Paper No. 6089

The determination of the collapse loads of rigidly jointed frameworks with members in which the axial forces are large †

by

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Mr D. H. Clyde (Research student, Engineering Laboratory, University of Cambridge) observed that the application of the simple methods of plastic analysis and design to triangulated structures was not possible.[15] Such methods required a monotonically increasing relation between load and deflexion for validity, and the unloading effect examined by the Author violated that. On the grounds of prohibitive labour, the Author ruled out a complicated non-linear analysis of the type performed on a digital computer by Foulkes[16] for a frame similar to those tested by the Author. The analysis suggested as an alternative considered two states (i) wholly elastic and (ii) rigid-plastic in a theory derived for conditions of symmetry and single curvature. Plasticity occurred at points which had also been the more highly stressed parts in the elastic range. For double-curvature conditions, however, the early plastic deformation took place at points other than those associated with the final collapse mechanism, especially when the slenderness ratios were low $\left(< \pi \sqrt{\frac{E}{f_y}}\right)$. The mechanism considered by the Author was attained after an unwrapping involving plastic deformation.[13, 14] It was thus to be expected that correlation with experimental results would not be so good for the double-curvature tests of Stevens and of Baker and Roderick as for the single-curvature tests of the Author and of Baker and Roderick.

In order to derive an unloading line such as the Author's plastic-collapse line, a structure was examined at various states of deformation and the load necessary to maintain plastic deformation was calculated. Plastic deformation occurred when the yield condition was satisfied at sufficient places to form a mechanism. The method known as the principle of virtual work or virtual displacements was a tool for writing out an equilibrium equation. A collapse mechanism was associated with that equation when the yield condition was satisfied at sufficient places (hinges) to form a mechanism. The elegance of the virtual-work approach lay in the fact that the imaginary mechanism used to write the equilibrium equation was the same as the real one associated with the breakdown of that equation so that intermediate steps might be omitted. By neglecting rigid body rotations CAA′ and CBB′ the Author formed equation (16) incorrectly but the correct form of the equilibrium equation from which equation (16) might be derived was used to form equation (17). Fig. 9 had been redrawn (as Fig. 14a and Fig. 14b) with caa′ and cbb′ added. From Fig. 14b:

$$W\frac{d_c}{4}\cot\alpha = (M_P')_{AB} - \{(M_P')_{AB} - (M_P')_{AC}\}\frac{2d_c}{4h} \quad . \quad . \quad . \quad (16a)$$

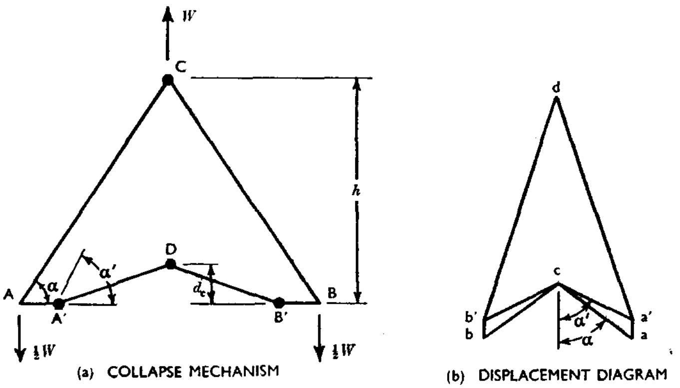
† Proc. Instn Civ. Engrs, Pt III, vol. 5, p. 213 (Apr. 1956).

[15] E. Chwalla, "Three contributions on the loading question of statically indeterminate steel trusses". Pubn Int. Assocn Bridge & Struct. Engg, vol. 2 (1933–34), p. 96.

[16] J. D. Foulkes, "The behaviour of a stiff jointed truss". Brit. Weld. Res. Assocn Report No. FE 1/44/54.

If d_c was very small compared with h:

$$\frac{Wd_c}{4}\cot\alpha = M_P' \quad \quad (16b)$$



(a) COLLAPSE MECHANISM (b) DISPLACEMENT DIAGRAM

Fig. 14

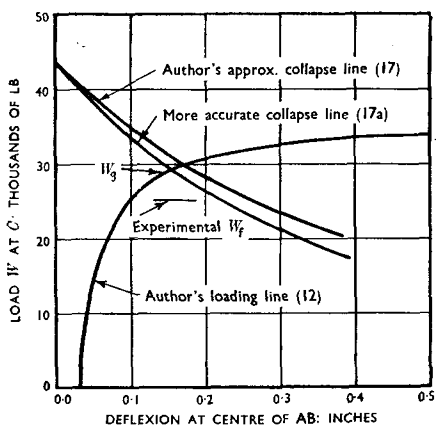


Fig. 15

In order to combine equation (16b) with equation (9) it was necessary to know the force P_1 in AB. Resolving forces at A (Fig. 8):

$$P_1 = \frac{W}{2}\cot\alpha \quad \quad (16c)$$

Hence, from equations (16b), (16c), and (9):

$$W = 2\,P_Y \tan\alpha\left(\sqrt{\left(\frac{d_c}{d}\right)^2 + 1} - \frac{d_c}{d}\right) \quad \quad (17a)$$

The alternative derivation was to consider A′D. There could be no vertical shear by symmetry so the change from M_P' sagging at A′ to M_P' hogging at D was brought about by the horizontal shear of P_1. Hence:

$$P_1 = \frac{2M_{P'}}{d_c} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16d)$$

Equation (16b) might be obtained by combining equations (16c) and (16d). It should be noted that the solution thus derived on a kinematic basis and from the equilibrium of a portion of the frame required checking to see whether a bending-moment diagram could be derived from it to cover the whole frame without violating the yield condition. The solution given was not more than a few per cent different from that of the Author. It would, however, give a consistent reduction in W_g and had been plotted for the Author's worked example of frame 11 to give a 3% reduction (Fig. 15). The Author's estimate of 80% for the ratio of $W_f : W_g$ was thus slightly conservative.

Mr W. Merchant and **Mr C. A. Rashid** (Department of Building and Structural Engineering, College of Technology, Manchester) observed that the use of the intersection of the elastic stability line with the collapse mechanism line to determine an approximation to the failure load of a structure was not new; it had been so used at the College of Technology, Manchester, for several years.

The original suggestion from the Author was that for triangulated frames, the influence of secondary stresses on the elastic stability line could be neglected by comparison with the effect of initial curvatures and end eccentricities of members. Since Mr Merchant and Mr Rashid were interested in the general problem they were grateful to the Author for permission and assistance in carrying out a further analysis of his test results.

Secondary moments, if stability effects were neglected, were proportional to the applied load W and in that case gave rise to an inward central deflexion of the strut also proportional to W. Supposing that the elastic deflexion calculated without reference to stability effects was αW, then, allowing for stability effects $\delta_s \approx \frac{\alpha W}{1 - W/W_c}$ would be taken for the deflexion; that had to be added to the deflexion resulting from initial curvature and end eccentricities $\delta_e \approx \frac{\delta_0 + e'}{1 - \frac{W}{W_c}}$ as used by the Author.

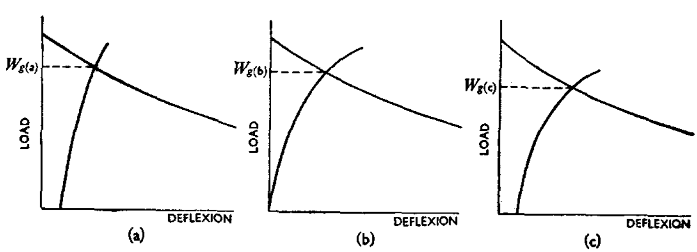


Fig. 16

Figs 16a, 16b, 16c showed three methods of obtaining W_g.
Fig. 16a showed the Author's method when the central deflexion taken was only δ_e.
Fig. 16b showed a similar method using only δ_s.
Fig. 16c showed a similar method using $\delta_s + \delta_e$.

To investigate the importance of δ_s, W_g had been evaluated by the three methods listed above and the results were given in Table 8. The Author seemed to assume that all moments arising from the end eccentricities were taken by the strut only whereas they

TABLE 8

Frame No.	Author's initial curvature $\delta_o \times 10^3$	Author's equivalent eccentricity $e \times 10^3$	Corrected equivalent eccentricity $e' \times 10^3$	Author's Y (central) $(\delta_o+e) \times 10^3$	Author's corrected Y (central) $(\delta_o+e') \times 10^3$	Author's W_g	Author's corrected W_g(a)	W_g(b) secondary effect	W_g(c) combined initial and secondary effects	W_y	W_c	W_f	$\frac{W_f}{W_g(a)}$	$\frac{W_f}{W_g(c)}$
1	−30	+ 3·5	+ 0·72	−26·5	−29·28	24,300	24,000	24,600	23,500	44,500	25,960	19,720	0·822	0·838
1′	−44	− 4·0	− 0·39	−48·0	−44·39	23,400	24,100	25,000	22,900	52,700	25,960	19,270	0·800	0·841
1″	−50	+29·5	+ 2·95	−20·5	−47·05	24,800	24,000	25,500	23,000	54,500	25,960	20,280	0·845	0·881
2	+12	−24·0	− 3·90	−12·0	+ 8·10	10,550	—	10,500	10,700	20,500	11,240	8,650	—	0·806
2″	+54	−59·0	− 9·88	− 5·0	+44·12	11,000	—	11,000	—	25,100	11,240	8,850	—	—
3	−43	− 1·0	− 0·10	−44·0	−43·1	33,600	34,250	38,000	32,700	46,900	44,570	24,900	0·726	0·761
5	−48	+12·0	+ 1·20	−36·0	−46·8	16,300	16,300	18,200	15,500	41,800	17,740	15,000	0·920	0·968
6	− 5	+42·0	+ 7·95	+37·0	+ 2·95	—	—	7,200	7,300	19,710	7,670	8,150	—	1·110
8	−35	− 7·0	− 0·77	−42·0	−35·77	42,300	43,000	45,000	41,000	46,900	94,400	35,130	0·816	0·856
9	−21	−18·5	− 3·07	−39·5	−24·07	11,200	11,350	11,600	11,000	11,850	43,400	9,250	0·815	0·841
10	−12	− 7·5	− 0·45	−19·5	−12·45	26,800	27,700	29,250	26,800	29,800	61,300	21,000	0·759	0·784
11	−20	−11·0	− 1·00	−31·0	−21·0	30,200	31,500	32,600	29,600	44,000	36,660	25,300	0·802	0·854
12	−20	+27·5	+ 5·89	+ 7·5	−14·11	—	14,200	13,800	13,000	20,300	16,250	12,380	0·871	0·952
13	−12	− 3·0	− 0·16	−15·0	−12·16	38,500	38,500	39,300	38,000	39,600	500,000	36,500	0·948	0·960
14	0	−41·0	− 4·08	−41·0	− 4·08	25,200	25,200	25,100	24,800	25,600	295,500	23,500	0·934	0·948
15	−11	+ 3·0	+ 0·82	− 8·0	−10·18	11,700	11,700	11,750	11,500	11,850	155,500	11,100	0·948	0·965
16	− 9	− 2·5	+ 0·38	−11·5	− 8·62	28,700	28,800	29,700	28,700	29,800	191,300	27,150	0·940	0·945
17	− 5	+ 6·0	+ 0·63	+ 1·0	− 4·37	19,000	19,000	19,200	18,800	19,200	115,000	16,600	0·874	0·878
18	0	−64·0	−11·95	−64·0	−11·95	8,780	7,600	8,600	7,400	8,860	52,800	7,030	0·924	0·948
19	−10	+16·0	+ 0·94	+ 6·0	− 9·06	—	38,200	39,500	36,250	42,900	52,300	27,200	0·712	0·750
20	− 7	+ 5·0	+ 0·57	− 2·0	− 6·43	25,400	25,000	25,000	23,200	27,750	32,100	20,200	0·808	0·871
21	0	−13·0	− 2·51	−13·0	− 2·51	10,900	11,700	10,200	9,860	12,790	13,200	9,070	0·778	0·920

Notes.—1. Frames 6, 12, and 19 in the Author's calculation failed on wrong side and now no longer failed in that manner.

2. 2 and 2″ are now failing in the wrong direction. They were the only two frames with positive initial curvature and hence there was a suspected error in the initial curvature.

3. In frames 2 and 6, initial positive $Y_{central}$ had been compensated by secondary effect.

4. Frame 6, W_c was less than experimental failure load.

should have been shared between the members. The Author's values had been corrected to take that into account. It reduced the equivalent eccentricities considerably, especially when the tension members were much stiffer than the strut.

The ratios $\frac{W_g(a)}{W_y}$, $\frac{W_g(b)}{W_y}$, $\frac{W_g(c)}{W_y}$ were plotted against $\frac{W_y}{W_c}$ in Figs 17a, 17b, and 17c. Comparison of Figs 17a and 17b showed that for that series of tests, the inclusion of

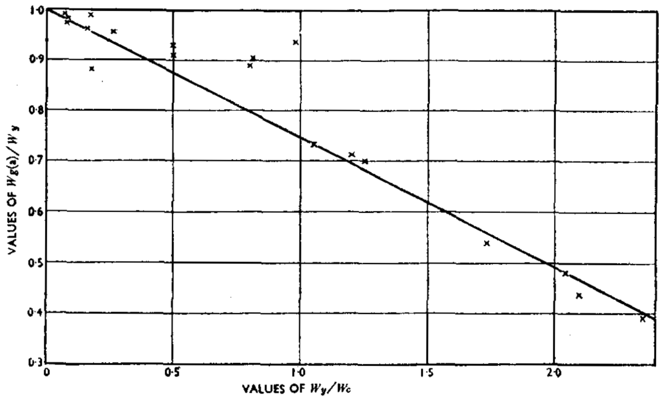


FIG. 17a

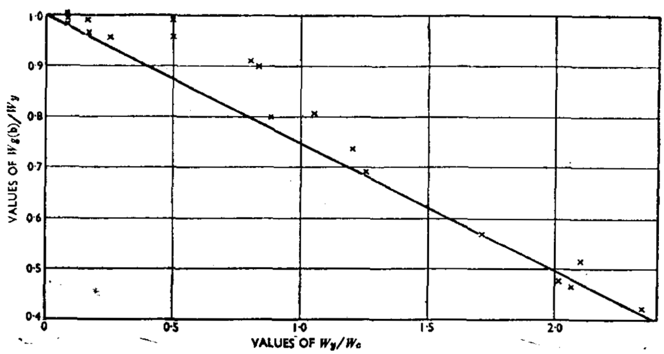


FIG. 17b

either δ_s or δ_e in the analysis would have a similar effect in determining values of W_g There was certainly no evidence that the secondary effects were negligible compared with the initial curvature effects.

The point was not so obvious from a comparison of Figs 17a and 17c where the addition of δ_s to δ_e only appeared to cause a further slight reduction in W_g. The same comparison, however, held between 17b and 17c and the apparent paradox was due to the non-linear nature of the problem.

The method of manufacture of the series of frames appeared sufficiently consistent to give rise to a fairly smooth relation between $\frac{W_g(a)}{W_y}$ and $\frac{W_y}{W_c}$.

The similar smooth relation between $\frac{W_g(b)}{W_y}$ and $\frac{W_y}{W_c}$ was not so surprising. The resultant $\frac{W_g(c)}{W_y}$ and $\frac{W_y}{W_c}$ relation was also therefore fairly smooth.

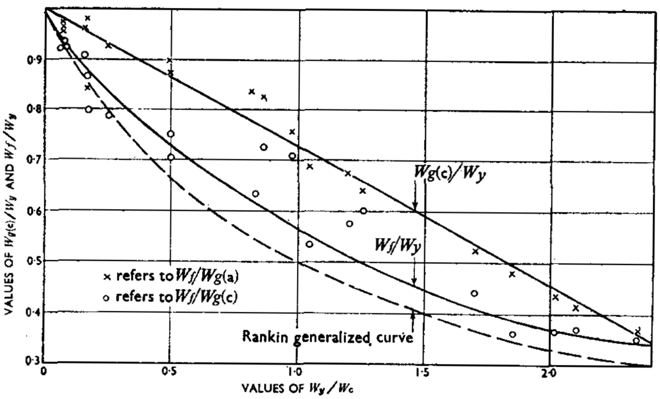


FIG. 17c

The revised ratio $\frac{W_f}{W_g(a)}$ and the ratio $\frac{W_f}{W_g(c)}$ were plotted against $\frac{W_y}{W_c}$ in Fig. 18. The depression of W_f below $W_g(c)$ resulted from the combined effects of deterioration of stability and form factor. It appeared greater for values of $\frac{W_y}{W_c} \approx 1{\cdot}0$ than for either

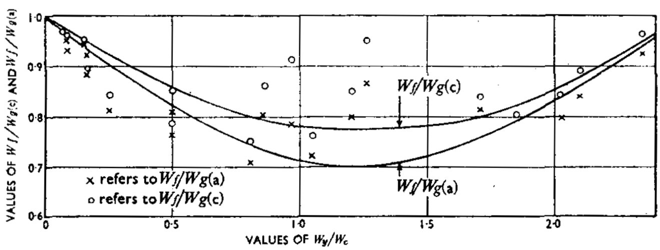


FIG. 18

slender or stocky trusses. Since $\frac{W_g(c)}{W_y}$ for these tests was fairly smooth it was worth plotting $\frac{W_f}{W_y}$ directly; that had also been done in Fig. 17c. The generalized Rankine curve $\frac{1}{W_f} = \frac{1}{W_y} + \frac{1}{W_c}$ was also shown for comparison.

It should be remembered that that valuable series of tests was made on annealed frames and accordingly there were no initial internal stresses. That difference from practical frames should be borne in mind.

Dr L. K. Stevens (Department of Civil Engineering, University of Melbourne) stated that the Paper showed that much more attention was now being given to the behaviour of rigid-jointed trusses. The mechanism by which collapse occurred was more fully understood, but theoretical methods of predicting collapse conditions still left much to be desired.

In Table 1 the minimum slenderness ratio was about 104. For the range shown an estimate of W_f equal to $0{\cdot}8W_g$ appeared to give reasonable results. However, as pointed out in Appendix II, W_f could be raised to $0{\cdot}95W_g$ for more stocky struts. The use of $0{\cdot}8W_g$ as an estimate of the collapse loads for rectangular sections might give conservative results, but an extension to non-rectangular sections would require considerable investigation before that attractively simple method could be accepted with confidence.

In Fig. 7 the presumably experimental points were shown as dots in the region where the experimental and predicted curves were in reasonable agreement. It was stated that for loads greater than 22,000 lb., "experimental deflexions are larger and the experimental curve becomes asymptotic to the plastic collapse line". Since the Author was using a loading mechanism which enabled controlled deformations to be applied, it was surprising that experimental points had been omitted in that region. It would be of interest to learn whether the broken line leading asymptotically to the plastic collapse line had been determined experimentally, or if it was based solely on theoretical grounds. In Fig. 11, the broken curves could be only an optimistic extrapolation of the experimental results, since the dead-weight and lever system, used by Baker and Roderick, did not allow the post-ultimate behaviour to be observed.

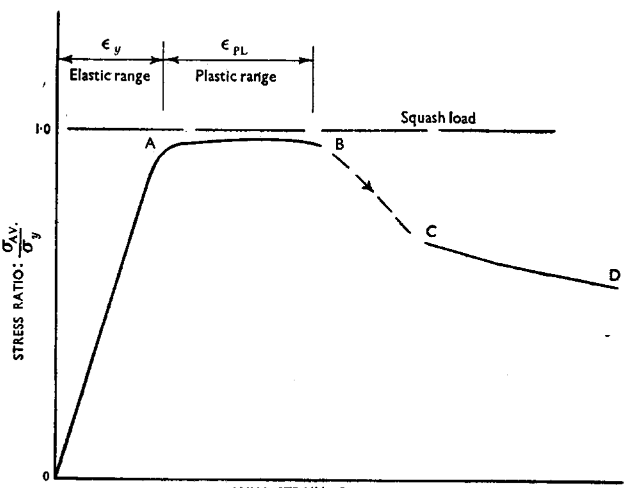


FIG. 19

The experimental verification of the behaviour after departure from the elastic stability line appeared rather doubtful. The verification of the theory would be more convincing if the Author were able to show experimental results in that region.

A further point on which the Author might care to give details was the large variation in yield point of the material used. In Table 7 that value ranged from 30,600 to 70,000 lb/sq. in. The shape of the stress/strain curve for those control specimens would also be of interest in determining the validity of applying the simple plastic theory to the prediction of the plastic-collapse line.

Further tests had recently been carried out by L. C. Schmidt on rigid-jointed trusses at the University of Melbourne. The load-deformation relation of the critical strut was found to have the form shown in Fig. 19, provided that the slenderness ratio was less than about 110.

The line OA corresponded to the elastic stability line, and it seemed possible that CD could be part of some plastic collapse line. However, the behaviour between OA and CD was characterized by a definite flat-topped plastic range, AB. The length of this plastic range, ϵ_{PL}, was dependent upon the slenderness ratio of the critical strut, and to a lesser extent on the combined stiffness of adjoining restraining members. At B, an increase of applied strain produced a sudden change of the strut configuration, and a new equilibrium position was reached at C, with a reduced load. The slope and extent of the line BC depended upon the elasticity of the whole system of truss and testing machine.

The logical development in rigid-jointed statically indeterminate truss design appeared to be the application of plastic-theory methods. For that to succeed, a knowledge of the length of the plastic range, as well as the collapse load, was required. If the Author had any results indicating the length and existence of a plastic range in his experiments, publication of them could be very helpful. Any explanation of it which could be made by his theory would be welcomed.

The Author, in reply, said that Mr Clyde had given the correct equation for the plastic collapse line in equation (17a). The Author had derived equation (17) by neglecting certain small terms in the equation of virtual displacements. (In Fig. 9b it had been intended that a and b should be shown as a′ and b′.) By adding the points a and b to the displacement diagram Mr Clyde had allowed the loads $\frac{W}{2}$ at A and B to move through the additional vertical distances a′a and b′b. Also the plastic hinges at A′ and B′ were rotated through angles which were slightly less than those considered by the Author. If those small differences were both taken into account then the value of W was reduced as Mr Clyde had shown. It appeared that the simplest way of developing the plastic collapse line for a general braced framework whose members were of rectangular cross-section was to consider the equilibrium condition as had been done in equations (16c) and (16d). The axial load P_1 in the strut which failed was always proportional to the applied load W:

$$P_1 = KW \qquad \text{(16e)}$$

and the general equation for the plastic collapse line became

$$W + \frac{P_Y}{K}\left(\sqrt{\left(\frac{d_c}{d}\right)^2 + 1} - \frac{d_c}{d}\right) \qquad \text{(17b)}$$

Dr Merchant and Mr Rashid had pointed out that the Author had erroneously assumed that all moments arising from end eccentricities were taken by the strut. It was true that they should be shared among the other members at the joint. A further refinement was the inclusion of the effects of secondary moments. The effect of including those modifications in the Author's analysis was shown in Fig. 20. Frame 1″ had been chosen to demonstrate those effects because it was the frame which had appeared to give least agreement between the elastic stability line and the experimental results. The curve ab_1 was that originally used by the Author assuming the equivalent strut shown in Table 1. By allowing the tension members to take their share of the moments arising from end eccentricities the equivalent strut was modified to that shown in the inset to Fig. 20 and the elastic stability line was moved to ab_2. In the case of frame 1″ the constant $\alpha = 0{\cdot}71 \times 10^6$ in/lb. and if secondary moments were included the elastic stability line was moved to ab_3. Fig. 20 showed that agreement with the measured deflexions was improved by modifying the analysis to include those two effects. The Author had shown[5] that deflexions were quite sensitive to initial imperfections when slender members were used. However, it was apparent from Table 8 that failure loads were comparatively insensitive to initial imperfections. It would therefore appear that beyond a certain degree of imperfection the collapse load of the structure was not materially affected by increases in those imperfections.

The struts of frames 2 and 2″ had both failed inwards. When applying the correct analysis to frame 2 the Author had found that an inwards failure of the strut was predicted

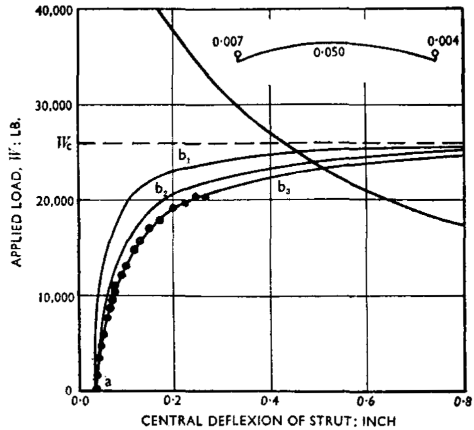


FIG. 20.—BEHAVIOUR OF FRAME 1″

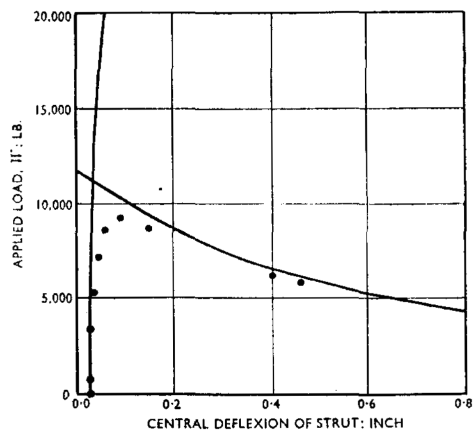


FIG. 21.—BEHAVIOUR OF FRAME 9

because of the high secondary moments ($\alpha = 2{\cdot}34 \times 10^{-6}$ in/lb.). However, when applying the analysis to frame 2″ an outwards failure had been predicted. Dr Merchant and Mr Rashid suspected an error in the measured value of the initial curvature of the strut. Although the measurements would not be now checked the calculations had been and no discrepancy was apparent. In an earlier elastic analysis[5] of that frame the Author had used a more exact analysis and had obtained good agreement with the experimental results. A further examination of the results of that frame was probably desirable. The explanation might lie in the curvature of the tension members. They have been neglected in the theory of the Paper under discussion but the measured values of curvature of the ties had been such that an inwards failure of the strut would be induced.

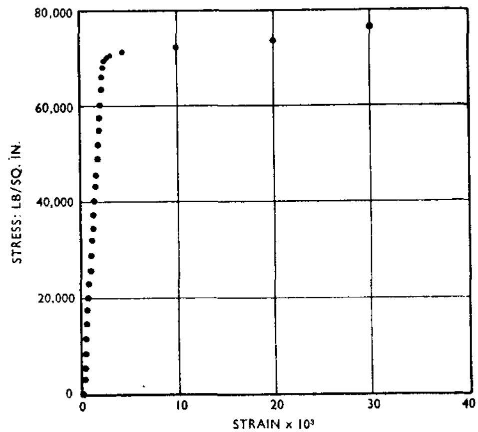


FIG. 22.—STRESS/STRAIN CURVE FOR A TENSILE SPECIMEN

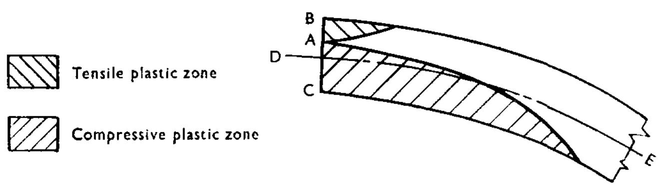


FIG. 23

Dr Stevens had mentioned that frames manufactured from bars of non-rectangular section might give results which differed considerably from those made from bars of rectangular section. The Author had had further frames of a larger size made with a view to investigating whether any appreciable variation did in fact occur. Results were not yet available.

Collapse of the frames had been, in most cases, quite sudden, particularly in the case of frame 11. In such cases only the broken curve had been sketched in. The sudden collapse of a frame was caused partly by the conversion of some of the elastic energy of the frame, load meter, and testing machine into work done upon the plastic hinge (and also by the sudden reduction in critical load when a plastic hinge formed at D). Equilibrium was not reached until the deflexions had become excessive and the frame had failed. The post-collapse behaviour of frame 9 had been observed at large deflexions at two loads and the results were illustrated in Fig. 21. It would be seen that those points lay very close to the plastic collapse line.

The material used for those frames was a bright drawn steel which must have had considerable work done on it in the drawing process and in its original state it had a high yield stress. Two types of heat treatment had been given. All of the first frames listed had been heat-treated for 1 hour at 550°C and that had not appreciably affected the yield stress. Because the strength of the stocky frames at that high yield stress would have exceeded the capacity of the testing machine the frames were heat-treated at 900°C for 1 hour. That reduced the yield stress to 30,600 lb/sq. in. Fig. 22 showed the behaviour of one of the specimens treated at 550°C for 1 hour. It appeared that the material did strain without appreciable increase in stress as yielding occurred.

The Author had noticed a plastic range (Fig. 19) in his test results when stocky frames had been tested. It was thought that that behaviour would show more on the type of graph which Dr Stevens had plotted than the type plotted by the Author. It was suggested that the explanation was that the axial deformation of the strut which failed was greatly affected by the location of the plastic zones. Fig. 23 showed a strut on one side of a plastic hinge at section BC. The axial deformation was the integrated sum of the deformations of elements along the centre-line DE. In stocky struts A tended to lie nearer to B than was the case for more slender struts and the centre-line DE passed through a comparatively larger zone of plastically compressed material. The plastic range would then be more apparent on a load-axial deformation graph than on a load-lateral deformation graph.

Paper No. 6100

The canals of the Gezira canalization scheme and the design of the Goneid pump scheme in the Sudan †

by

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Correspondence

Mr H. F. Ayres (formerly of the Irrigation Department of the Egyptian Ministry of Public Works) observed that under the heading "Minor canals" the Author had stated that those canals were designed to store night flow not passed to the fields and had proceeded to describe how it was done.

Behind those simple statements lay the work of the late Mr A. D. Butcher, C.B.E., who was in the Irrigation Service of the Egyptian Government.

Shortly after the first world war, Mr Butcher had been appointed Director of the Hydraulic Research station at the Delta Barrage and carried out research into problems connected with irrigation in Egypt and the distribution of water in the Gezira area.

As a result of his investigations and especially his model experiments he had been able to advise the adoption of the system and devices referred to in the Paper.

Mr A. A. Middleton (Sir Murdoch MacDonald and Partners) stated that the Paper was useful in drawing attention to some considerations in detail design and construction of an irrigation scheme. He hoped that details would be given of designs and methods of construction adopted elsewhere for comparison.

As the Author had remarked, methods of design and construction adopted in the Sudan had been developed during many years. The spacing of the minor canals and field channels had, however, remained unchanged since the original scheme. It was a great tribute to the original designers that no modifications were desirable.

The Gezira irrigation scheme was one of the few large irrigation schemes where the water was supplied to the cultivators on demand. Agriculturally that was ideal but it

† Proc. Instn Civ. Engrs, Part III, vol. 5, p. 233 (Apr. 1956).