily dealt with by resolving for a minimum cost solution in the presence of the stress constraints only. This is, again, a zero degree of difficulty problem. Graphical representation of the two solutions, or otherwise, then yields a complete description of the minimum cost solution.

114. Proceeding with this solution, and identifying weights δ_1 and δ_2 with the terms of the, unchanged, primal function, and δ_3 and δ_4 with the terms of equations (50) and (51) respectively, leads to the following results:

normality condition: $\delta_1 + \delta_2 = 1$

orthogonality conditions:

$$\left. \begin{array}{l} \text{for } t: \quad \delta_1 - 2\delta_4 = 0 \\ \text{for } n: \quad \delta_2 - \delta_3 - 2\delta_4 = 0 \\ \text{for } Z: \quad 2\delta_2 - 3\delta_3 = 0 \end{array} \right\} \dots \dots \dots (52)$$

The solution for equations (52) yields the results:

$$\delta_1 = 1/4; \quad \delta_2 = 3/4; \quad \delta_3 = 1/2; \quad \delta_4 = 1/8$$

115. Note that if stress governs the design rather than deflexion, the proper allocation of our total resources, in order to obtain a minimum cost solution, is changed. The actual minimum cost, d_B , with a stress constraint, now follows:

$$d_B = 0.81824\gamma A^2 B \left\{ \frac{K_F^2 K_B^8 K_1^8 W^6}{\sigma_B^4 \cdot \sigma_F} \right\}^{1/8} \dots \dots \dots (53)$$

Again, it may be noted that it is wise to span the beams in the shorter direction.

116. If the deflexion constraint is now defined in terms of the span of the beams, A , as A/k , where $k=360$ in the Paper, equations (49) and (53) are particularly conveniently expressed in terms of the actual minimum cost per unit floor area per unit beam span, say C_D and C_B respectively, i.e:

$$C_D = \frac{d_D}{A^2 B} = \eta_D \cdot k^{6/11} W^{6/11} \dots \dots \dots (54)$$

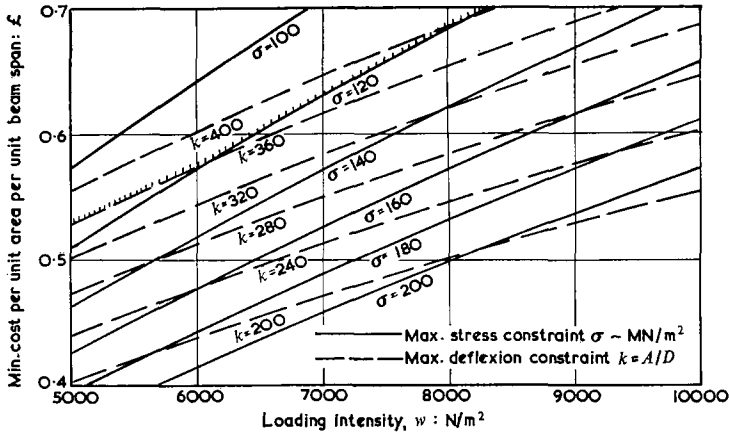


Fig. 7

and:

$$C_B = \frac{d_S}{A^2 B} = \eta_S \cdot \frac{W^{5/8}}{\sigma_B^{1/2} \sigma_F^{1/8}} \dots \dots \dots (55)$$

where η_D and η_S are numerical coefficients depending only on the specific values of the unit cost and design parameters listed in equation (28).

117. These equations give a complete description of all minimum cost solutions for any floor area carrying any intensity of loading. There seems to be no reason why similar expressions cannot be derived in somewhat more practical conditions. The results are ideal for graphical representation, in which the 'envelope' of minimum costs for any W is completely defined for any given stress and deflexion constraints. Fig. 7 does this for the values of η_D and η_S given by the data in the Paper, and with $\sigma_F = \sigma_B$, showing only a typical part of this 'minimum cost chart'. It will be noted, for example, that the shaded boundary represents the lower bound of costs for a specification of $\sigma_F = \sigma_B = 120 \text{ MN/m}^2$ and $D = A/360$.

A. B. Templeman

Dr Swannell's contribution is most interesting. Geometric programming is an extremely simple technique to understand and carry out once the user has passed the stumbling block of the mathematical formulation. The posynomial formulation referred to earlier and outlined by Dr Swannell has the advantage of being comparatively easy to comprehend but is unfortunately not a general formulation. It requires the coefficients of all polynomial terms to be positive. The formulation given in the Paper is general and applies to both positive and negative coefficients, but is more daunting in aspect. The choice between being mathematically precise and being easily understood yet mathematically imprecise is always a difficult one. I chose the general formulation because I was at that time writing the computer program which I wanted to be as comprehensive as possible.

119. The need for the program is partly seen in Dr Swannell's considerable extension of my second example. When stress constraints are added to the deflexion constraints already present a problem having two degrees of difficulty is encountered. As it stands this is difficult to solve without recourse to a computer but Dr Swannell overcomes it by considering the case of slack deflexion constraints. He is then able to solve the problem and finally draw the interesting minimum cost chart.

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120. The chart enables the minimum cost of any floor area to be virtually read off once the load intensity, deflexion and stress constraints are known. As such it would appear to be potentially very useful: the main objection to it would seem to be that the assumptions made when I set up the problem are too restrictive. In this case progress may also be made. Assumption (b) relates to the floor plates being simply supported between beams. This is obviously a great over-simplification and further examination has shown that a section composed of deck plates and beams acting compositely and spanning in the A direction may be formulated in geometric programming form. Assumption (e) may be omitted by including both deflexion and stress constraints *ab initio*. Assumption (d) may also be removed at the cost of a considerably more complicated problem. This is hardly worthwhile, and the remaining assumptions are quite innocuous.

121. This new, more representative design problem has several degrees of difficulty and preliminary investigations show it to be of polynomial and not posynomial form. With the aid of the computer program solutions should be extremely rapid. I therefore concur with Dr Swannell's view that minimum cost designs and charts could be produced rapidly for much more practical problems. Small scale investigations have already begun on several types of deck design problems and they show that the technique of geometric programming is easily applicable to reinforced concrete design and built-up steel sections as well as to the Universal sections used in example 2. The technique therefore shows promise of being a very useful design tool for a wide range of structural design problems and further lines of research are indicated in this field.

122. I would finally like to thank all who have contributed to the discussion and the many people who have shown an interest in possible applications of geometric programming in civil engineering. The technique is now being used as a basis for the development of an interactive computer-aided design system for minimum cost drainage systems. Investigation also appears to be warranted into hydraulic network design as well as several other structural design applications in addition to those described above. I hope that with continuing research and development along these lines geometric programming may prove to be a powerful design tool for the practising civil engineer.