Analysis of orthotropic steel bridge decks by a stiffness method

M. S. TROITSKY & A. K. AZAD

Yew Chaye Loo and **Professor A. R. Cusens,** Department of Civil Engineering, University of Dundee

The Authors have presented a stiffness method of analysis for an orthotropic steel deck system with rigid edge beams spanning continuously over flexible transverse floor beams. An eighth order differential equation is derived for the solution of an equivalent plate representing the actual steel deck with eccentric torsionally soft stiffeners.

41. As an equivalent plate system is adopted, it is apparent that the solution cannot predict the actual distributions of transverse and longitudinal stress at joints between deck plate and stiffeners and at joints between deck and floor beams. The accurate assessment of these local stresses is important in a fatigue analysis of a bridge deck.

42. In the Paper Fourier series functions are assumed for the displacement fields (w, u and v). How many harmonics are necessary for an accurate solution?

43. In deriving the various stiffness coefficients, the Authors assume both deck and floor beams to be resting on rigid simple supports. In actual construction, the main girders supporting the beams and the deck deflect under load. This casts doubt on the validity of the method. The assumption of rigid simple edge supports also prohibits the method of analysis from being used in cases where the deck overhangs the main girders and for decks with other than two main girders.

44. As a result of the flexibility of the main girders, the deck, as well as distributing loads to the girders (local effects), also acts as a top flange to the girders. Thus, in addition to the local effects, there are overall in-plane stresses induced in the deck. A design which omits these overall effects is incomplete.

45. There are existing methods of analysis capable of including these overall effects for a wider range of problems than the Authors' method. A full three-dimensional finite element analysis would be the most versatile but is costly. However, a two-step procedure can be adopted to give efficient solutions. In this procedure the bridge is first analysed as a folded plate structure[20–22] with the equivalent deck system and supporting girders as constituent plates. The overall stresses in the deck as well as in the supporting (plate or box) girders are determined in this preliminary analysis. This is followed by a local finite element analysis of the relevant part of the deck where constructional details can be accounted for accurately. The preliminary analysis provides the necessary displacement boundary conditions to ensure that, during local analysis, the deck is accurately treated as a part of the overall structure.

Professor Troitsky and Mr Azad

The primary object of the Paper is to present a method of analysis of conventional orthotropic steel bridge decks having open ribs, by incorporating the basic theory of eccentrically stiffened plates, which may be useful in evaluating a design method for its degree of accuracy and reliability. The idealization of the deck as being simply supported by the main carrying members simplifies considerably the mathematical complexities.

47. The effect of bending of the main girders which are free to deflect can be taken into account separately by treating the deck as the top flange and following conven-

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tional methods. This procedure of evaluating the in-plane stress in the deck is satisfactory for practical purposes as long as the width of the deck is relatively small compared with the span, which is the case in medium and long span bridges. The stresses in the deck due to the bending of the main girders are superimposed on those obtained from the independent analysis of the simply supported deck structure. The validity of such superposition of stresses for design purposes has been examined theoretically and experimentally.[12,15,23] The design method of Pelikan–Esslinger, which is still the most commonly used method because of its simplicity and sufficient accuracy, is also based on this two-step procedure.

48. Although a method of analysis capable of including the overall effects is definitely desirable, the advantage of such a method disappears when encountered with an actual design of a deck that is used in cable-stayed, suspension or arch type bridges. The complexity of such bridge systems compels the designers to isolate the two actions and treat the deck structure separately. Furthermore, the loading criteria for the design may be too complicated to include in a particular one-step method for analysing an orthotropic deck.

49. The proposed method of analysis cannot be used to analyse a cantilevered orthotropic deck. However, for decks supported by box girders or by several plate girders some approximations can be made without any significant loss of accuracy. It has been observed[24] that the width of decks between the assumed simple supports of main girders has small influence on the critical longitudinal stresses in the loaded central rib provided there are sufficient numbers of ribs on either side of the loaded rib and that the floor beams have identical flexibilities in all cases. Thus a suitable substitute of simple spans of floor beams can be used which becomes also the substitute width of the deck.

50. It is true that a well formulated three-dimensional finite element analysis would be more accurate for predicting stresses in any complex structure for which a rigorous mathematical solution is difficult to undertake. However, that does not discourage any attempt to present such approaches based on certain idealizations which do not adversely affect the overall structural behaviour of the structure. The results from such an analysis may be used for a good comparison with the solutions from a finite element method.

51. The number of cycles necessary for a satisfactory convergence of the stresses depends on the width of the deck. For the example bridge decks considered in the Paper, it was obtained at $m=25$ when the two consecutive amplitudes of the sinusoidal series were less than 0·1 kip/sq. in. The convergence in deflexion is rapid and after only a few cycles the change becomes insignificant.

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Corrigenda

On page 449 of the Paper, equation (1b) *should read*

$$\sigma_y = \frac{E}{1-\mu^2}\left[\frac{\partial v}{\partial y} - z\frac{\partial^2 w}{\partial y^2} + \mu\left(\frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial x^2}\right)\right]$$

and equation (1d) *should read*

$$\sigma_y = E\left(\frac{\partial v}{\partial y} - z\frac{\partial^2 w}{\partial y^2}\right)$$

On page 450 equation (5a) *should read*

$$M_x = -EI\left(\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 w}{\partial y^2}\right)$$

and equation (5b) should read

$$M_y = -E\left(\bar{I}_y\frac{\partial^2 w}{\partial y^2} + \mu I\frac{\partial^2 w}{\partial x^2} - S_y\frac{\partial v}{\partial y}\right)$$