17. J. F. Baker and J. W. Roderick, "The Behaviour of Stanchions Bent in Double Curvature." *Welding Research*, vol. 2, p. 2 (Feb. 1948).
18. J. W. Roderick and J. Heyman, "Approximate Methods of Calculating Collapse Loads of Stanchions Bent in Double Curvature." *Welding Research*, vol. 2, p. 63 (Aug. 1948).
19. J. F. Baker and J. W. Roderick, "Further Tests on Stanchions." *Welding Research*, vol. 2, p. 110 (Dec. 1948).
20. J. W. Roderick and M. R. Horne, "The Behaviour of a Ductile Stanchion Length when Loaded to Collapse." Report FE1/11, Brit. Weld. Res. Assocn, 1948.
21. J. F. Baker, M. R. Horne, and J. W. Roderick, "The behaviour of continuous stanchions." Proc. Roy. Soc., Series A, vol. 198 (1949), p. 493.
22. J. W. Roderick, "Tests on Stanchions Bent in Single Curvature about Both Principal Axes." British Welding Journal, vol. 2, p. 217 (May 1955).
23. J. Heyman, "Tests on I-Section Stanchions Bent about the Major Axis." Report FE1/40, Brit. Weld. Res. Assocn, 1953.
24. A. Robertson, "The Strength of Struts." Selected Engineering Paper No. 28, Instn Civ. Engrs, 1925.
25. J. F. Baker and E. L. Williams, "The Design of Stanchions in Building Frames." Final Report, Steel Structures Research Cttee, H.M.S.O., 1936, p. 511.
26. M. R. Horne, "The Flexural-Torsional Buckling of Members of Symmetrical I-Section under Combined Thrust and Unequal Terminal Moments." Quart. J. Mech. App. Math., vol. 7 (1954), p. 410.
27. M. G. Salvadori, "Lateral Buckling of I-Beams." Trans. Amer. Soc. Civ. Engrs, vol. 120, p. 1165 (1955).
28. B.S. 4, 1932. "Dimensions and Properties of British Standard Channels and Beams for Structural Purposes." British Standards Institution.
29. W. F. Cassie and W. B. Dobie, "The Torsional Stiffness of Structural Sections." *Structural Engineer*, vol. 26, p. 154 (Mar. 1948).
30. J. F. Baker and E. L. Williams, "A Study of the Critical Loading Conditions in Building Frames." Final Report, Steel Structures Research Cttee, H.M.S.O., 1936, p. 471.

The Paper, which was received on 24 September, 1955, is accompanied by twenty-three sheets of diagrams, from which the Figures in the text have been prepared.

Discussion

Dr R. H. Wood (Principal Scientific Officer, Building Research Station), after complimenting the Author, said that designers by the elastic theory had become accustomed to the alternative arrangements of loading which could be obtained on the same frame, and both he and Professor Baker before him on the Steel Structures Research Committee had tried to follow those alternatives. Dr Wood had come to the conclusion that what was good for the goose was good for the gander, and in the present case he thought that if the live load was taken off certain floors, so as to promote the critical arrangement corresponding to the lowest mode of buckling, it might be possible, even with the same frame, to change the $P_x \,.\, P_y$ case into a $P_x \,.\, E_y$ or an $E_x \,.\, E_y$ case, with the probability of a lower collapse load. He thought, therefore, that it was a question of finding the absolute minimum of various cases mentioned in the Paper.

He then referred to a stanchion that had collapsed as an $E_x \,.\, E_y$ case (Fig. 26). The Cambridge team had done well to point out that that sort of thing was possible. It would

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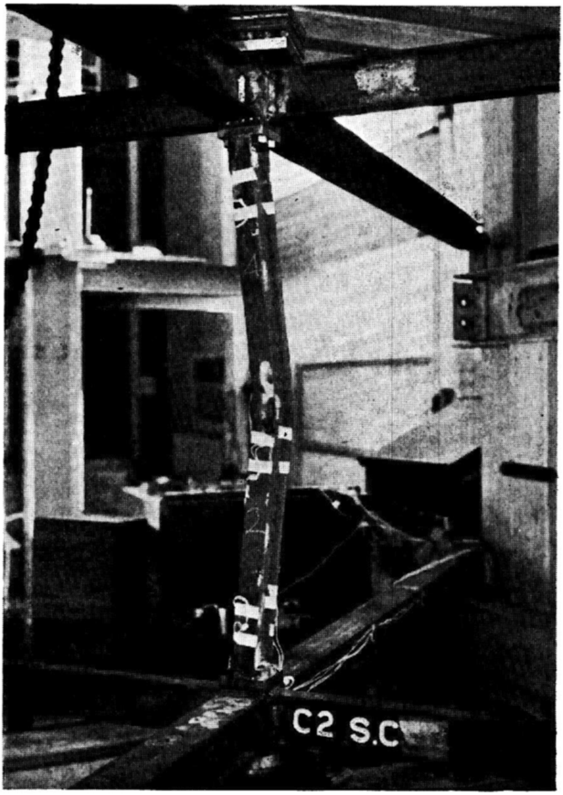

FIG. 26.—STANCHION COLLAPSED AS AN $E_x \, . \, E_y$ CASE

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be noticed that to make use of the plastic hinges at the ends it was necessary to have stiff beams. The load was not shown on the beams, but it developed the kind of collapse shown even with load on the beams. That particular stanchion would support nearly twice the Euler load about the minor axis.

Dr Wood then referred, by means of slides, to the analysis of elasto-plastic states of stanchions by means of a differential analyser,[31] and showed a set of elastic-plastic moment-rotation relationships for the integrated effects from one end to the other. Such generalized results for $P/P_E = 0, 0{\cdot}6$, and $1{\cdot}0$ had already been obtained.[32] At 60% of the Euler load there was positive rotational stiffness of the stanchion, if still elastic, which might eventually become negative stiffness if plasticity developed.

After extracting in Fig. 27 from those known results the properties dealing with the single-curvature type of beam loading at various direct loads, the question arose of how

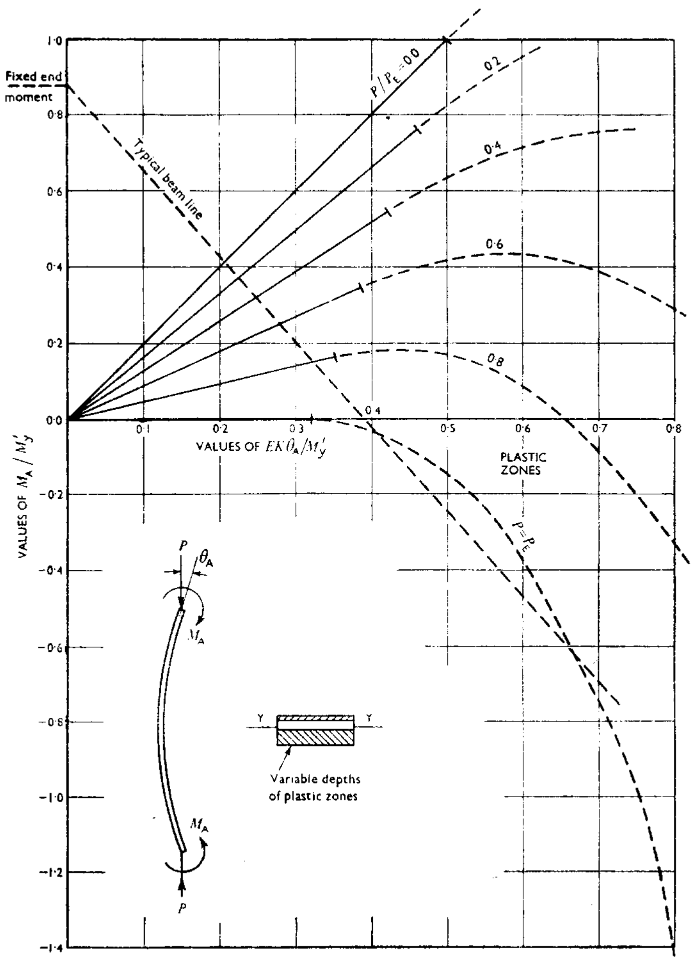

FIG. 27.—EQUILIBRIUM STATES FOR RECTANGULAR STANCHIONS WITH EQUAL AND OPPOSITE END MOMENTS

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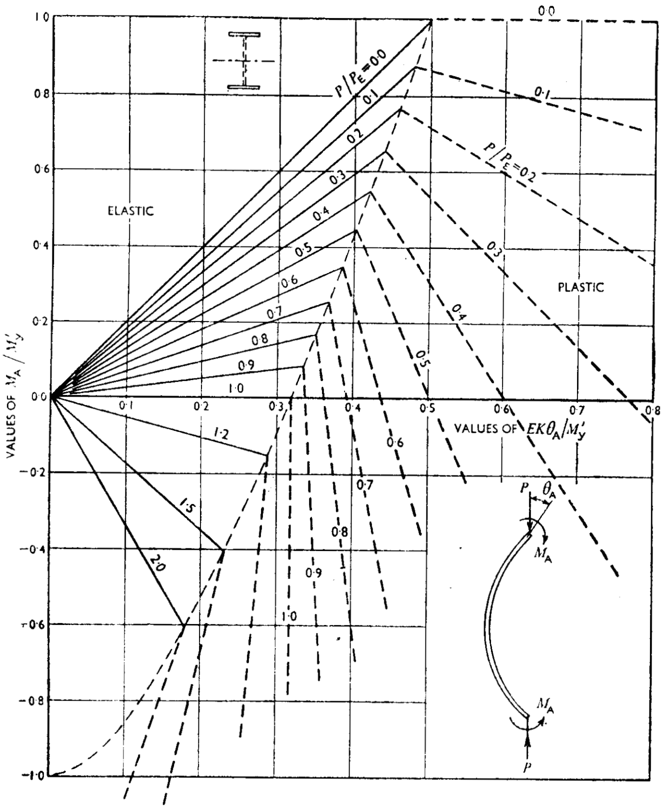

	Minimum values	
P/P_E	$\frac{fy}{E}\left(\frac{L}{k}\right)^2$	$\frac{L}{k}$ for $fy = 45{,}000$ lb/in^2 $E = 30 \times 10^6$
0	0	0
0·4	3·95	51·3
0·8	7·90	72·6
1·0	9·87	81·1
1·2	11·84	88·9
1·6	15·79	102·6
2·0	19·74	114·7
4·0	39·48	162·2

Note. Direct stress requires the following limitation:

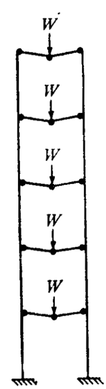

Fig. 28.—Equilibrium states for a pair of parallel plates with equal and opposite end moments

Fig. 29

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advanced stages of plasticity corresponding to those shown in Fig. 26 could be used. It was possible to start with zero rotation and corresponding fixed-end moment on the beams, and relaxing the system would then give a "beam line" with a positive stiffness, which would be a straight line (Fig. 27). If, however, at any stage the beam went plastic the corresponding beam line would then shoot off horizontally. It would be seen at once, since the dotted lines represented plastic states and the full lines elastic states, that if the beam became plastic the chances of getting anything superior to elastic design for the stanchion were very remote, and that agreed with the Author's conclusions. To use the advanced elasto-plastic states at heavy direct loads would necessitate a beam which was still elastic.

Fig. 28 showed corresponding states for a pair of parallel plates. It was interesting, because the form factor was unity, whilst in the previous case it had been 1·5. There was positive stiffness to start with and an abrupt change to negative stiffness. At the Euler load the stiffness of such an elastic stanchion was zero, whilst plasticity resulted instantly in a stiffness of minus infinity. That showed the effect of plasticity when the form factor was very small but the effect would be limited since it applied usually only to the strong axis.

Commenting on the subject of overall frame stability, Dr Wood said that that was a feature which was probably not very important at the moment, because the Author's method applied to no-sway conditions; but Dr Wood wished to show what happened when what he would call a deterioration of the stiffness matrix took place with any kind of plasticity developing anywhere. It would depend, of course, on where it did develop. In Fig. 29 which showed vertical loads only carried by the beams and no wind loads, it was found that even in the elastic region there was a critical load for the whole frame. The best work on that subject was that by Dr Merchant and Dr Chandler at Manchester University. The corresponding stanchion loads for the overall critical load would be about twice the Euler load if side-sway was prevented. If plastic hinges developed on the various beams in Fig. 29, bearing in mind that there were only vertical loads on the frame and that side-sway was prevented, the critical load was gradually brought down until at the most there was approximately the Euler load in each stanchion length. If plasticity developed in the stanchion lengths the critical load would come lower still.

He would now assume that side-sway was not prevented. That was going to be a difficulty in the future, though it did not apply to the present Paper. The upper critical load for the whole frame would probably correspond to about $0{\cdot}4P_E$, i.e., about 40% of the Euler load for any individual stanchion. It would be noted that that was already a very considerable drop of about 5:1 even if the frame stayed elastic. As the plastic zones developed in the beams with side-sway not prevented, the bottom limit for the critical load in the stanchions would reduce to nearly zero, because, taking the left-hand stanchion with the hinges as shown, the equivalent length of that stanchion was then about twice the height of the building. It had practically no restraint left, and no stanchion would stand up to that, not even about the strong axis. It meant that the frame could never get into that kind of collapse mode at all. The structure would collapse sideways instead. In general, as the moment-distribution process carried on, raising the carrying capacity, the upper critical load of the whole structure was continuously reduced by the development of plastic zones, and the collapse of the whole frame would take place when those tendencies met.

Two conclusions might be made. First, it had been a wise decision on the part of the Steel Structures Research Committee to limit simple design methods to cases not involving side-sway, where the side-sway was prevented by the walls and floors. Secondly, there were obviously various ways in which it was possible to deal with the acute problem of elastic-plastic states. At the Building Research Station their terms of reference were slightly different, in that they were expected to produce the simplest possible design consistent with increasing economy.

Dr Jacques Heyman (University Lecturer in Engineering, University of Cambridge) that the Author had adopted an excellent classification system when reducing the

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possible forty-five different cases of end conditions for stanchions to the nine shown in Fig. 5. It would probably be agreed that those covered most of the stanchion terminal conditions within the limits which the Author had imposed on himself, and Dr Heyman was pleased to hear that the Author had promised a new Paper on the plastic-hinge terminal condition.

Dr Heyman was also impressed with the efficient way in which the Author had made certain simplifying assumptions, not only to enable him to solve the problem but to give results which appeared always to be on the safe side. Dr Heyman believed that the results given in the Paper could be used with perfect confidence in design. The Author had intimated that he was not presenting a design process, but Dr Heyman believed that it was possible even at the present stage to design stanchions from the curves given in the Paper, and offered a spot check on the proposals in the form of a very simple design example.

At first glance it did not appear to fit into the Author's nine terminal conditions, but on further analysis it would be found that it could be fitted in. What he wanted to consider was a continuous heavy plate-girder system designed by the plastic theory, in which each span had a full plastic moment at the ends and also at the centre; i.e., each span became statically determinate at collapse. On the drawing-board it was possible to work out the reactions at each support and therefore find the loads transmitted to those supports. The loads coming on to the stanchions used with that plate-girder system would be known, and it should be possible to design the stanchions plastically by the methods which the Author had proposed. "On the drawing-board" the forces on the stanchions would be acting nicely on the centre-lines, but in practice a centre-line was rather an intangible thing and various codes differed in the eccentricity which could be allowed in designing, say, a capped stanchion with no true moment connexion to the beam system which it supported. In fact the reaction on the stanchion could shift by a few inches without in the least affecting the collapse system for the continuous plate girder, so that although one had a statically determinate system there was a degree of indeterminacy about where the reaction came on the stanchion.

In Fig. 30 Dr Heyman had taken as an example a very heavy plate-girder section, perhaps slightly unrealistic, 16 in. sq. with 2-in. flanges and 1-in. web and a length of 20 ft. The Author's torsion function was extremely high, 200 (over 100 there was virtually no need to worry about instability due to torsion). It was basically a 1,000-ton stanchion at collapse. The figure of 1,032, calculated by the methods proposed by the Author, corresponded to axial loading. Dividing that figure of 1,032 by 1·75, one obtained a working value corresponding to a load factor of 1·75 of 590 tons, corresponding to the value given in B.S. 449 of 475. The Code of Practice said that an eccentricity of one-third the width of the stanchion shaft should be allowed for—in the present case 5·3 in. That led to a working load of 294 on the stress system of analysis of the stanchion. The Author's method, fitting the stanchion into the $P_x . P_y$ classification, gave a load of 583 tons at which the yield stress was reached at the top of the stanchion. Dividing by 1·75 gave 333 tons, which was not very different from the figure of 294 tons. The British Standard asked for an eccentricity of half the shaft width and led to a value of 246 tons; the Author gave a value of 467 tons (or 266 tons when divided by the load factor).

Going to a still further eccentricity, the 9·38 in., which Dr Heyman had labelled (i), there was a still further reduction to 423 tons. The figure of 509 tons in the line beginning 9·38 (ii) corresponded to some unpublished work by the Author in which he had allowed a full plastic moment to develop at the top of the stanchion. It was possible to read off from a curve the axial stress of a stanchion at collapse, in the case in question 6·7 tons/sq. in., leading to the figure of 509 tons, and from that figure of axial load one could work out the reduced full plastic moment and therefore the eccentricity of loading, which came out to 9·38 in. One therefore got a measure of the safeness of the Author's work in the present Paper. The figure of 1,032 tons was reduced progressively as the eccentricity was increased, but jumped up again when full plasticity was developed. The attainment of the yield stress at the end of the stanchion corresponding to the figure of 423 tons did not represent the ultimate carrying capacity of the stanchion; there was a margin of safety, and the load could go up to 509 tons.

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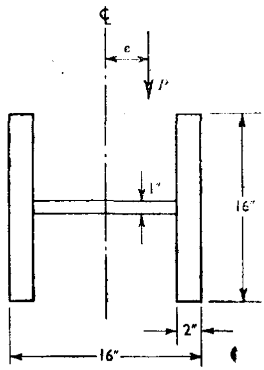

$l = 20$ ft $l/r_x = 36{\cdot}5$ $l/r_y = 55{\cdot}6$ $T = 202$ tons/sq. in.

Eccentricity e, (in.)	Values of P (tons) and p (ton/sq. in.)			
	B.S. 449	C.P. 113	Horne (collapse)	Horne ($\div$ 1·75)
0	475 (6·25)	475 (6·25)	1,032 (13·6)	590 (7·8)
5·33	—	294 (3·87)	583 (7·78)	333 (4·38)
8	246 (3·24)	—	467 (6·15)	266 (3·51)
9·38 (i)	—	—	423 (5·57)	242 (3·18)
9·38 (ii)	—	—	509 (6·7)	291 (3·8)

Fig. 30

A further interesting point came out of that. The figure of 1,032 tons for the zero eccentricity corresponded to failure of the stanchion by instability at the centre, the condition being $p + N_x f_x < f$, the function f coming from the chart which the Author gave. At the eccentricity of 5·3 in., the collapse condition was determined by the attainment of the yield stress at the end of the stanchion, and the stanchion itself did not become truly unstable. Similarly, the greater eccentricities also corresponded to the attainment of yield stress at the end of the stanchion. If one wrote the condition that the yield stress should be reached at the end of the stanchion and also that the Author's function f should just be reached, that corresponded to a very small eccentricity of about 2 in. in the present example, well below the values specified by the British Standard and by the Code of Practice. The corresponding axial load for that condition at collapse was about 11 tons/sq. in., or 850 tons in round figures.

In practice Dr Heyman thought that what would happen with the particular design example which he had chosen of a continuous plate-girder system was that the load might at first be applied at some fairly large eccentricity, and that would presumably cause yield at the top of the stanchion; there would then be some relaxing of the eccentricity, the point of application of the load moving in towards the centre-line; and the ultimate

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condition would be reached when in the present example the load became about 2 in. eccentric and the stanchion then began to fail by true instability rather than by reaching the yield stress at the end.

Dr. Heyman did not wish to make too much of that example or conclusion, but it brought out another point of excellence in the Paper, which was that the Author had pointed out not only the load at which a stanchion collapsed but also *how* the stanchion collapsed, whether by attainment of the yield stress at the end or by overall torsional buckling combined with lateral bending instability. That might open up the way to tackle problems such as the one which Dr Heyman had presented, of eccentricity of loading, and a host of others which up to the present had been intractable.

Mr Eric Ingerslev (Director, Alderton Construction Co. Ltd) expressed some hesitation in commenting on the excellent Paper which the Author had presented, because it put the whole matter so clearly, and because of the great simplicity at which it aimed. He wished to comment, however, on the main equation in the Paper, equation (4), which was $\frac{1}{F}\left(\frac{M_x'}{M_E}\right)^2 + \frac{P}{P_E} = 1$. He had been interested to read that equation, because 10 years ago he had looked into those problems and had come to the same equation, and in 1948 he had presented a Paper in which that was mentioned.[33]

First, that equation was an approximation. For the case of a column with a constant moment in the column it was exact, but for any other moment distribution it was not, but it was on the safe side. The equation was based on the deformation due to buckling from moments only being roughly the same as the Euler deformation, and that was approximately the case when the moment distribution was symmetrical. In the case of antisymmetrical moment distribution, however, it no longer held good.

Starting with the same moment applied top and bottom turning the same way, but with no load on the column, the angle of twist would increase from zero at one end to its maximum half way and then down again to zero at the other end, i.e., describing a symmetrical curve. But since $EI_y\frac{d^2y}{ds^2} = \frac{I_x - I_y}{I_x} M\beta$ where y denoted sideways deflexion, M the moments, and β the twist; a symmetrical twist curve $\times$ antisymmetrical moments must produce antisymmetrical deflexion, i.e., the strut would deflect in an S-shape and the corresponding critical column load would be four times the Euler load and the basic equation would become $\left(0{\cdot}391\frac{M}{M_E}\right)^2 + \frac{P}{4P_E} = 1$.

If, on the other hand, one started with a column load but no moments, that would produce a symmetrical deflexion and consequently an antisymmetrical distribution of the angle of twist, i.e., the angle of twist would be zero halfway and the beam, as far as buckling from moments, would act as if consisting of two beams of half length each with triangular moment distribution, i.e., according to Table 1 on p. 124 the corresponding buckling moment became $M = \frac{2}{0{\cdot}565}M_E = 3{\cdot}55M_E$ and the basic equation turned into $\left(0{\cdot}282\frac{M}{M_E}\right)^2 + \frac{P}{P_E} = 1$. Fig. 31 showed graphically the various basic equations—in thin line the Author's equation, and in thick line the combined effect of the two abovementioned equations.

The Figure illustrated the large margin available at certain load combinations. It would be most valuable if the series of experiments now carried out at Cambridge could be made to include tests to check that curve and even more the intermediate cases, where the top and bottom movements were not equal, and where the analytical investigation was considerably more complicated.

Obviously simplicity must not be lost sight of if a general workable solution was to emerge, but there were two more factors, which contained a considerable further margin in certain cases and which could easily be included without spoiling the simple picture.

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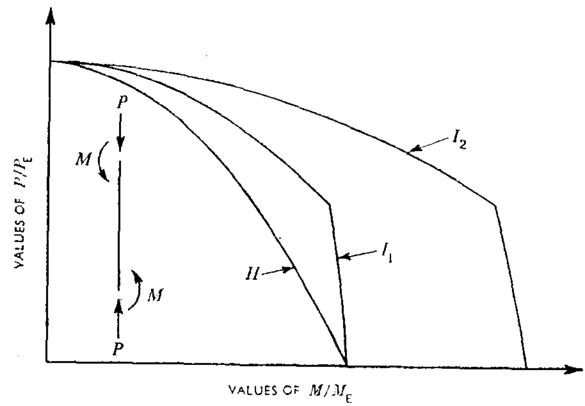
According to the Paper, $M_E{}^2 = \frac{\pi^2}{l^2}(EI_y)(GK)$ or $M_1 = \frac{k}{l}\sqrt{EI_yGK}$ in which $k = \pi\sqrt{F}$ where M_1 denoted the critical moment for $P = 0$. Now, in the literature on the subject, in the first instance in 1899 independently by Michell and Prandtl, that equation was invariably derived by investigating a narrow rectangular section where I_y was very small compared with I_x. A close investigation would show that a factor $\frac{I_x}{I_x - I_y}$ should be added to the expression for M_1 which checked with the fact, that for $I_x = I_y$ it was impossible to make a beam fail by buckling sideways. For an I-section, that factor might be quite considerable, e.g., a 10 in. × 8 in. R.S.J. would have added as much as $23\frac{1}{2}\%$.



Basic equation
H according to Horne
I_1 according to Ingerslev
I_2 according to Ingerslev, but taking in resistance to warping and factor $\frac{I_x}{I_x - I_y}$

FIG. 31

Further, it was stated in the Paper that the resistance to warping was neglected, but he felt that that could be taken into account fairly easily, and by indicating the point of warping it might be possible to improve by simple means the capacity of the column very considerably. Given a beam and column subject to a constant moment, he would represent the factor of stiffness to warping by $\alpha^2 = \frac{\frac{1}{2}Dh^2}{Cl^2}$, where D denoted the stiffness of one flange, h the height of the joist, C the torsional rigidity, and l the length of the column. It would then be found that the expression for M_1 should have yet another factor, $\sqrt{1 + (\pi\alpha)^2}$. When the load varied between two equal moments and two opposite moments the factor, which for a constant moment was π, would vary within the limits of 4 and 2·9, i.e., it was reasonably constant. D was almost identical with half the rigidity of the column, so that it was possible to write $\sqrt{1 + 0{\cdot}85\frac{EI_yk^2}{4GKl^2}}$. That could be written to a fairly accurate approximation and in a form easy to use as $\sqrt{1 + \frac{3}{4}\left(\frac{hb}{lt}\right)^2}$, where h,

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again, denoted the height of the section, b the width, l the column length, and t the thickness of the flange. That was a simple extra factor to put on. He had worked it out for a 10 in. × 8 in. × 55 lb.-joist with a column length of 9 ft, and it gave another 30% strength to the column. Taking in the two extra factors, they added up to 60%, so that quite a considerable amount could be saved by adding those fairly simple factors to the basic moment.

The full expression for M would therefore be:

$$M_1 = \frac{\pi\sqrt{F}}{L}\sqrt{EI_yGK} \times \frac{I_x}{I_x - I_y} \times \sqrt{1 + \frac{3}{4}\left(\frac{hb}{lt}\right)^2}$$

and the basic equation $\left(\frac{M}{M_1}\right)^2 + \frac{P}{P'} = 1$ in which either $\pi\sqrt{F} = 8{\cdot}03$ and $P_1 = 4P_E$ or $\pi\sqrt{F} = 11{\cdot}12$ and $P_1 = P_E$.

Now the last term in the formula for M_1 was based on free rotation of the flanges at the ends. In actual fact there would be a certain restraint, and that made it possible by simple means to increase the resistance to warping considerably.

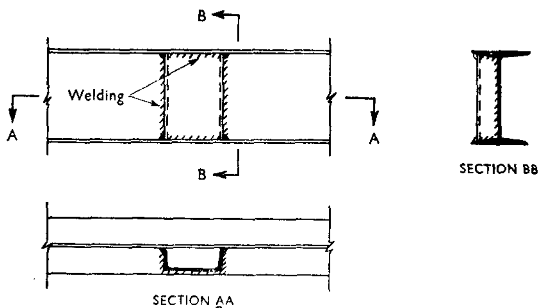

FIG. 32

Mr Ingerslev drew attention to a means of stiffening up the torsional rigidity published in Stockholm by Mr Nylander.[34] Taking an I-section, what Mr Nylander suggested was to weld in a channel between the flanges at intervals along the web (Fig. 32). If that was done the additional stiffness was enormous, as the box formed by the channel and the web prevented relative twist between the flanges, and it was obtained at very little cost.

Mr H. S. L. Harris (Lecturer in Engineering, University of Cambridge) said that when designing a structure by B.S. 449, any restraint on the beams from the stanchion was effectively being ignored. That had been shown to be quite inaccurate so far as the resulting moments coming into the stanchions were concerned, but led to the use of relatively slender stanchions. Would the Author go a little further in discussing his reasons for rejecting the stiff-beam-whippy-column approach and taking in its place the failing-beam-elastic-column condition. Anyone who had not put a great deal of work into understanding plastic design methods was bound to compare the difficulty of a proper analysis of the collapse of a stanchion with the very simple approach in B.S. 449. Mr Harris wondered whether it was possible that, of the many frames designed to B.S. 449, a few in which the joints between beams and stanchions happened to be strong had fortuitously been designed so that their collapse loads were approximately the same as if the structure had been designed on a collapse basis, but in the opposite mode to that set out in the Paper. Had the Author analysed any B.S. 449 frames, with strong joints, to find their true load factor?

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A second point concerned elastic restraints coming into one side of a stanchion when the other side of that stanchion was subjected to a known moment, i.e., the beam coming into the other side of the stanchion had become fully plastic. He was not clear what one should assume as the least moment that might arise on the side of the stanchion remote from the beam which had reached its fully plastic moment. Would the Author say a little more on that subject?

Mr E. Goodwin (Experimental Officer, Building Research Station) said he had been particularly impressed by the way in which the Author had reduced his complicated information to simple charts. Combined flexural-torsional buckling had been given considerable prominence in the Paper, and it was a new feature to crop up in building design. Fig. 23 showed that in the particular case represented there the new method allowed far lower bending stresses than had been recommended previously, and that was because torsional effects had been brought into consideration, whereas before they had been ignored. The case represented in Fig. 23 happened to be a bad case from the torsional point of view.

It seemed to him that a general impression was being created that torsional effects were likely to exert considerable influence on advanced design methods generally, but he would point out that that was not necessarily so. At the Building Research Station they were very interested in torsional effects and had been studying the extra stresses involved with the aid of their differential analyser. That machine was a mechanical analogue computer, and it was very useful in studying the question of torsional-flexural buckling, because the basic equations were capable of analytical solution only in the simple case when the stanchion was pin-ended. With their machine they had studied the more complicated cases where it had a partial restraint at the ends. The extra stresses which came into the picture were due to the warping of the cross-sections and to increased bending about the weak axis. The extra stresses were much reduced if the stanchion was encastered by elastic restraints at the ends. In the particular case shown in Fig. 23, he had been able to show that at a slenderness ratio of 200, if the stanchion were encastered and prevented from warping at the ends, the allowable bending stresses could be raised from $4\frac{1}{2}$ to 11 tons/sq. in.

With regard to the degree of restraint available at the ends of a stanchion in a rigid frame, it seemed to him likely that, with regard to bending about the weak axis, it was likely to be very nearly encastered, particularly in the case of the slenderer sections which gave trouble in that respect. With regard to restraint on warping, there seemed to be no definite evidence on which to base an assessment, and that was something on which they would very much like information. Mr Ingerslev had given them a very useful line to follow there, and Mr Goodwin thought that if extra stiffeners of any kind could be employed to prevent end warping they would be very well worthwhile.

In conclusion, it should be emphasized that the beneficial effects of end restraint applied only to design methods or design cases where the adjoining beams had not been allowed to go wholly plastic.

*** **Mr L. G. Johnson** said that Dr Horne was to be congratulated on producing a stanchion design method with such a logical foundation and which had clearly involved a tremendous amount of work. His Paper represented a real and important step forward in the design of steel structures, especially as the day was rapidly approaching when multi-storey buildings would no longer be able to rely on panel walls to resist wind forces and ideas on beam and stanchion design would have to be revised accordingly.

The design method described in Part 2 of the Paper was used to determine the sections of the stanchions in a new 17-storey block of flats having a height-to-base ratio of about 2·5:1. Walls and partitions were too flimsy to stiffen the structure against the action of wind and so the framing was designed as fully rigid about the major axes of the stanchions,

*** This contribution was received in writing upon the closure of the oral discussion.—Sec.

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and bracing was provided along the minor axes, i.e., conditions which corresponded to those set-out in section (10) (Case $P_x \cdot O_y$).

The following facts came to light as a result of the analysis. Axial stress p was constant for a given stanchion length and therefore $p + f_x'' \leqslant 15{\cdot}25$ did not have to be checked because $f_x' > f_x''$ by definition.

It was noted that $\dfrac{l}{r_x}$ was always appreciably greater than $40/\sqrt{p}$ and therefore equation (31) did not hold for that particular structure. The function N_x was obtained from Fig. 14 much more quickly than from the expression given by equation (27) and its value varied between 1·05 and 1·60, but for normal storey heights of 9 ft it did not exceed 1·10.

Partly owing to the fact that the ratio of live load to dead load was of the order of 1:4 and partly because wind forces produced double curvature bending, in no case was a stanchion length found to be bent in single curvature. That meant generally that $1/\sqrt{F}$ remained at approximately 0·4. T was almost invariably greater than 100 and, since bending stresses were smaller than the axial stresses, f_x was usually quite small and f_x^2/T could be neglected as it never exceeded 0·2. It was sufficiently accurate therefore to assume that $p' = p$.

In all but one or two isolated instances, maximum end stress was the criterion for design and even in the odd cases when stability was critical, the combined stresses were always greater than 14·6 tons/sq. in.

The value of f was never less than 13·5 and usually $> 14{\cdot}0$.

The actual design of one 17-storey stanchion, after all moments and loads had been tabulated, was achieved in about one day by one man, which speed could obviously be greatly increased when the designer became more familiar with the method.

All those facts seemed to indicate that that logical design method could be applied successfully to multi-storey structures and it soon became apparent to the designer that the calculations involved were by no means difficult; numerous short cuts simplified the method and reduced the amount of labour involved.

The Author, in reply, first dealt with the remarks concerned entirely with elastic stability, since that was an aspect which could be dealt with in isolation. Mr Ingerslev had discussed equation (4) at some length. It was not claimed as exact by the Author, who had elsewhere[26] dealt fully with the problem, comparing his own exact results, which were in agreement with results also obtained by Salvadori,[27] with equation (4), and showing conclusively that it invariably gave results on the safe side. All the factors mentioned by Mr Ingerslev increased the instability load, but Mr Ingerslev had omitted a further factor, known as the Wagner effect, which decreased that load. To take account of all the factors involved at all ratios of end moments would be impracticable. Mr Ingerslev's final equation did not take account of the Wagner effect; it only applied (after an important correction, to be dealt with later) to symmetrical and antisymmetrical end moments; and, with antisymmetrical moments, it was necessary to try the two conditions $\pi\sqrt{F} = 8{\cdot}03$ and $P_1 = 4P_E$ or $\pi\sqrt{F} = 11{\cdot}12$ and $P_1 = P_E$ to find which gave the smaller answer. Furthermore, the derivation of the elastic critical load was but one step in the formulation of a design method; the idea that it might be possible to introduce all those refinements into a method which sought to limit the maximum stress occurring anywhere in an initially imperfect member to the yield value was, the Author considered, quite unrealistic.

Mr Ingerslev had given the wrong solution for the effect of flexure about the major axis on the critical load. The factor $\dfrac{I_x}{I_x - I_y}$ in his final equation should be replaced by $\sqrt{\dfrac{I_x}{I_x - I_y}}$. Considering the 10-in. × 8-in. R.S.J. quoted by Mr Ingerslev, the increase in critical moment became 11% in place of 23½%. Even the figure of 11% was misleading, since flexural-torsional instability only became of importance in steel members when one moment of inertia was many times the other. Thus a 12-in. × 5-in. R.S.J.

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showed an increase in critical moment as a result of that factor of only 2·3%, whilst a very slender member such as a 20-in. × 6½-in. R.S.J. showed an increase of only 1·4%. In some cases it might even give unsafe results if the term under discussion were taken into account. It arose because of the curvature which existed about the major axis just prior to buckling; if that curvature were offset by pre-cambering, as was sometimes done, then the term could not be admitted.

The neglect of warping resistance by the Author, discussed by Mr Ingerslev, was of greater potential importance. If, however, warping resistance were taken into account, it might be necessary to allow also for the Wagner effect. The Author had shown [26] that in practical sections resistance to warping would always at least compensate the Wagner effect, and it was therefore a very convenient simplification to neglect both. Moreover, if one did actually allow for warping rigidity in the calculation of the critical load for instability, then logically one ought also to take account of the greater increase in extreme fibre stresses in the compression flange as compared with the tension flange. The net result would probably be that the benefit of taking warping rigidity into account was not very great. The Author felt that the design method presented in the Paper was perhaps too complicated, even as it stood, to be used widely in practice and it might still have to be simplified, although he hoped that the attempt to use it would be made. While, therefore, it was true that some of the effects which Mr Ingerslev had mentioned would be advantageous if taken into account, it was impracticable, in the Author's view, to introduce them at a stage when one was trying to see one's way through a forest of difficult problems towards a practicable design method. In the sifting process, as the attempt was made to apply the results, he thought that those additional effects might well be taken into account, perhaps in a semi-empirical and not directly obvious manner.

The Author agreed with Mr Ingerslev that the use of battens or channels to prevent warping could significantly increase the critical load. He ventured to doubt, however, whether the benefit was necessarily so great in relation to the cost as Mr Ingerslev seemed to indicate.

The Author certainly agreed with Mr Goodwin that it was important to take account of every restraint condition that could conveniently be obtained, particularly end restraints on warping. He had stated in the Paper that the case P_xE_y was of considerable importance, and restraint against warping at the ends was one reason why that was so. In such cases torsional failure would not be as important as might appear from the present Paper. The Author had, however, placed great emphasis on torsional failure because of his observation of unclad structures which had collapsed. In full-scale tests and in tests on small-scale stanchions the potential importance of torsional failure had been evident, and although it might ultimately be possible to return to a method in which it did not have to be taken into account, it was unwise to neglect it in the intermediate stages of an attempt to produce an entirely new design method. In the past the chief factor which had enabled torsional failure to be neglected was the encasing of the stanchions by material sufficiently rigid to prevent twisting. Where that was still done there was probably more hope of achieving economy by the use of composite action theory [35, 36] than by means of the methods suggested in the Paper. There was, however, a growing tendency to use fire casing and floor systems which contributed only slightly if at all to the strength of the structure. Under such circumstances it might be found that existing codes were in some cases dangerous, and torsional failure ought certainly to be considered in research devoted to the production of more rational and dependable design methods. That fact had been fully confirmed in some recent tests carried out in the United States.[37]

Dr Wood had quite rightly raised the question of the most critical load arrangements. The closely related problem of the safe minimum value to assume for the moment of resistance of the beam on the side of the stanchion opposite to the fully plastic beam had been raised by Mr Harris. Considering the stanchion length BE in Fig. 33a, the critical loading condition was not likely to be reached with full live loading on all four beams, but rather with conditions which led either to the most severe case of double curvature, as in Fig. 33b, or the most severe case of single curvature, as in Fig. 33c. The beams carrying

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dead load only would probably be elastic for their entire length, but in some cases the moments of resistance in those beams at B and E might exceed the yield moment, thus reducing very considerably the bending moments applied to the stanchion length BE by the other beams. How could one hope to arrive at a simple method of assessing the moments of resistance of the beams carrying dead load only, when it was realized that they must depend on the elastic stiffnesses of the beams and stanchions in the vicinity of the joints under consideration? One might be tempted to despair if one thought of the great labour entailed in the work of the Steel Structures Research Committee, and in the methods of analysis developed by Wood,[7, 9] directed towards the solution of that same problem for entirely elastic structures. Might it not therefore be necessary to abandon the ideal of a rational and reasonably simple design method, even with the assistance of the plastic theory? The Author did not think so, for the following reasons.

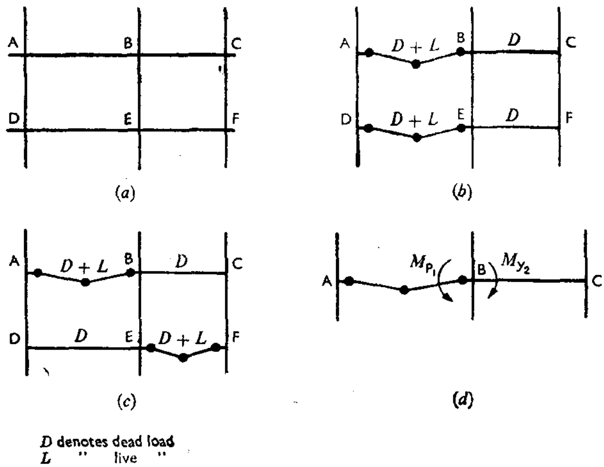

FIG. 33

In the tests[22] carried out on the $E_x . E_y$ case (Fig. 8) the major axis beams applied large bending moments to the stanchion in the early stages of the test. As soon as the stanchion yielded at all extensively, however, and before it was really in its collapse condition, those bending moments were relaxed away completely and did not appear to affect at all radically the collapse load of the stanchion. Similar behaviour might well occur in the beam systems ABC and DEF in relation to the stanchion length BE in Figs 33b and c. The two beams AB and BC might be replaced, so far as their effect on the stanchion length was concerned, by a suitably loaded single elastic beam, provided one of the actual members AB or BC was just on the point of collapse, the other remaining entirely elastic. Hence, when considering any joint B, as in Fig. 33d, the maximum out-of-balance beam moment which had to be taken into account was $M_{p_1} - M_{y_2}$, where M_{p_1} was the full plastic moment of AB, assumed to be the larger member, and M_{y_2} was the yield moment of the smaller member BC. That result followed since any larger out-of-balance moment would be relaxed away before the stanchion reached its final state of incipient collapse. Further work was required to establish that approach, and it might be necessary to impose limits for its application depending on the relative elastic rigidities of the members concerned. It did, however, represent a promising line of attack on that difficult problem.

The Author was interested to see the stanchion curves presented by Dr Wood in Figs 27 and 28, and to receive Dr Wood's confirmation that, with fully plastic beams, there was

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little likelihood of obtaining anything superior to elastic design for the stanchions. Curves similar to those in Fig. 27 had been obtained by the Author himself, and a description of their application was to be published shortly.[38] The Author agreed with Dr Wood that the question of frame stability was extremely important. It could fairly be said that no existing design methods really tackled that aspect and effectively depended for overall rigidity on the cladding. With the advent of tall lightly clad buildings the necessity of finding a solution to the frame-stability problem was becoming acute and remained a challenge to those concerned with structural research.

Mr Harris had asked why the Author had dismissed as unpromising the stiff-beam-whippy-column approach. The Author would not say that he had rejected it altogether but it had the disadvantage that it was rather intractable as a design method. In order to sustain the stanchions the beams would be continuous and elastic and the collapse loads of the stanchions would depend on the rigidities of the beams. The design of the stanchion itself influenced the bending moments which it exerted on the beam system at the point of collapse, so that there was a complex interaction between the beam design and the design of the stanchions. That made the problem intractable—it led, in fact, to a process of double trial and error. It was necessary to assume stanchion sections and the ratio of beam-to-stanchion stiffnesses, to check the adequacy of the assumed stanchion sections, to calculate suitable beam sizes to take the bending moments applied to them by the stanchions in addition to the usual floor loads, and finally to recalculate the stiffness ratios. The calculations had to be repeated where the actual ratio of beam-to-stanchion stiffness was less than that previously assumed. In the cases which the Author had considered, no great economy resulted from the process as compared with the design obtained by the direct application of the simple design method according to B.S.449, and there was thus no great incentive to pursue that approach further. So far as the Author could tell, a frame designed according to the simple method and then constructed with rigid joints would at collapse fall into the category $E_x . E_y$ for inside stanchions, into one of the categories $O_x . E_y$ or $E_x . O_y$ for outside stanchions, and into category $O_x . O_y$ for stanchions at outside corners, although the classification would be approximate only. It was probable that some of the beams—particularly those in outside bays—would be partially plastic at collapse, and the nominal B.S.449 design would not be so extreme in sacrificing beam economy in the interest of economical stanchions as would the $E_x . E_y$ approach mentioned in the Paper. The partial plasticity of the beams in a nominal B.S.449 design would make a collapse analysis extremely difficult, and it had not hitherto been attempted by the Author.

Dr Heyman had raised a problem in connexion with the use of the stanchion curves which was related to the question of what was the most critical loading condition. The beam-column interaction in his example would necessarily depend on the relative rigidities of the members up to the collapse load of the columns, but he was inclined to agree with Dr Heyman that the condition at collapse could be argued from other considerations. Dr Heyman's approach, in which he suggested that the effective eccentricity was that for which the critical load, calculated for yield at the end, coincided with that calculated on the basis of the instability behaviour of the member, was very convincing, and seemed likely to be the correct solution.

Dr Heyman had remarked that he did regard the Paper as presenting a design method for multi-storey frames. Whilst the Author hoped that, in a sense, that was so, he wished to emphasize that he had not sought to establish a well-defined method such as would be expected, for example, in a Code of Practice. The Paper could only have real value if attempts were made to apply it and if the experience so obtained were used both to simplify the process and to expand its scope to take account of factors not previously covered. It was the Author's hope that that might be done.

The Author was very grateful to Mr Johnson for placing on record his experience in the use of the stanchion design curves. He found that, in practice, f always exceeded 13·5 tons/sq. in. Should that prove to be a rule only very rarely broken, it might be profitable to replot the curves in Fig. 13 to give a wider spacing in that region. Mr Johnson was

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quite correct in pointing out the redundancy of the inequality $p + f_x'' \leqslant 15{\cdot}25$ in the conditions (30) which governed the design when bending was about the major axis only. Mr Johnson's remarks concerning the non-appearance of single-curvature bending and his experience that, in almost all cases, the critical conditions for the stanchions in his problem were for end stress and not instability were of particular interest. Used under such circumstances the design curves would certainly show considerable economies as compared with the requirements of B.S.449, as might be seen from Figs 22 and 25. There would therefore appear to be some incentive for using the curves in the Paper, in conjunction with a suitable load factor, in elastic design methods for rigid frames, with the prospect of considerable economies without excessive complication of the design process.

Further References

31. "The Building Research Station: Open Days for Civil and Structural Engineers." *Structural Engineer*, vol. 33, p. 329 (Oct. 1955).
32. E. H. Bateman, "The Development of the Elastic Theory of Continuous Frames." *Structural Engineer*, vol. 33, p. 286 (*Discussion*) (Sept. 1955).
33. E. Ingerslev, "Lateral stability of I-beams." International Ass. for Bridge and Structural Eng. Final Report, p. 309 (1948).
34. H. Nylander, "*Die Einw. Aussteif. a. Dreh. vorg, etc.*" Diss, Stockholm, 1942.
35. R. H. Wood, "Studies in composite construction, Part I—The composite action of brick panel walls supported on reinforced concrete beams." National Building Studies Research Paper No. 13. H.M. Stationery Office, London, 1952.
36. R. H. Wood, "Studies in composite construction, Part II—The interaction of floors and beams in multi-storey buildings." National Building Studies Research Paper No. 22. H.M. Stationery Office, London, 1955.
37. R. L. Ketter, E. L. Kaminsky, and L. S. Beedle, "Plastic Deformation of Wide-Flange Beam-Columns." Trans A.S. Civ. Engrs, vol. 120 (1955), p. 1028.
38. M. R. Horne, "The Elastic-Plastic Theory of Compression Members." J. Mech. & Phys. Solids, vol. 4, part 2 (January 1956).

The closing date for Correspondence on the foregoing Paper was 15 March, 1956. No contribution received after that date will be printed in the Proceedings.—Sec.

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