Observations on an unusual parameter for grain fall velocity

A. S. PATEMAN

Professor K. J. Ives, University College, London

The 'unusual parameter' $N=(\rho_s/\rho-1)gd^3/\nu^2$ reported by the Author cannot be considered as new, for it appears in *Principles of chemical engineering* (1937),[3] in *Water supply and waste water disposal* (1954),[4] in *Fluidization and fluid-particle systems* (1960)[5] and *Water and its impurities* (1963).[6]

11. In this latter publication by T. R. Camp the Author's log N vs log Re plot of Fig. 1 occurs. In addition to the parameter N which does not contain a velocity, w, term, there is also another dimensionless parameter C_d/Re which does not contain a diameter, d, term, which is also plotted on Camp's graph.

12. This is particularly useful as observations of fall velocity, w, enable C_d/Re to be calculated.

$$\frac{C_d}{Re}=\frac{4g(\rho_s-\rho)\nu}{3\rho w^3} \qquad (8)$$

From the Camp curve the corresponding value of Re can be determined, and hence the diameter value

$$d=\frac{\nu Re}{w} \qquad (9)$$

This is the diameter of a sphere of the same material, with the same fall velocity. For sieve-fractionated material there is a size d_s which is the mean of the passing and retaining sieves. For spheres, $d=d_s$. This we have confirmed with spherical glass beads (ballotini). For non-spheres $d<d_s$ and the ratio d/d_s is a measure of sphericity.

13. Since the early 1960s we have been using this method of characterizing the shape of granular material. Typical results for d/d_s are ballotini 1·0, magnetite 0·90, Leighton Buzzard sand 0·85, crushed flint sand 0·78, anthracite 0·75, and crushed ceramic 0·71. These materials range in density from 1·4 to 4·9 g/cm^3, and sizes from 0·4 mm to over 2 mm have been tested. Further information is given in a paper by Mohanka.[7]

14. This test, and the use of the Camp curves, is considered so standard that it has been a routine undergraduate laboratory test at University College London since 1910.

Dr J. J. Sharp, Memorial University, Newfoundland, Canada

The Author's use of the non-dimensional parameter $gd^3(\rho_s/\rho-1)$ is interesting and he is to be commended for endeavouring to achieve a convenient solution rather than accepting a solution which is merely correct. However, the parameter is not altogether new although it may not have been used before in this particular field. It is a valid parameter for any fluid system involving viscous action and 'reduced' gravitational action, and has been used extensively in the analysis of small density difference phenomena.[8-10]

16. The method of synthesis[11] provides a simple method of formulating a dimensionally homogeneous equation in n terms, which leads, by inspection, to all possible correct forms of $(n-1)$ term non-dimensional solutions. In the present case three actions are involved, namely: viscosity, ν, velocity, w, and 'reduced' gravitational acceleration, $g'=g(\rho_s-\rho)/\rho$. (The term 'reduced' could be misleading but has

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become accepted terminology and implies that the net effect of gravity is reduced due to, in this instance, buoyancy.) Thus

$$\Phi[\underbrace{w^2/g',\ \nu/w,\ \nu^{2/3}/(g')^{1/3}}_{2^*\text{ from}},\ d] = 0 \qquad (10)$$

where 2* indicates that any two linear terms may be chosen provided each action (w, ν and g') is included at least once.

17. Equation (10) contains all possible forms of correct solution, and convenient solutions for d and w are obtained by ensuring that these variables appear only once in the final non-dimensional equation. The choice of term nos 2 and 3 leads to

$$\Phi\left[\frac{d}{\nu/w}, \frac{\nu^{2/3}/(g')^{1/3}}{d}\right] = 0 \qquad (11)$$

and since

$$\frac{\nu^{2/3}}{(g')^{1/3}d} = \left[\frac{gd^3(\rho_s/\rho - 1)}{\nu^2}\right]^{-1/3}$$

equation (11) is identical to the functional equation providing the basis for Fig. 2. An alternative equation which is equally convenient could be achieved by the choice of term no. 1 instead of term no. 2.
Thus

$$\Phi\left[\frac{w^2}{g'd}, \frac{\nu^{2/3}}{(g')^{1/3}d}\right] = 0 \qquad (12)$$

18. A parameter similar to that derived by the Author has also found applicability in consideration of flow resistance in pipes and channels.[12-14] Again three actions are involved: velocity, w, viscosity, ν, and 'reduced' gravitational acceleration, $g' = sg$, where s = slope of piezometric line.
Thus

$$\Phi[\underbrace{w^2/sg,\ \nu/w,\ \nu^{2/3}/(sg)^{1/3}}_{2^*\text{ from}},\ D,\ k] = 0 \qquad (13)$$

where D = pipe diameter
and k = size of roughness elements.

19. The common, and inconvenient solution, is that which provides the basis for the Stanton diagram, i.e.

$$\Phi\left[\frac{w^2}{(sgD)}, \frac{wD}{\nu}\right] = 0 \qquad (14)$$

However, by choosing term nos 1 and 2 rather than 1 and 3, and using k as the repeating variable

$$\Phi\left[\frac{w^2}{sgk}, \frac{\nu^{2/3}}{(sg)^{1/3}k}, \frac{D}{k}\right] = 0 \qquad (15)$$

Equation (15) gives a direct solution for w, ν and D as compared with equation (14) which provides direct solutions only for ν and k.

20. It can be seen that the second parameter of equation (15) is similar to that used by the Author, in so far as both are forms of

$$N = \frac{\nu^{2/3}}{(g')^{1/3}l}$$

where g = 'reduced' gravitational acceleration

$$\left(\frac{g(\rho_s - \rho)}{\rho} \quad \text{or} \quad sg\right)$$

and l = relevant length (d or k).

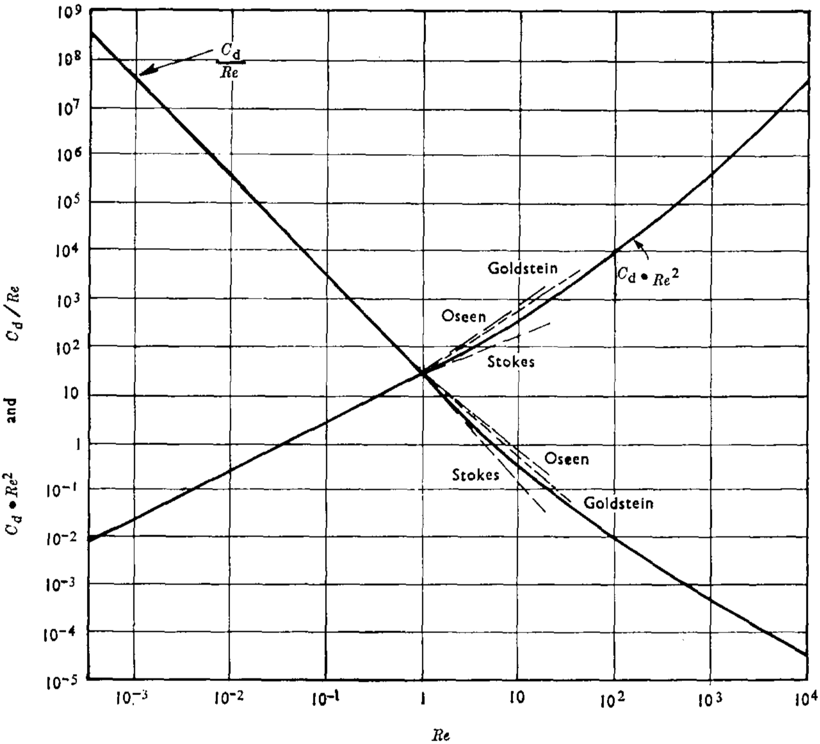

Fig. 3. $C_d . Re^2$ and C_d/Re vs Reynolds number, Re, for spheres (after Schiller and Naumann[15])

Professor W. H. Graf, Lehigh University, Bethlehem, Pennsylvania

The purpose of this contribution is to point to the fact that the non-dimensional parameter, resulting from 'an unorthodox combination of physical quantities', is not a novel concept.

22. Under the influence of gravity, a particle of $d = 2a$ will ultimately attain a steady terminal velocity, w. This may be expressed by

$$C_d a^2 \pi \frac{\rho w^2}{2} = \frac{4a^3\pi}{3}(\rho_s - \rho)g \quad . \; . \; . \; . \; . \; . \; . \quad (16)$$

where hydrodynamic forces counterbalance gravitational ones.

23. To find the terminal velocity, w, for a given particle dropped in a fluid of known properties, Schiller and Naumann[15] multiplied equation (16) by $(2a/\nu)^2$ and obtained

$$C_d Re^2 = \frac{4}{3} g \frac{\rho_s - \rho}{\rho} \cdot \frac{(a/2)^3}{\nu^2} \quad . \; . \; . \; . \; . \; . \; . \quad (17)$$

Since the right-hand side of equation (17) is readily computable, the value of the left-hand side is obtained. With help of a curve, such as that given in Fig. 3 where $C_d Re^2$ is plotted against Re, the Reynolds number Re is obtained, and in turn the desired settling velocity is found.

24. If the particle diameter, $d=2a$, is in question and the particle was dropped with a known settling velocity in a liquid of known properties, equation (16) should be multiplied by (w/ν) or

$$\frac{C_d}{Re}=\frac{4}{3}g\frac{\rho_s-\rho}{\rho}\cdot\frac{\nu}{w^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Since the right-hand side of equation (18) is calculable, consulting Fig. 3, where C_d/Re is plotted against Re, the particle Reynolds number, and furthermore the particle diameter, are obtained.

25. To my knowledge, equations (17) and (18) were proposed by Schiller and Naumann;[15] they are reviewed in Graf.[16] The two relationships facilitate an explicit approach for the terminal settling velocity, w, and/or the particle diameter, $d=2a$, respectively.

26. Credit should be given to the Author who apparently was not aware of the existence of such a relationship and developed the parameter, N, given with equation (17).

Dr J. F. Riddell, Lecturer, University of Strathclyde

The use of the conventional curve of drag coefficient against Reynolds number to predict the steady terminal settling velocity of a particle results in the same practical disadvantage to the user as that found with the Stanton diagram when it is desired to determine the velocity of flow, V, within a pipe. Because V appears in both the ordinate and abscissa parameters, that is $V/(sgD)^{1/2}$ and VD/ν respectively, where s is the piezometric slope, D is the pipe diameter and ν is the coefficient of kinematic viscosity, a trial and error solution is required. With pipe flow, however, this problem has been recognized for some time and alternative arrangements of parameters have been prepared that allow an explicit solution for V. Such an explicit solution requires that V should appear in only one parameter, or non-dimensional group, the alternative arrangement being easily obtained by compounding of the original groups.

28. Thus if a, b, c are dimensionally similar terms, and

$$\Phi(a/b, b/c) = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

then the non-dimensional groups a/b and b/c can be compounded by multiplication to eliminate b. The compounded term a/c then appears in the alternative functional arrangement together with one of its two constituent terms. Thus

$$\left.\begin{array}{ll} & \Phi(a/b\,.\,b/c,\ a/b \text{ or } b/c) = 0 \\ \text{or} & \Phi(a/c,\ a/b \text{ or } b/c) \qquad = 0 \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

In the case of pipe flow, V has been eliminated by a compounding of the groups $V/(sgD)^{1/2}$ and VD/ν

$$\Phi\{[V/(sgD)^{1/2}]^{-1}.[VD/\nu],\ V/(sgD)^{1/2} \text{ or } VD/\nu\} = 0 \quad . \quad . \quad . \quad (21)$$

giving as an alternative two term non-dimensional functional arrangement

$$\Phi\{(sg)^{1/2}D^{3/2}/\nu,\ V/(sgD)^{1/2}\} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

A plot developed on the above arrangement forms the basis of the Johnson–Langhaar diagram for pipe flow.[12]

29. Considering the non-dimensional group $(sg)^{1/2}.D^{3/2}/\nu$ (or sgD^3/ν^2, the power to which all the terms of a group are raised being immaterial), the terms of the group can be classified as:

- (*a*) sg a modified or effective gravitational acceleration, that is one applicable to acceleration along a line of slope s
- (*b*) D^3 (a linear size measure)^3
- (*c*) ν^{-2} (the kinematic viscosity coefficient)$^{-2}$

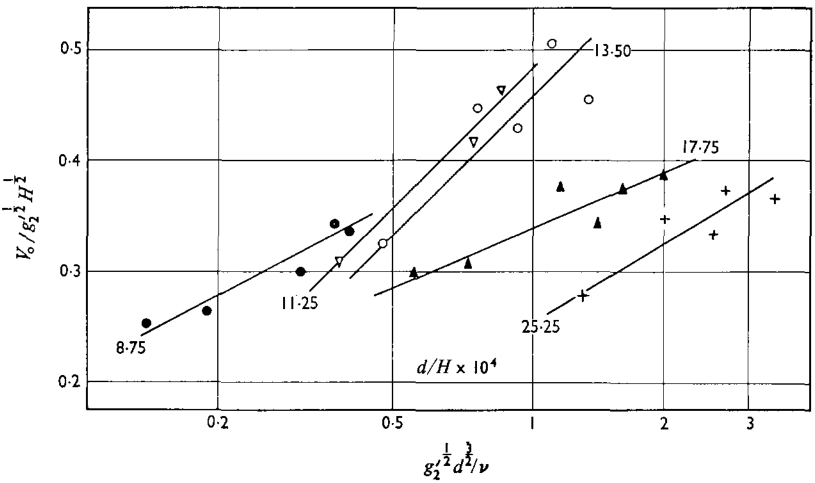

Fig. 4. Non-dimensional plot of initial underflow velocity, $V_0/g'_2{}^{1/2}H^{1/2}$, against relative particle settling size, $g'_2{}^{1/2}d^{3/2}/\nu$, for values of particle size, d/H

An alternative classification can also be made:

(*a*) $(sgD)^{1/2}$ a representative velocity giving a measure of the effective gravitational force

(*b*) ν/D a representative velocity giving a measure of the viscous resistance force

30. Thus the non-dimensional group $(sgD)^{1/2}/(\nu/D)$ is a ratio of a gravitational driving force measure to a viscous resistance force measure.

31. On examining the Author's non-dimensional parameter

$$N = \left(\frac{\rho_s - \rho}{\rho}\right) gd^3/\nu,$$

the terms of this group may be classified as:

(*a*) $\frac{\rho_s - \rho}{\rho} g$ a modified or effective gravitational acceleration, that is, one applicable to a body of fluid of mass density, ρ, that is displaced by the movement through it of a solid particle of mass density ρ_s

(*b*) d^3 (a linear size measure)^3

(*c*) ν^{-2} (the kinematic viscosity coefficient)$^{-2}$

Alternatively:

(*a*) $\left(\frac{\rho_s - \rho}{\rho} gd\right)^{1/2}$ a representative velocity giving a measure of the effective gravitational force

(*b*) ν/d a representative velocity giving a measure of the viscous resistance force

Apart from the form of the modification to the gravitational acceleration measure, the terms of the new group are seen to be similar to that adopted in pipe flow. The group itself expresses the ratio of a particle driving force measure to a particle resistance force measure.

32. The form of effective gravitational acceleration used in the parameter N is again not unknown, its being fairly standard for correlations of studies involving

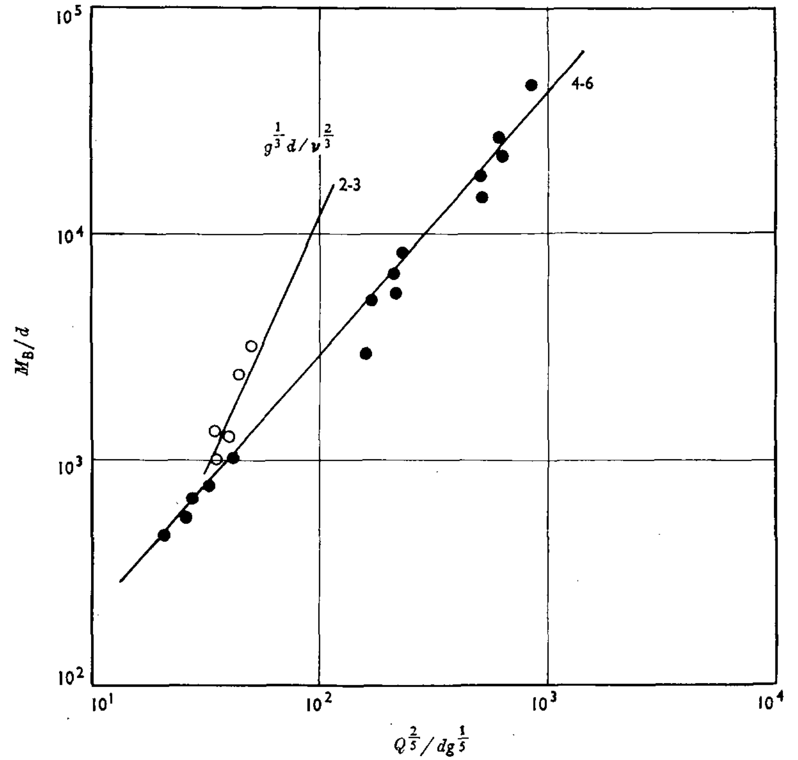

Fig. 5. Non-dimensional plot of meander width, M_B/d, against flow, $Q^{2/5}/dg^{1/5}$. for values of relative particle settling size, $g^{1/3}d/\nu^{2/3}$ (after Shahjahan[19])

particles suspended in a fluid or involving the movement of one fluid under, over or through another. It is generally denoted by g'_1, or g'_2 where

$$g'_1 = \frac{\rho_1 - \rho_2}{\rho_1} \cdot g, \; g'_2 = \frac{\rho_1 - \rho_2}{\rho_2} \cdot g,$$

and ρ_1 and ρ_2 are the mass densities of the more and less dense fluids, respectively. In studies of the interchange of salt and fresh water, Keulegan[17] used the term in the group $g'_2{}^{1/2}H^{3/2}/\nu$ (H is the overall depth of flow), this being given the name 'densimetric Reynolds number'. Subsequently termed the 'Archimedes number', the group has been found extremely useful in the correlation of data from density difference phenomena experiments.

33. Use, too, has previously been made of the group with the linear size term as a nominal particle diameter. A correlation has been made,[18] for example, of the initial velocity V_0 of a suspension effect density current (or turbidity current) formed from a suspension of particles of size, d, flowing in a channel of overall depth, H, based on the arrangement (Fig. 4)

$$\Phi\{V_0/g'_2{}^{1/2}H^{1/2}, \; g'_2{}^{1/2}d^{3/2}/\nu, \; d/H\} = 0 \quad . \; . \; . \; . \; . \quad (23)$$

Other forms of Archimedes number found effective in sediment transport correlations include $(sg)^{1/3}d/\nu^{2/3}$ and $g^{1/3}d/\nu^{2/3}$, the latter being used in alluvial sand plain studies[19] to correlate meander geometry terms such as the meander breadth M_B, Fig. 5.

34. I would thus suggest that the Author's parameter is neither new nor unusual but is a quite well established non-dimensional group. The effectiveness of the correlation undertaken by the Author merely demonstrates the benefits to be obtained by the use of this and other arrangements of terms which for some strange reason are considered unorthodox.

Mr M. R. Gourlay, Senior Lecturer in Civil Engineering, University of Queensland, St Lucia, Queensland, Australia

I wish to comment on two points raised by the Author: first, the avoidance of a trial and error solution for evaluating the fall velocity from the particle characteristics and second, the effects of particle shape on the fall velocity.

36. With regard to the direct calculation of the fall velocity, w, the use of the N–Re diagram is to be commended since the calculation is very simple. It is worth noting that the parameter N has also been used for the same purpose in replotting the Shield's diagram for the initial movement of sediment particles. For instance, Bonnefille[20, 21] has produced N–Re^* graphs, where $Re^* = v_* d/\nu$ for the initial motion condition and also for separating the zones of occurrence of various bed forms such as ripples, dunes, etc.

37. Direct calculation of w can also be made from a graph in which N is plotted against the parameter

$$M = \frac{w^3}{\left(\frac{\rho_s}{\rho} - 1\right) g\nu} = \frac{4}{3}\frac{Re}{C_d} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

38. A graph of this form, but with $(\rho_s/\rho - 1)$ as a parameter, has been produced by Bewtra.[22] Again an analogous graph can be produced for the initial movement of sediment.[23] Such a graph has the apparent advantage that the particle size only appears in the parameter N and the fall velocity in the parameter M. Further, if $M^{1/3}$ is plotted against $N^{1/3}$, the resulting graph is a universal dimensionless relationship between w and d with particle shape being the only parameter. It is, however, not quite so easy to use for practical computations of w as the Author's diagram.

39. Returning to the Author's Fig. 2 and the question of particle shape, I have added to it curves for isometric particles of various shapes derived from the published data of Pettyjohn and Christiansen.[24] These include spheres, cube octahedrons, cubes and tetrahedrons (data for octahedrons are also given by these authors, but have not been used by me). Pettyjohn and Christiansen found that the shape effects of such particles could be represented by the sphericity, ψ, of the particles; ψ is defined as the ratio of the surface area of a sphere of the same volume as the particle to the surface area of the particle. The size of the particles is represented by the spherical or nominal diameter which is the diameter of a sphere of the same volume as the particles. For the isometric particles used these quantities can be very easily calculated for a given shape.

40. Figure 6 shows a plot of N versus Re for these data together with the Author's data. To avoid confusion, the individual points have not been shown on the graph. The effect of shape, as predicted by the Author, is quite marked. The smaller the sphericity the greater the divergence from the curve for spheres at higher values of Re. On the other hand, the shape effects are not nearly so great at low Re. An interesting point which can be gleaned from the remarks column of the data tables of Pettyjohn and Christiansen[24] is that the point where the curves for the angular particles diverge from that for spheres corresponds to a significant change in the mode of fall of the particles. At low Re the particle oriented itself with one of its surfaces

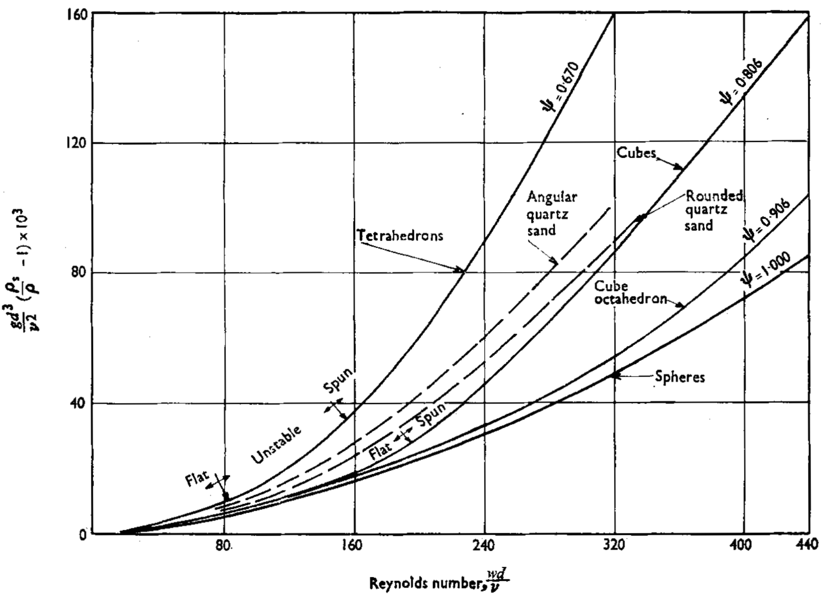

Fig. 6

in the horizontal plane, indicated by the word 'flat' on Fig. 6. On the other hand, at higher values of Re the particle revolved about its axis and followed a spiral path in its descent, indicated by the word 'spun' on Fig. 6. This difference of behaviour naturally does not occur with spheres and apparently also not with cube octahedrons. It did occur with cubes where the transition occurs fairly abruptly at about $Re = 180$. Once the particle commences to spin, the shape effect upon the fall velocity increases significantly. The behaviour is similar for tetrahedrons except that there is a longer transition between the two modes of fall during which the particle is unstable.

41. The Author's natural particles lie between the cubes and tetrahedrons, but have a somewhat smoother transition between the two regimes. This is to be expected since the various grains of a given sieve size from which the average values were calculated presumably had a range of shapes, i.e. different sphericities. The average sphericity of the quartz particles apparently lies between about 0·7 for the angular particles at $Re = 150$ and about 0·8 for the rounded particles at $Re = 300$. It would be of very great interest if it were possible to measure the sphericity of the natural particles and see if it fits consistently into the pattern shown by the isometric particles. In this regard it may be noted that for large Re (<2000) Pettyjohn and Christiansen found that C_d was a simple function of the sphericity, ψ, i.e.

$$C_d = 5{\cdot}31 - 4{\cdot}88\psi. \qquad (25)$$

42. Comparison with the corresponding value of C_d for natural sediments, approximately 1·35–1·4 according to Engelund and Hansen,[25] gives an average sphericity for large particles of about 0·8 or a little greater.

43. I believe that the direct determination of the sphericity of natural sediment particles would be a very useful step in the understanding of the influence of particle shape on particle motion.

Mr D. Kane, Engineering Assistant, Babtie, Shaw & Morton, Glasgow

By using dimensional analysis and equating the drag force, F_D, on a particle to the downward weight force the Author has derived a new parameter for the fall velocity of grains. As is so often the case the dimensional analysis approach has been effective in working to a known or anticipated result. Had a similitude analysis been employed then all possible arrangements of the relevant variables would have been readily available.

45. Adopting the Method of Synthesis, a form of similitude analysis in which the relevant variables in a system are arranged into a dimensionally homogeneous functional equation which is further arranged into a non-dimensional form, the new parameter can be obtained.

46. To produce the initial functional equation it is often convenient to work in linear terms and hence the measures of actions in the system are combined in pairs to form a ratio which is subsequently reduced to linear terms.

47. Thus, in this case, for the terminal velocity, w, of a particle of diameter, d, and mass density, ρ_s, settling in a fluid of mass density, ρ, and coefficient of viscosity, ν, three proportionalities may be produced. These are:

$$\frac{w^2}{\dfrac{\rho_s-\rho}{\rho}\cdot g}$$ combination of a system action with the effective gravitational action

$$\frac{\nu^{2/3}}{\left(\dfrac{\rho_s-\rho}{\rho}\cdot g\right)^{1/3}}$$ combination of viscous force action with effective gravitational action

$$\frac{\nu}{w}$$ combination of viscous force action with the system action.

Since there are three force actions in the system only two proportionalities are required along with d to define the system. In making the choice it is necessary to ensure that each action is included at least once, although in this case no special care need be taken.

48. Thus, the system is defined by

$$F\left[\nu^{2/3}\Big/\left(\frac{\rho_s-\rho}{\rho}\cdot g\right)^{1/3}, \nu/w, d\right] = 0 \quad . \; . \; . \; . \; . \; . \quad (26)$$

which on rearrangement into a non-dimensional form gives

$$F\left[\nu^{2/3}\Big/\left(\frac{\rho_s-\rho}{\rho}\cdot g\right)^{1/3} d, \nu/wd\right] = 0 \quad . \; . \; . \; . \; . \; . \quad (27)$$

or

$$F\left[\frac{\rho_s-\rho}{\rho}\cdot gd^3/\nu^2, wd/\nu\right] = 0 \quad . \; . \; . \; . \; . \; . \; . \quad (28)$$

49. Equation (28) is therefore seen to contain the two parameters used for the plot of Fig. 2. Had the influence of the other effective gravitational force, i.e. $g(\rho_s-\rho)/\rho_s$ as applied to the settling particle, been introduced in equation (26) the relative density parameter ρ_s/ρ would also have appeared in equation (28).

50. By selecting the terms $\nu^{2/3}/(g(\rho_s-\rho)/\rho)^{1/3}$ and $w^2/g(\rho_s-\rho)/\rho$ for inclusion in equation (26) together with d, the following relationship is obtained

$$F\left[w\Big/\left(\frac{\rho_s-\rho}{\rho}\cdot gd\right)^{1/2}, \frac{\rho_s-\rho}{\rho}\cdot gd^3/\nu^2\right] = 0 \quad . \; . \; . \; . \; . \quad (29)$$

51. The new parameter $((\rho_s-\rho)/\rho)gd^3/\nu^2$ is therefore seen to be a non-dimensional group formed by the ratio of a gravity force measure and a resistance force measure. It is, therefore, a combination of the Froude number, $w/(((\rho_s-\rho)/\rho)gd)^{1/2}$, and the Reynolds number wd/ν arranged such that the settling velocity is eliminated.

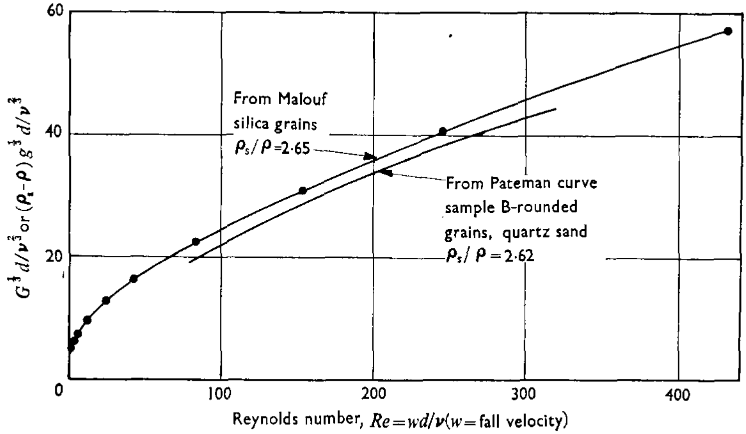

Fig. 7

52. Similitude analysis, by exhibiting all the possible arrangements of the variables, is thus seen as a more powerful tool than dimensional analysis and is particularly suited to problems in sediment movement.

Professor K. M. Malouf, Partner, Associated Consulting Engineers, Beirut, and Professor-Lecturer in Hydraulic Engineering, American University of Beirut, Lebanon

The non-dimensional parameter for the solution of the problem of estimating the fall velocity of particles given by the Author could well form the basis for a generalized solution for particles of all shapes.

54. Some standard method for establishing a representative diameter for particles of irregular shape could perhaps be developed. I have carried out experiments[26] on Perspex and silica particles in which the grains were first sieved and then a representative diameter for each grade obtained by the average of two methods: travelling microscope and settling velocity using previously established curves.[27] It would be interesting to know by how much this dimension would differ from that of the sieve aperture used by the Author.

55. If the law of resistance is written as drag force = submerged weight of grain, then

$$C_d \rho A w^2 = (\rho_s - \rho) g V \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

where A and V are the representative cross-sectional area and the volume of the grain, respectively. Also, since it has been shown that the coefficient of drag, C_d, is a function of the particle Reynolds number, wd/ν, then the relationship may be written in the form

$$C_d = C_s G d / w^2 = \phi(wd/\nu) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

where $G = (\rho_s/\rho - 1)g$ and
C_s = a shape factor defined by the relationship $C_s d = (V/A)$ and with an obvious value of 2/3 for a sphere.

56. Since the function ϕ is not as yet established, suitable manipulation of equation (31) can yield two non-dimensional parameters[26]

$$N_1 = C_s^{1/3} \frac{dG^{1/3}}{\nu^{2/3}} = \Phi_1(wd/\nu) = \Phi_1(Re) \quad . \quad . \quad . \quad . \quad . \quad (32)$$

and another important but less practical parameter

$$N_2 = C_s^{1/3}\frac{\nu^{1/3}G^{1/3}}{w} = \Phi_2(wd/\nu) = \Phi_2(Re) \quad . \quad . \quad . \quad . \quad (33)$$

57. It is apparent that one of these non-dimensional parameters N_1 is practically identical with the Author's N since

$$N_1^3 = C_s\frac{d^3G}{\nu^2} = C\frac{gd^3(\rho_s/\rho - 1)}{\nu^2} = C_sN \quad . \quad . \quad . \quad . \quad (34)$$

the only difference being an exponent and the dropping of the shape factor C_s. Dropping the shape factor renders a series of results applicable only to particles of a similar degree of roundness or angularity—whatever that may mean. Also, in view of the inaccuracy which would invariably enter when attempting to represent a particle of irregular shape by one single linear dimension, d, the parameter N_1 in which d appears in the first power should prove less inaccurate than N where d has been cubed.

58. In Fig. 7, results that I have previously obtained[26] are shown plotted along with one of the curves in Fig. 2. The relative position of the two curves suggests a ratio of the shape factor C_s between the two types of particles of about 1·26.

59. The relationships in equations 32 and 33 may be used to determine either the terminal speed or the representative grain diameter when either one is known. Also, the equations could become universally applicable once a method (perhaps photographic) can be devised to render an irregular particle shape sufficiently recognizable to permit a predetermined shape factor to be assigned to it. I hope that the Author will succeed in developing the necessary empirical curves to enable this to be done.

Mr A. S. Pateman

On reflection, describing the parameter evolved as new or unusual may have been somewhat rash but nevertheless it is apparently not as widely known as it might be, especially in the field of loose boundary hydraulics, e.g. Raudkivi does not mention it.[28]

61. In particular, the contributions of **Dr Sharp** and **Mr Kane** and also of **Dr Riddell** illustrate very clearly the use of the method of synthesis described by Barr[11] and its value as a powerful tool in this field enabling all possible arrangements of the relevant variables to be determined.

62. **Mr Gourlay** has, by adding the curves for other shapes from the results of Pettyjohn and Christiansen,[24] shown that for each shape of particle there is (as may be expected) an individual curve and that the plotting of the curves for natural sands shows to which isometric particle shape their behaviour most nearly corresponds. As regards sphericity, this is an admirable measure of the shape of a particle but is difficult to measure (the fluorometer method of Alger and Simons[29] may be of interest here but it was only applied by them to particles larger than 2·5 cm) and define (at least four definitions are known, all of which give somewhat differing results). Under these circumstances it is perhaps of more practical value to compare the behaviour of natural shapes with that of isometric particles of known or calculable sphericity.

63. **Professor Ives** has drawn attention to the work of Camp[6] and earlier work. The method of characterizing the shape of granular material he describes is obviously useful and provides a further measure of sphericity. It would be of interest to have available the collective results for the many materials of various shapes that must have been tested at University College since this test was introduced.

64. As suggested by **Professor Malouf**, some such method as was described could well form a basis for a generalized solution describing both the effect of shape and manner of fall of a particle by using a 'shape factor' or measure of 'sphericity' (providing an agreed definition of these terms could be obtained) or by comparing the falling characteristics of natural particles with those of known geometric properties.

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